MATHEMATICS (EM)

9TH CLASS

PUNJAB BOARD NOTES

Presented by:

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Exercise 1.1

Q.1 Find the order of the following matrices.

$$A = \begin{bmatrix} 2 & 3 \\ -5 & 6 \end{bmatrix}$$

It has 2 rows & 2 columns that's why its order is 2 - by -2

$$B = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$$

It has 2 rows & 2 columns. So, its order is 2- by -2

$$C = \begin{bmatrix} 2 & 4 \end{bmatrix}$$

It has 1 row and 2 columns. So, its order is 1 – by -2

$$D = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$$

It has 3 rows and 1 column. So, its order is 3 – by -1

$$E = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$$

It has 3 rows and 2 columns. So, its order is 3 – by –2

$$F = \begin{bmatrix} 2 \end{bmatrix}$$

It has 1 row & 1 column. So, its order is 1- by -1

$$G = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$$

It has 3 rows and 3 columns. So, its order is 3 –by -3

$H=\begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 6 \end{bmatrix}$

It has 2 rows & 3 columns. So, its order is 2- by -3

Q.2 Which one of the following matrices are equal?

1) $A=[3]$, **2)** $B=\begin{bmatrix} 3 & 5 \end{bmatrix}$,

3) $C=[5-2]$ **4)** $D=\begin{bmatrix} 5 & 3 \end{bmatrix}$

5) $E=\begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$ **6)** $F=\begin{bmatrix} 2 \\ 6 \end{bmatrix}$

7) $G=\begin{bmatrix} 3-1 \\ 3+3 \end{bmatrix}$ **8)** $H=\begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$

9) $I=\begin{bmatrix} 3 & 3+2 \end{bmatrix}$ **10)** $J=\begin{bmatrix} 2+2 & 2-2 \\ 2+4 & 2+0 \end{bmatrix}$

Solution:

Order of $A=[3]$ is equal to Order of $C=[5-2]$

Order of $B=\begin{bmatrix} 3 & 5 \end{bmatrix}$ is equal to Order of $I=\begin{bmatrix} 3 & 3+2 \end{bmatrix}$

Order of $C=[5-2]$ is equal to Order of $A=[3]$

$D=\begin{bmatrix} 5 & 3 \end{bmatrix}$ has no equal matrix.

$E=\begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$ has equal matrices.

Order of $\Rightarrow H=\begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$ is equal to Order of $J=\begin{bmatrix} 2+2 & 2-2 \\ 2+4 & 2+0 \end{bmatrix}$

Order of $F=\begin{bmatrix} 2 \\ 6 \end{bmatrix}$ is equal to Order of $G=\begin{bmatrix} 3-1 \\ 3+3 \end{bmatrix}$

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- گروپ میں معزز، پڑھے لکھے، سلجھے ہوئے ممبرز موجود ہیں اخلاقیات کی پابندی کریں اور گروپ رولز کو فالو کریں بصورت دیگر معزز ممبرز کی بہتری کی خاطر ریموو کر دیا جائے گا۔
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- ہمارے کسی بھی گروپ میں سیاسی و فرقہ واریت کی بحث کی قطعاً کوئی گنجائش نہیں ہے۔
- اگر کسی کو بھی گروپ کے متعلق کسی قسم کی شکایت یا تجویز کی صورت میں ایڈمن سے رابطہ کیجئے۔
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گروپ میں کسی بھی قادیانی، مرزائی، احمدی، گستاخِ رسول، گستاخِ امہات المؤمنین، گستاخِ صحابہ و خلفائے راشدین حضرت ابو بکر صدیق، حضرت عمر فاروق، حضرت عثمان غنی، حضرت علی المرتضیٰ، حضرت حسنین کریمین رضوان اللہ تعالیٰ اجمعین، گستاخ اہلبیت یا ایسے غیر مسلم جو اسلام اور پاکستان کے خلاف پراپیگنڈا میں مصروف ہیں یا ان کے روحانی وذہنی سپورٹرز کے لئے کوئی گنجائش نہیں ہے لہذا ایسے اشخاص بالکل بھی گروپ جوائن کرنے کی زحمت نہ کریں۔ معلوم ہونے پر فوراً ریموو کر دیا جائے گا۔

- تمام کتب انٹرنیٹ سے تلاش / ڈاؤنلوڈ کر کے **فری آف کاسٹ** وٹس ایپ گروپ میں شیئر کی جاتی ہیں۔ جو کتاب نہیں ملتی اس کے لئے معذرت کر لی جاتی ہے۔ جس میں محنت بھی صَرف ہوتی ہے لیکن ہمیں آپ سے صرف دعاؤں کی درخواست ہے۔
- عمران سیریز کے شوقین کیلئے علیحدہ سے عمران سیریز گروپ موجود ہے۔
- **لیڈیز کے لئے الگ گروپ کی سہولت موجود ہے جس کے لئے ویریفکیشن ضروری ہے۔**
- اردو کتب / عمران سیریز یا سٹڈی گروپ میں ایڈ ہونے کے لئے ایڈمن سے وٹس ایپ پر بذریعہ میسج رابطہ کریں اور جواب کا انتظار فرمائیں۔ برائے مہربانی اخلاقیات کا خیال رکھتے ہوئے موبائل پر کال یا ایم ایس کرنے کی کوشش ہر گز نہ کریں۔ ورنہ گروپس سے تو ریموو کیا ہی جائے گا بلاک بھی کیا جائے گا۔

نوٹ: ہمارے کسی گروپ کی کوئی فیس نہیں ہے۔ سب فی سبیل اللہ ہے

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اللہ تبارک تعالیٰ ہم سب کا حامی وناصر ہو

Q.3 Find the values of a, b, c & d.

$$\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & +2d \end{bmatrix}$$

Solution:

As Matrices are equal so their corresponding entries are same.

$a+c=0 \rightarrow (1)$

$a+2b=-7 \rightarrow (2)$

$c-1=3 \rightarrow (3)$

$4d-6=+2d \rightarrow (4)$

Solving 3rd equation

$c - 1 = 3$

$c = 3 + 1$

$c = 4$

Solving 2nd equation

$a + 2b = -7$

$-4 + 2b = -7$

$2b = -7 + 4$

$2b = -3$

$b = \frac{-3}{2}$

Solving 1st equation

$a + c = 0$

$a + 4 = 0$

$a = -4$

Solving 4th equation

$4d - 6 = 2d$

$-6 = 2d - 4d$

$-6 = -2d$

$d = \frac{+\cancel{6}_3}{+\cancel{2}_1}$

$d = 3$

Exercise 1.2

Q.1 Identify the following matrices.

$$A=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

It's all members are 0. So, it's a null matrix.

$$B=\begin{bmatrix}2 & 3 & 4\end{bmatrix}$$

It has only 1 row. So, it's a row matrix.

$$C=\begin{bmatrix}4\\0\\6\end{bmatrix}$$

It has only 1 column. So, it's a column matrix.

$$D=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

It is an identity matrix because its diagonal entries are 1 and non-diagonal entries are zero.

$$E=\begin{bmatrix}0\end{bmatrix}$$

It has only 0. So, it's a null matrix.

$$F=\begin{bmatrix}5\\6\\7\end{bmatrix}$$

It has only 1 column. So, it's a column matrix.

Q.2 Identify the following matrices.

(1) $\begin{bmatrix}-8 & 2 & 7\\12 & 0 & 4\end{bmatrix}$

Its number of rows & columns are not equal. So, it's a rectangular matrix.

(2) $\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$

It has only one column. So, it's a column matrix.

(3) $\begin{bmatrix} 6 & -4 \\ 3 & -2 \end{bmatrix}$

The number of rows & columns are equal. So, it's a square matrix.

(4) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Identity matrix – Because Diagonal entries are 1 and non-diagonal entries are 0.

(5) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

Number of rows & columns are not equal. So, it's a rectangular matrix.

(6) $\begin{bmatrix} 3 & 10 & -1 \end{bmatrix}$

It's a row matrix because it has only 1 row.

(7) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Column matrix because it has only one column.

(8) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Square matrix because number of rows & columns are equal.

(9) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

Null matrix because all elements are 0.

Q.3 Identify the matrices.

(1) $A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

Scalar- matrix because it non-diagonal entries are 0 & diagonal entries are same.

(2) $B = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$

Diagonal matrix because its non-diagonal entries are 0.

(3) $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Unit matrix because diagonal-entries are 1.

(4) $D = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$

Diagonal matrix because non-diagonal are 0.

(5) $E = \begin{bmatrix} 5-3 & 0 \\ 0 & 1+1 \end{bmatrix}$

Scalar- because diagonal are same.

Q.4 Find the negative of matrices.

(1) $A = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$$-A = -\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

(2) $B = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$

$$-B = -\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$$

$$-B = \begin{bmatrix} -3 & +1 \\ -2 & -1 \end{bmatrix}$$

(3) $C = \begin{bmatrix} 2 & 6 \\ 3 & 2 \end{bmatrix}$

$$-C = -\begin{bmatrix} 2 & 6 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -6 \\ -3 & -2 \end{bmatrix}$$

(4) $D = \begin{bmatrix} -3 & 2 \\ -4 & 5 \end{bmatrix}$

$$-D = -\begin{bmatrix} -3 & 2 \\ -4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} +3 & -2 \\ +4 & -5 \end{bmatrix}$$

(5) $E = \begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix}$

$$-E = -\begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & +5 \\ -2 & -3 \end{bmatrix}$$

Q.5 Find the transpose.

(1) $A = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$

$$A^t = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}^t$$

$$A^t = \begin{bmatrix} 0 & 1 & -2 \end{bmatrix}$$

(2) $B = \begin{bmatrix} 5 & 1 & -6 \end{bmatrix}$

$$B^t = \begin{bmatrix} 5 & 1 & -6 \end{bmatrix}^t$$

$$B^t = \begin{bmatrix} 5 \\ 1 \\ -6 \end{bmatrix}$$

(3) $C = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$

$$C^t = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}^t$$

$$C^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \end{bmatrix}$$

(4) $D = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$

$$D^t = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}^t$$

$$D^t = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$$

(5) $E = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$

$$E^t = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}^t$$

$$E^t = \begin{bmatrix} 2 & -4 \\ 3 & 5 \end{bmatrix}$$

(6) $F = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$F^t = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^t$$

$$F^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Q.6 Verify if $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ **and** $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$

(i) $(A^t)^t = A$

Solution: $(A^t)^t = A$

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t$$

$$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$\left(A^t\right)^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}^t$$

$$\left(A^t\right)^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$\left(A^t\right)^t = A$$

Hence Proved.

(ii) $\left(B^t\right)^t = B$

Solution: $\left(B^t\right)^t = B$

$$B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}^t$$

$$B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$\left(B^t\right)^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}^t$$

$$\left(B^t\right)^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$\left(B^t\right)^t = B$$

Hence proved

Exercise 1.3

Q.1 Which of the following are conformable for addition?

$A=\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$, $B=\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$C=\begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & -2 \end{bmatrix}$, $D=\begin{bmatrix} 2+1 \\ 3 \end{bmatrix}$

$E=\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, $F=\begin{bmatrix} 3 & 2 \\ 1+1 & -4 \\ 3+2 & 2+1 \end{bmatrix}$

Solution:
In the above matrices following matrices are suitable for addition.

(i) A and E are conformable for addition because their order is same and both are square matrix.

(ii) B and D are conformable for addition because the order is same i.e. they have two rows and 1 Columns and both are rectangular matrices.

(iii) C and F are conformable for addition because their order is same i.e. they have three 3 rows and 2 columns and they are a rectangular matrix.

Q.2 Find the additive inverse of the following matrices:

(1) $A=\begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}$

Solution:
Additive inverse of a matrix is negative matrix.

$A=\begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}$ is

$$-A=-\begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}=\begin{bmatrix} (-1)\times 2 & (-1)4 \\ (-1)(-2) & (-1)1 \end{bmatrix}$$

$$-A=\begin{bmatrix} -2 & -4 \\ 2 & -1 \end{bmatrix}$$

(2) $B=\begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$

Solution: $B=\begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$

Its additive inverse is

$$-B=-\begin{bmatrix} +1 & 0 & -1 \\ +2 & -1 & 3 \\ +3 & -2 & 1 \end{bmatrix}$$

$$-B=\begin{bmatrix} -1 & 0 & 1 \\ -2 & 1 & -3 \\ -3 & 2 & -1 \end{bmatrix}$$

(3) $C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

Solution: $C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

$$-C = -\begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} -1\times 4 \\ -1\times -2 \end{bmatrix}$$

The additive inverse is

$$-C = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$$

(4) $D = \begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix}$

Solution: $D = \begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix}$

The additive inverse is

$$-D = -\begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -1\times 1 & -1\times 0 \\ -1\times -3 & -1\times -2 \\ -1\times 2 & -1\times 1 \end{bmatrix}$$

$$-D = \begin{bmatrix} -1 & 0 \\ 3 & 2 \\ -2 & -1 \end{bmatrix}$$

(5) $E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Solution: $E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

The additive inverse of the given matrix is:

$$-E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1\times 1 & -1\times 0 \\ -1\times 0 & -1\times 1 \end{bmatrix}$$

$$-E = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

(6) $F = \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$

Solution: $F = \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$

Its additive inverse is

$$-F = -\begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} -1\times\sqrt{3} & -1\times 1 \\ -1\times -1 & -1\times\sqrt{2} \end{bmatrix}$$

$$-F = \begin{bmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{2} \end{bmatrix}$$

Q.3 **If** $A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$

$C = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix},\ D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix},$

then find.

(i) $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Solution: $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

As $A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$

So, $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

The order of matrix A and the given matrix order is same. So, they can be added easily.

$$= \begin{bmatrix} -1+1 & 2+1 \\ 2+1 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 \\ 3 & 2 \end{bmatrix}$$

(ii) $B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Solution: $B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

As $B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

So, $B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

The order of both above matrices are same, so, they can be easily added.

$$= \begin{bmatrix} 1+(-2) \\ -1+3 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -1 & +3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

(iii) $C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

Solution: $C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

As $C = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$

So, $C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$$

Their orders are same so they can added

$$= \begin{bmatrix} 1+(-2) & -1+(1) & 2+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & -1+1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 5 \end{bmatrix}$$

(iv) $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

Solution: $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

As $D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$

So, $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

Their orders are same. So, they can be added.

$$= \begin{bmatrix} 1+0 & 2+1 & 3+0 \\ -1+2 & 0+0 & 2+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 3 \\ 1 & 0 & 3 \end{bmatrix}$$

(v) $\mathbf{2A}$

Solution: $2A$

As $A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$

So, $2A$

$$= (2) \times \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2(-1) & 2\times 2 \\ 2\times 2 & 2\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 4 \\ 4 & 2 \end{bmatrix}$$

(vi) $(-1)B$

Solution: $(-1)B$

As $B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

So, $(-1)B$

$$= (-1) \times \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} (-1)\times 1 \\ (-1)\times(-1) \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

(vii) $(-2)C$

Solution: $(-2)C$

As C = $\begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$

So, (-2) C

$$= (-2) \times \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (-2)(1) & (-2)(-1) & (-2)(2) \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 2 & -4 \end{bmatrix}$$

(viii) $\mathbf{3D}$

Solution: $3D$

As $D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$

So, $3D$

$$= (3) \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times 1 & 3\times 2 & 3\times 3 \\ 3\times -1 & 3\times 0 & 3\times 2 \end{bmatrix}$$

$$=\begin{bmatrix} 3 & 6 & 9 \\ -3 & 0 & 6 \end{bmatrix}$$

(ix) $3C$

Solution: $3C$

As $C=\begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$

So, 3C

$$=(3)\times\begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$$
$$=\begin{bmatrix} 3\times 1 & 3\times -1 & 3\times 2 \end{bmatrix}$$
$$=\begin{bmatrix} 3 & -3 & 6 \end{bmatrix}$$

Q.4 Perform the indicated operations and simplify the following:

(i) $\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}+\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}\right)+\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

Solution: $\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}+\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}\right)+\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$$=\begin{bmatrix} 1+0 & 0+2 \\ 0+3 & 1+0 \end{bmatrix}+\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$
$$=\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}+\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$
$$=\begin{bmatrix} 1+1 & 2+1 \\ 3+1 & 1+0 \end{bmatrix}$$
$$=\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$$

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}+\left(\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}-\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\right)$

Solution:

$$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}+\left(\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}-\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\right)$$
$$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}+\begin{bmatrix} 0-1 & 2-1 \\ 3-1 & 0-0 \end{bmatrix}$$
$$=\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}+\begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$$
$$=\begin{bmatrix} 1-1 & 0+1 \\ 0+2 & 1+0 \end{bmatrix}$$
$$=\begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$$

(iii) $\begin{bmatrix} 2 & 3 & 1 \end{bmatrix}+\left(\begin{bmatrix} 1 & 0 & 2 \end{bmatrix}-\begin{bmatrix} 2 & 2 & 2 \end{bmatrix}\right)$

Solution:

$$=\begin{bmatrix} 2 & 3 & 1 \end{bmatrix}+\begin{bmatrix} 1-2 & 0-2 & 2-2 \end{bmatrix}$$
$$=\begin{bmatrix} 2 & 3 & 1 \end{bmatrix}+\begin{bmatrix} -1 & -2 & 0 \end{bmatrix}$$
$$=\begin{bmatrix} 2-1 & 3-2 & 1-0 \end{bmatrix}$$
$$=\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

(iv) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -1 \\ 0 & 1 & 2 \end{bmatrix}+\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$

Solution:

$$=\begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -1 \\ 0 & 1 & 2 \end{bmatrix}+\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$
$$=\begin{bmatrix} 1+1 & 2+1 & 3+1 \\ -1+2 & -1+2 & -1+2 \\ 3+0 & 1+2 & 2-1 \end{bmatrix}$$
$$=\begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 3 & 4 & 5 \end{bmatrix}$$

(v) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}+\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$

Solution:

$$=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}+\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$$
$$=\begin{bmatrix} 1+1 & 2+1 & 3+1 \\ -1+2 & -1+2 & -1+2 \\ 3+0 & 1+2 & 2-1 \end{bmatrix}$$
$$=\begin{bmatrix} 2 & 2 & 1 \\ 0 & 2 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

(vi) $\left(\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}+\begin{bmatrix}2 & 1\\1 & 0\end{bmatrix}\right)+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$

Solution:

$$=\begin{bmatrix}1+2 & 2+1\\0+1 & 1+0\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3\\1 & 1\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}3+1 & 3+1\\1+1 & 1+1\end{bmatrix}$$

$$=\begin{bmatrix}4 & 4\\2 & 2\end{bmatrix}$$

Q.5 For the matrices $A=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$,

$B=\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$ **and**

$C=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$, **verify the following rules:**

(i) $A+C=C+A$

Solutions:

L.H.S = A+C

R.H.S = C+A

LHS= A+C

$$=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1-1 & 2+0 & 3+0\\2+0 & 3-2 & 1+3\\1+1 & -1+1 & 0+2\end{bmatrix}$$

$$=\begin{bmatrix}0 & 2 & 3\\2 & 1 & 4\\2 & 0 & 2\end{bmatrix}$$

RHS= C+A

$$=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-1+1 & 0+2 & 0+3\\0+2 & -2+3 & 3+1\\1+1 & 1-1 & 2-0\end{bmatrix}$$

$$=\begin{bmatrix}0 & 2 & 3\\2 & 1 & 4\\2 & 0 & 2\end{bmatrix}$$

$A+C=C+A$

Hence proved

L.H.S = R.H.S

(ii) $A+B=B+A$

Solution: $A+B=B+A$

L.H.S = A+B

R.H.S = B+A

LHS= A+B

$$=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & 2-1 & +3+1\\2+2 & 3-2 & 1+2\\1+3 & -1+1 & 0+3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix}$$

RHS= B+A

$$=\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\3 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & -1+2 & 1+3\\2+2 & -2+3 & 2+1\\3+3 & 1-1 & 3-0\end{bmatrix}$$

$$=\begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\6 & 0 & 3\end{bmatrix}$$

$A+B=B+A$

Hence proved

L.H.S = R.H.S

(iii) $B+C=C+B$

Solution: $B+C=C+B$

L.H.S = B+C

R.H.S = C+B

L.H.S = B+C

$$=\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1-1 & -1+0 & 1+0\\ 2+0 & -2-2 & 2+3\\ 3+1 & 1+1 & 3+2\end{bmatrix}$$

$$=\begin{bmatrix}0 & -1 & 1\\ 2 & -4 & 5\\ 4 & 2 & 5\end{bmatrix}$$

R.H.S = C+B

$$=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1+1 & 0-1 & 0+1\\ 0+2 & -2-2 & 3+2\\ 1+3 & 1+1 & 2+3\end{bmatrix}$$

$$=\begin{bmatrix}0 & -1 & 1\\ 2 & -4 & 5\\ 4 & 2 & 5\end{bmatrix}$$

L.H.S = R.H.S

B+C=C+B

Hence proved

(iv) $A+(B+A)=2A+B$

Solution: $A+(B+A)=2A+B$

L.H.S = A+ (B+A)

R.H.S = 2A+B

L.H.S= A+ (B+A)

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 3\\ 4 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3 & 7\\ 6 & 4 & 4\\ 5 & -1 & 3\end{bmatrix}$$

R.H.S = 2A+B

$$=2\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 4 & 6\\ 4 & 6 & 2\\ 2 & -2 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3 & 7\\ 6 & 4 & 4\\ 5 & -1 & 3\end{bmatrix}$$

L.H.S =R.H.S

A+ (B+A) =2A+B

Hence proved

(v) $(C-B)+A=C+(A+B)$

Solution: $(C-B)+A=C+(A+B)$

L.H.S = (C-B) +A

R.H.S = C+(A-B)

L.H.S = (C-B) + A

$$=\left(\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right)+\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-2 & 1 & -1\\ -2 & 0 & 1\\ -2 & 0 & -1\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3 & 2\\ 0 & 3 & 2\\ -1 & -1 & -1\end{bmatrix}$$

RHS= C+ (A-B)

$$=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}+\left(\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}+\begin{bmatrix}0 & 3 & 2\\ 0 & 5 & -1\\ -2 & -2 & -3\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3 & 2\\ 0 & 3 & 2\\ -1 & -1 & -1\end{bmatrix}$$

L.H.S = R.H.S

(C-B)+A=C+ (A-B)

Hence proved

(vi) $2A+B=A+(A+B)$

Solution: $2A+B=A+(A+B)$

L.H.S = 2A+B

R.H.S = A+ (A+B)

LHS= 2A+B

$$= 2\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \\ 2 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 7 \\ 6 & 4 & 4 \\ 5 & -1 & 3 \end{bmatrix}$$

RHS= A + (A+B)

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 7 \\ 6 & 4 & 4 \\ 5 & -1 & 3 \end{bmatrix}$$

L.H.S = R.H.S

2A+B=A+ (A+B)

Hence proved

(vii) $(C-B)-A=(C-A)-B$

Solution: $(C-B)-A=(C-A)-B$

L.H.S = (C-B) –A

R.H.S = (C-A) –B

LHS= (C-B) – A

$$= \left(\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \right) - \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & - & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 & -1 \\ -2 & 0 & 1 \\ -2 & 0 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -1 & -4 \\ -4 & -3 & 0 \\ -3 & 1 & -1 \end{bmatrix}$$

RHS= (C-A) –B

$$= \left(\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} \right) - \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 & -3 \\ -2 & -5 & 2 \\ 0 & 2 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -1 & -4 \\ -4 & -3 & 0 \\ -3 & 1 & -1 \end{bmatrix}$$

L.H.S = R.H.S

(C-B)-A=(C-A)-B

Hence proved

(viii) $(A+B)+C=A+(B+C)$

Solution: (A+B) + C = A+ (B+C)

L.H.S = (A+B) + C

R.H.S = A + (B+C)

LHS= (A+B) +C

$$= \left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \right) + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 4 \\ 4 & -1 & 6 \\ 5 & 1 & 5 \end{bmatrix}$$

R.H.S = A+ (B+C)

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$=\begin{bmatrix}1&1&4\\4&-1&6\\5&1&5\end{bmatrix}$$

L.H.S = R.H.S

(A+B)+C=A+ (B+C)

Hence proved

(ix) $A+(B-C)=(A-C)+B$

Solution: $A+(B-C)=(A-C)+B$

L.H.S = A+ (B-C)

R.H.S = (A-C) + B

L.H.S = A+ (B-C)

$$=\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\left(\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}-\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}\right)$$

$$=\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\begin{bmatrix}2&-1&1\\2&0&-1\\2&0&1\end{bmatrix}$$

$$=\begin{bmatrix}3&1&4\\4&3&0\\3&-1&1\end{bmatrix}$$

RHS= (A-C) +B

$$=\left(\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}-\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}\right)+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$=\begin{bmatrix}1+1&2-0&3-1\\2-0&3+2&1-3\\1-1&-1-1&0-2\end{bmatrix}+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$=\begin{bmatrix}2&2&3\\2&5&-2\\0&-2&-2\end{bmatrix}+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$=\begin{bmatrix}3&1&4\\4&3&0\\3&-1&1\end{bmatrix}$$

L.H.S = R.H.S

A+ (B-C) = (A-C) +B

Hence proved

(x) $2A+2B=2(A+B)$

Solution: $2A+2B=2(A+B)$

L.H.S = 2A + 2B

R.H.S = 2(A + B)

L.H.S = 2A + 2B

$$=2\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+2\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$=\begin{bmatrix}2&4&6\\4&6&2\\2&-2&0\end{bmatrix}+\begin{bmatrix}2&-2&2\\4&-4&4\\6&2&6\end{bmatrix}$$

$$=\begin{bmatrix}4&2&8\\8&2&6\\8&0&6\end{bmatrix}$$

RHS= 2 (A+B)

$$=2\left(\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}\right)$$

$$=\begin{bmatrix}1+1&2-1&3+1\\2+2&3-2&1+2\\1+3&-1+1&0+3\end{bmatrix}$$

$$=2\begin{bmatrix}2&1&4\\4&1&3\\4&0&3\end{bmatrix}$$

$$=\begin{bmatrix}4&2&8\\8&2&6\\8&0&6\end{bmatrix}$$

L.H.S = R.H.S

2A+2B=2(A+B)

Hence proved

Q.6 **If** $A=\begin{bmatrix}1&-2\\3&4\end{bmatrix}$ **and** $B=\begin{bmatrix}0&7\\-3&8\end{bmatrix}$

find:

(i) $3A-2B$

Solution: $3A-2B$

$$3A-2B=3\begin{bmatrix}1&-2\\3&4\end{bmatrix}-2\begin{bmatrix}0&7\\-3&8\end{bmatrix}$$

$$=\begin{bmatrix}3&-6\\9&12\end{bmatrix}-\begin{bmatrix}0&14\\-6&16\end{bmatrix}$$

$$=\begin{bmatrix}3 & -20\\15 & -4\end{bmatrix}$$

(ii) $2A^t - 3B^t$

Solution: $2A^t - 3B^t$

When we take transpose of any matrix we change rows into columns or columns into rows.

$$A^t = \begin{bmatrix}1 & 3\\-2 & 4\end{bmatrix}$$

$$B^t = \begin{bmatrix}0 & -3\\7 & 8\end{bmatrix}$$

$$2A^t - 3B^t = 2\begin{bmatrix}1 & 3\\-2 & 4\end{bmatrix} - 3\begin{bmatrix}0 & -3\\7 & 8\end{bmatrix}$$

$$=\begin{bmatrix}2 & 6\\-4 & 8\end{bmatrix} - \begin{bmatrix}0 & -9\\21 & 24\end{bmatrix}$$

$$=\begin{bmatrix}2 & 15\\-25 & -16\end{bmatrix}$$

Q.7 If

$$2\begin{bmatrix}2 & 4\\-3 & a\end{bmatrix} + 3\begin{bmatrix}1 & b\\8 & -4\end{bmatrix} = \begin{bmatrix}7 & 10\\18 & 1\end{bmatrix}$$

Solution:

$$2\begin{bmatrix}2 & 4\\-3 & a\end{bmatrix} + 3\begin{bmatrix}1 & b\\8 & -4\end{bmatrix} = \begin{bmatrix}7 & 10\\18 & 1\end{bmatrix}$$

$$=\begin{bmatrix}4 & 8\\-6 & 2a\end{bmatrix} + \begin{bmatrix}3 & 3b\\24 & -12\end{bmatrix} = \begin{bmatrix}7 & 10\\18 & 1\end{bmatrix}$$

$$=\begin{bmatrix}7 & 8+3b\\18 & 2a+(-12)\end{bmatrix} = \begin{bmatrix}7 & 10\\18 & 1\end{bmatrix}$$

$8 + 3b = 10$ ________ (i)

$2a - 12 = 1$ ________ (ii)

By solving equation (ii) we get the value of a

$2a - 12 = 1$

$2a = 1 + 12$

$2a = 13$

$$a = \frac{13}{2}$$

By solving equation (i) we get the value of b

$8 + 3b = 10$

$3b = 10 - 8$

$3b = 2$

$$b = \frac{2}{3}$$

Q.8 If $A = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$ **and** $B = \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}$

Then verify that

(i) $(A+B)^t = A^t + B^t$

Solution: $(A+B)^t = A^t + B^t$

L.H.S = $(A + B)^t$

R.H.S = $A^t + B^t$

To solve L.H.S

L.H.S = $(A + B)^t$

$$=(A+B)^t = \left(\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} + \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}\right)^t$$

$$=\begin{bmatrix}2 & 3\\2 & 1\end{bmatrix}^t$$

$$\text{R.H.H} = (A+B)^t = \begin{bmatrix}2 & 2\\3 & 1\end{bmatrix}$$

To solve R.H.S

R.H.S = $A^t + B^t$

$$A^t = \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$

$$B^t = \begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$

$$\text{RHS} = A^t + B^t = \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix} + \begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}2 & 2\\3 & 1\end{bmatrix}$$

L.H.S = R.H.S $\Rightarrow (A+B)^t = A^t + B^t$

Hence Proved

(ii) $(A-B)^t = A^t - B^t$

Solution: $(A-B)^t = A^t - B^t$

L.H.S = $(A - B)^t$

R.H.S = $A^t - B^t$

LHS= $(A - B)^t$

$(A - B)^t = \left(\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} - \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}\right)^t$

$= \begin{bmatrix}0 & 1\\-2 & 1\end{bmatrix}^t$

$(A - B)^t = \begin{bmatrix}0 & -2\\1 & 1\end{bmatrix}$

R.H.S = $A^t - B^t$

$= \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t - \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}^t$

$= \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix} - \begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$

$= \begin{bmatrix}1-1 & 0-2\\2-1 & 1-0\end{bmatrix}$

$= \begin{bmatrix}0 & -2\\1 & 1\end{bmatrix}$

L.H.S = R.H.S

$(A-B)^t = A^t - B^t$

Hence proved

(iii) **$A + A^+$ is a symmetric**

Solution:

$A + A^+$ is a symmetric

To show that $A + A^t$ is symmetric, we will show that

$(A + A^t)^t = (A + A^t)$

$A + A^t = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} + \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t$

$= \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} + \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$

$= \begin{bmatrix}1+1 & 2+0\\0+2 & 1+1\end{bmatrix}$

$A + A^t = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}$

$(A + A^t)^t = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}^t$

$= \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}$

$(A + A^t)^t = (A + A^t)$

Hence Proved

$A + A^t$ symmetric

(iv) **$A - A^t$ is a skew symmetric**

Solution: $A - A^t$

To show that A-A^t is skew symmetric we will show that

$(A - A^t)^t = -(A - A^t)$

$A - A^t = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t - \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t$

$= \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} - \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$

$= \begin{bmatrix}1-1 & 2-0\\0-2 & 1-1\end{bmatrix}$

$A - A^t = \begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}$

$(A - A^t)^t = \begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}^t$

$= \begin{bmatrix}0 & -2\\2 & 0\end{bmatrix}$

$= -\begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}$

$(A - A^t)^t = -(A - A^t)$

Hence proved

$A - A^t$ is a skew symmetric

(v) **$B + B^t$ is a symmetric**

Solution: $B + B^t$

The show that $B + B^t$ is symmetric we will show that

$(B + B^t)^t = (B + B^t)$

$B + B^t = \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix} + \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}^t$

$= \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix} + \begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$

$= \begin{bmatrix}1+1 & 1+2\\2+1 & 0+0\end{bmatrix}$

$B + B^t = \begin{bmatrix}2 & 3\\3 & 0\end{bmatrix}$

$(B + B^t)^t = \begin{bmatrix}2 & 3\\3 & 0\end{bmatrix}^t$

$= \begin{bmatrix}2 & 3\\3 & 0\end{bmatrix}$

$(B + B^t)^t = (B + B^t)$

Hence proved
$B+B^t$ is a symmetric

(vi) $B-B^t$ **is a skew symmetric**
Solution: $B-B^t$
To show that $B - B^t$ is skew symmetric, we will show that
$(B-B^t)^t = -(B-B^t)$

$$B-B^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 1-2 \\ 2-1 & 0-0 \end{bmatrix}$$

$$B-B^t = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$(B-B^t)^t = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$- \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$(B-B^t)^t = -(B-B^t)$
Hence proved
$B-B^t$ is a skew symmetric.

Exercise 1.4

Q.1 Which of the following product of matrices if conformable for multiplication?

(i) $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1^{st} matrix is equal to number of rows of 2^{nd} matrix.

(ii) $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -2 & -1 \\ 1 & 3 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1^{st} matrix is equal to number of rows of 2^{nd} matrix.

(iii) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$

No, these matrices cannot be multiplied because number of columns of 1^{st} matrix is not equal to the number of rows of 2^{nd} matrix.

(iv) $\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1^{st} matrix is equal to number of rows of 2^{nd} matrix.

(v) $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 3 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1^{st} matrix is equal to number of rows of 2^{nd} matrix.

Q.2 If $A=\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$, $B=\begin{bmatrix} 6 \\ 5 \end{bmatrix}$ **find**

(i) AB

Solution: $AB=\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \end{bmatrix}$

$$AB=\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$$=\begin{bmatrix} (3\times 6)+(0\times 5) \\ (-1\times 6)+(2\times 5) \end{bmatrix}$$

$$=\begin{bmatrix} 18+0 \\ -6+10 \end{bmatrix}=\begin{bmatrix} 18 \\ 4 \end{bmatrix}$$

(ii) BA (if possible)

Solution:

BA is not possible became number of columns of B not equal to number of rows of A.

Q.3 Find the following products

(i) $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

Solution: $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

$=\begin{bmatrix}(1\times4)+(2\times0)\end{bmatrix}$

$=\begin{bmatrix}4+0\end{bmatrix}$

$=\begin{bmatrix}4\end{bmatrix}$

(ii) $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}5\\-4\end{bmatrix}$

Solution: $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}4\\-0\end{bmatrix}$

$=\begin{bmatrix}(1\times5)+(2\times-4)\end{bmatrix}$

$=\begin{bmatrix}5+(-8)\end{bmatrix}$

$=[5-8]$

$=[-3]$

(iii) $\begin{bmatrix}-3 & 0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

Solution: $\begin{bmatrix}-3 & 0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

$=\begin{bmatrix}(-3\times4)+(0\times0)\end{bmatrix}$

$=[-12+0]$

$=[-12]$

(iv) $\begin{bmatrix}6 & 0\end{bmatrix}\begin{bmatrix}4\\-0\end{bmatrix}$

Solution: $\begin{bmatrix}6 & 0\end{bmatrix}\begin{bmatrix}4\\-0\end{bmatrix}$

$\begin{bmatrix}6 & +0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

$=\begin{bmatrix}6\times4+(-0)(0)\end{bmatrix}$

$=\begin{bmatrix}24-0\end{bmatrix}$

$=\begin{bmatrix}24\end{bmatrix}$

(v) $\begin{bmatrix}1 & 2\\-3 & 0\\6 & -1\end{bmatrix}\begin{bmatrix}4 & 5\\0 & -4\end{bmatrix}$

Solution: $\begin{bmatrix}1 & 2\\-3 & 0\\6 & -1\end{bmatrix}\begin{bmatrix}4 & 5\\0 & -4\end{bmatrix}$

$=\begin{bmatrix}1\times4+2\times0 & 1\times5+2\times(-4)\\-3\times4+0\times0 & -3(5)+0\times(-4)\\6(4)+(-1)(0) & 6(5)+(-1)(-4)\end{bmatrix}$

$=\begin{bmatrix}4+0 & 5-8\\-12+0 & -15-0\\24-0 & 30+4\end{bmatrix}$

$=\begin{bmatrix}4 & -3\\-12 & -15\\24 & 34\end{bmatrix}$

Q.4 Multiply the following matrices.

(a) $\begin{bmatrix}2 & 3\\1 & 1\\0 & -2\end{bmatrix}\begin{bmatrix}2 & -1\\3 & 0\end{bmatrix}$

Solution: $\begin{bmatrix}2 & 3\\1 & 1\\0 & -2\end{bmatrix}\begin{bmatrix}2 & -1\\3 & 0\end{bmatrix}$

$\begin{bmatrix}2\times2+(3\times3) & (2\times-1)+(3\times0)\\(1\times2)+(1\times3) & (1\times-1)+(1\times0)\\(0\times2)+(-2\times3) & (0\times-1)+(-2\times0)\end{bmatrix}$

$=\begin{bmatrix}4+9 & -2+0\\2+3 & -1+0\\0+-6 & 0+0\end{bmatrix}$

$$=\begin{bmatrix} 13 & -2 \\ 5 & -1 \\ 0-6 & 0 \end{bmatrix}$$

$$=\begin{bmatrix} 13 & -2 \\ 5 & -1 \\ -6 & 0 \end{bmatrix}$$

(b) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}$

Solution: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}$

$$=\begin{bmatrix} (1\times1)+(2\times3)+(3\times-1) & (1\times2)+(2\times4)+(3\times1) \\ (4\times1)+(5\times3)+(6\times-1) & (4\times2)+(5\times4)+(6\times1) \end{bmatrix}$$

$$=\begin{bmatrix} 1+6+(-3) & 2+8+3 \\ 4+15+(-6) & 8+20+6 \end{bmatrix}$$

$$=\begin{bmatrix} 7-3 & 13 \\ 19-6 & 34 \end{bmatrix}$$

$$=\begin{bmatrix} 4 & 13 \\ 13 & 34 \end{bmatrix}$$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Solution: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

$$=\begin{bmatrix} (1\times1)+(2\times4) & (1\times2)+(2\times5) & (1\times3)+(2\times6) \\ (3\times1)+(4\times4) & (3\times2)+(4\times5) & (3\times3)+(4\times6) \\ (-1\times1)+(1\times4) & (-1\times2)+(1\times5) & (-1\times3)+(1\times6) \end{bmatrix}$$

$$=\begin{bmatrix} 1+8 & 2+10 & 3+12 \\ 3+16 & 6+20 & 9+24 \\ -1+4 & -2+5 & -3+6 \end{bmatrix}$$

$$=\begin{bmatrix} 9 & 12 & 15 \\ 19 & 26 & 33 \\ 3 & 3 & 3 \end{bmatrix}$$

(d) $\begin{bmatrix} 8 & 5 \\ 6 & 4 \end{bmatrix}\begin{bmatrix} 2 & -\frac{5}{2} \\ -4 & 4 \end{bmatrix}$

Solution: $\begin{bmatrix} 8 & 5 \\ 6 & 4 \end{bmatrix}\begin{bmatrix} 2 & -\frac{5}{2} \\ -4 & 4 \end{bmatrix}$

$$=\begin{bmatrix} (8\times2)+(5\times-4) & \left(8\times-\frac{5}{2}\right)+(5\times4) \\ (6\times2)+(4\times-4) & \left(6\times-\frac{5}{2}\right)+(4\times4) \end{bmatrix}$$

$$=\begin{bmatrix} 16+(-20) & \frac{-40}{2}+20 \\ 12+(-16) & \frac{-30}{2}+16 \end{bmatrix}$$

$$=\begin{bmatrix} 16-20 & -20+20 \\ 12-16 & -15+16 \end{bmatrix}$$

$$=\begin{bmatrix} -4 & 0 \\ -4 & 1 \end{bmatrix}$$

(e) $\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Solution: $\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$=\begin{bmatrix} (-1\times0)+(2\times0) & (-1\times0)+(2\times0) \\ (1\times0)+(3\times0) & (1\times0)+(3\times0) \end{bmatrix}$$

$$=\begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Q.5 **Let** $A=\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$

and $C=\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ **verify whether**

(i) $AB = BA$

Solution: $AB = BA$

L.H.S = AB

R.H.S = BA

L.H.S =AB

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(3\times-5)\\ (2\times1)+(0-3) & (2\times2)+(0-5)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & -2+(-15)\\ 2+0 & 4+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}$$

$$\text{R.H.S} = \text{BA} = \begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\times\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1\times(-1)+2\times2 & 1\times3+2\times0\\ -3\times(-1)+(-5)2 & -3\times3+(-5)(0)\end{bmatrix}$$

$$=\begin{bmatrix}-1+4 & 3+0\\ 3-10 & -9-0\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3\\ -7 & -9\end{bmatrix}$$

Since L.H.S $\neq$ R.H.S

L.H.S $\neq$ R.H.S

L.H.S $\neq$ R.H.S

(ii) $A(BC)=(AB)C$

Solution: $A(BC)=(AB)C$

L.H.S = A (BC)

R.H.S = (AB) C

L.H.S

L.H.S=A(BC)

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}2+2 & 1+6\\ -6+(-5) & -3+(-15)\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}4 & 7\\ -6-5 & -3-15\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}4 & 7\\ -11 & -18\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times4)+(3\times-11) & (-1\times7)+(3\times-18)\\ (2\times4)+(0\times-11) & (2\times7)+(0\times-18)\end{bmatrix}$$

$$=\begin{bmatrix}-4+(-33) & -7+(-54)\\ 8+0 & 14+0\end{bmatrix}$$

$$=\begin{bmatrix}-4-33 & -7-54\\ 8 & 14\end{bmatrix}$$

$$=\begin{bmatrix}-37 & -61\\ 8 & 14\end{bmatrix}$$

R.H.S = (AB)C

$$=\left(\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\right)\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(-3\times-5)\\ (2\times1)+(0\times-3) & (2\times2)+(0\times-5)\end{bmatrix}$$

$$=\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & -2+(-15)\\ 2+0 & -4+0\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & 4\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-10\times2)+(-17\times1) & (-10\times1)+(-17\times3)\\ (2\times2)+(4\times1) & (2\times1)(4\times3)\end{bmatrix}$$

$$=\begin{bmatrix}-20+(-17) & -10+(-51)\\ 4+4 & 2+12\end{bmatrix}$$

$$=\begin{bmatrix}-20-17 & -10-51\\ 8 & 14\end{bmatrix}$$

$$=\begin{bmatrix}-37 & -61\\ 8 & 14\end{bmatrix}$$

Since

L.H.S = R.H.S $\Rightarrow$ A (BC) = (AB)C

Hence proved

(iii) $A(B+C)=AB+AC$

Solution: $A(B+C)=AB+AC$

L.H.S = A (B+C)

R.H.S = AB+AC

L.H.S

LHS=A (B+C)

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}+\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1+2 & 2+1\\ -3+1 & -5+3\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}3 & 3\\ -2 & -2\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times3)+(3\times-2) & (-1\times3)+(3\times-2)\\ (2\times3)+(0\times-2) & (2\times3)+(0\times-2)\end{bmatrix}$$

$$=\begin{bmatrix}-3+(-6) & -3+(-6)\\ 6+0 & 6+0\end{bmatrix}$$

$$=\begin{bmatrix}-3-6 & -3-6\\ 6 & 6\end{bmatrix}$$

$$=\begin{bmatrix}-9 & -9\\ 6 & 6\end{bmatrix}$$

R.H.S=AB+AC

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}+\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(3\times-5)\\ (2\times1)+(0\times-3) & (2\times+2)+(0\times-5)\end{bmatrix}+\begin{bmatrix}(-1\times2)+(3\times1) & (-1\times1)+(3\times3)\\ (2\times2)+(0\times1) & (2\times1)+(0\times3)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-3) & -2+(-15)\\ 2+0 & +4+0\end{bmatrix}+\begin{bmatrix}-2+3 & -1+9\\ 4+0 & 2+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & +4\end{bmatrix}+\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & +4\end{bmatrix}+\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-10+1 & -17+8\\ 2+4 & +4+2\end{bmatrix}$$

$$=\begin{bmatrix}-9 & -9\\ 6 & 6\end{bmatrix}$$

Since LHS=RHS

A (B+C) = AB+AC

Hence proved

(iv) A(B–C) = AB–AC

Solution: A (B–C) = AB–AC

L.H.S = A (B–C)

R.H.S = AB–AC

L.H.S=A(B–C)

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}-\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1-2 & 2-1\\ -3-1 & -5-3\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}-1 & 1\\ -4 & -8\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times-1)+(3\times-4) & (-1\times1)+(3\times-8)\\ (2\times-1)+(0l-4) & (2\times1)+(0\times-8)\end{bmatrix}$$

$$=\begin{bmatrix}+1+(-12) & -1+(-24)\\ -2+0 & 2+0\end{bmatrix}$$

$$=\begin{bmatrix}1-12 & -1-24\\ -2 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-11 & -25\\ -2 & 2\end{bmatrix}$$

R.H.S=AB–AC

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}-\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+3\times-5)\\ (2\times1)+(0\times-3) & (2\times2)+(0\times-5)\end{bmatrix}-\begin{bmatrix}(-1\times2)+(3\times1) & (-1\times1)+(3\times3)\\ (2\times2)+(0\times1) & (2\times1)+(0\times3)\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)-(3\times3) & (-1\times2)+(3\times-5)\\ (2\times1)+(0\times-3) & (2\times2)+(0\times-5)\end{bmatrix}-\begin{bmatrix}(-1\times2)+(3\times1) & (-1\times1)+(3\times3)\\ (2\times2)+(0\times1) & (2\times1)+(0\times3)\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2+0 & 4+0\end{bmatrix}-\begin{bmatrix}-2+3 & -1+9\\ 4+0 & 2+0\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}-\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-10-1 & -17-8\\ 2-4 & 4-2\end{bmatrix}$$

$$=\begin{bmatrix}-11 & -25\\ -2 & 2\end{bmatrix}$$

Since L.H.S = R.H.S

A (B-C) =AB-AC, Hence **proved.**

Q.6 **For the matrices** $A=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$ **and** $C=\begin{bmatrix}-2 & 6\\ 3 & -9\end{bmatrix}$

Verify that

(i) $(AB)^t = B^t A^t$

Solution: $(AB)^t = B^t A^t$

L.H.S $= (AB)^t$

R.H.S $= B^t A^t$

$$(AB)=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(3\times-5)\\ (+2\times1)+(0\times-3) & (2\times2)+(0\times-5)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & -2+(-15)\\ 2+0 & 4+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}$$

LHS= $(AB)^t$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}^t$$

$$=\begin{bmatrix}-10 & 2\\ -17 & 4\end{bmatrix}$$

$$B^t=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}^t$$

$$B^t=\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}$$

$$A^t=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$$

$$A^t=\begin{bmatrix}-1 & 2\\ 3 & 0\end{bmatrix}$$

L.H.S=B^tA^t

$$=\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}\times\begin{bmatrix}-1 & 2\\ 3 & 0\end{bmatrix}$$

$$=\begin{bmatrix}(1\times-1)+(-3\times3) & (1\times2)+(-3\times0)\\ (2\times-1)+(-5\times3) & (2\times2)+(-5\times0)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & 2+0\\ -2+(-15) & 4+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & 2\\ -2-15 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-10 & 2\\ -17 & 4\end{bmatrix}$$

Since L.H.S = R.H.S

$(AB)^t = B^tA^t$

Hence proved

L.H.S = R.H.S

(ii) $\mathbf{(BC)^t= C^tB^t}$

Solution: $(BC)^t= C^tB^t$

L.H.S $= (BC)^t$

R.H.S $= C^t B^t$

To solve L.H.S

$$BC=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\times\begin{bmatrix}-2 & 6\\ 3 & -9\end{bmatrix}$$

$$=\begin{bmatrix}(1\times-2)+(2\times3) & (1\times6)+(2\,x\times-9)\\ (-3\times-2)+(-5\times3) & (-3\times6)+(-5\times-9)\end{bmatrix}$$

$$=\begin{bmatrix}-2+6 & 6+(-18)\\ 6+(-15) & -18+45\end{bmatrix}$$

$$=\begin{bmatrix}4 & 6-18\\ 6-15 & 27\end{bmatrix}$$

$$=\begin{bmatrix}4 & -12\\ -9 & 27\end{bmatrix}$$

Taking transpose of BC:-

$$(BC)^t=\begin{bmatrix}4 & -12\\ -9 & 27\end{bmatrix}^t$$

$$LHS = (B\,C)^t = \begin{bmatrix}4 & -9\\ -12 & 27\end{bmatrix}$$

To solve R.H.S =

Taking transpose of matrix C

$$C^t=\begin{bmatrix}-2 & 3\\ 6 & -9\end{bmatrix}$$

Taking transpose of matrix B

$$B^t=\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}$$

Now, multiplying matrices, $B^t C^t$

$$R.H.S = C^t B^t=\begin{bmatrix}-2 & 3\\ 6 & -9\end{bmatrix}\times\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}$$

$$=\begin{bmatrix}(-2\times1)+(3\times2) & (-2\times-3)+(3\times-5)\\ (6\times1)+(-9\times2) & (6\times-3)+(-9\times-5)\end{bmatrix}$$

$$=\begin{bmatrix} -2+6 & 6+(-15) \\ 6+(-18) & -18+45 \end{bmatrix}$$

$$=\begin{bmatrix} 4 & 6-15 \\ 6-18 & 27 \end{bmatrix}$$

$$=\begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix}$$

Hence proved

L.H.S = R.H.S

Exercise 1.6

Q.1 Use of matrices, if possible to solve the following systems of linear equations.

(i) The matrices inversion method

(ii) The Cramer's rule

(i) $2x-2y=4$

$3x+2y=6$

By matrices inversion method

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

Let $A=\begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix} X=\begin{bmatrix} x \\ y \end{bmatrix} B=\begin{bmatrix} 4 \\ 6 \end{bmatrix}$

$$A=\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$$

$$|A|=\begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix}$$

$$|A|=(2)(2)-(-2)(3)$$

$$|A|=4+6$$

$$|A|=10$$

Then, solution is possible because A is non-singular matrix.

$$AdjA=\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$$

As we know that

$$AX=B$$

$$X=A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\frac{1}{10}\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$=\frac{1}{10}\begin{bmatrix} 2\times4 & +2\times6 \\ -3\times4+2\times6 \end{bmatrix}$$

$$=\frac{1}{10}\begin{bmatrix} 8+12 \\ -12+12 \end{bmatrix}$$

$$=\frac{1}{10}\begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} \frac{20}{10} \\ \frac{0}{10} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$x=2, y=0$

Solution Set = {(2, 0)}

By Cramer's rule

$A=\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix} B=\begin{bmatrix} 4 \\ 6 \end{bmatrix}$

$$|A|=\begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix}$$

$$=(2)(2)-(-2)(3)$$

$$=4-(-6)$$

$$=4+6$$

$$=10$$

$$|A_x|=\begin{vmatrix} 4 & -2 \\ 6 & 2 \end{vmatrix}$$

$$=(4)(2)-(-2)(6)$$

$$=8+12$$

$$=20$$

$$|A_y|=\begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix}$$

$$=(2)(6)-(4)(3)$$

$$=12-12$$

$$=0$$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{20}{10}$$

$$x=2$$

$$y=\frac{|A_y|}{|A|}$$

$y=\frac{0}{10}$

$y=0$

Solution Set = {(2, 0)}

(ii) $2x+y=3$

$6x+5y=1$

Matrices inversion method

$$\begin{bmatrix}2 & 1\\6 & 5\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}3\\1\end{bmatrix}$$

Let $A=\begin{bmatrix}3 & -2\\3 & 2\end{bmatrix} X=\begin{bmatrix}x\\y\end{bmatrix} B=\begin{bmatrix}4\\6\end{bmatrix}$

$$A=\begin{bmatrix}2 & 1\\6 & 5\end{bmatrix}$$

$$|A|=\begin{vmatrix}2 & 1\\6 & 5\end{vmatrix}$$

$=(2)(5)-(1)(6)$

$=10-6$

$=4$

Solution is possible because A is non-singular matrix.

$$AdjA=\begin{bmatrix}5 & -1\\-6 & 2\end{bmatrix}$$

$AX=B$

$X=A^{-1}B$

$X=\frac{1}{|A|}\times AdjA\times B$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}5 & -1\\-6 & 2\end{bmatrix}\begin{bmatrix}3\\1\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}5\times3+(-1\times1)\\-6\times3+2\times1\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}15+(-1)\\-18+2\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}14\\-16\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{14}{4}\\\frac{16}{4}\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{7}{2}\\-4\end{bmatrix}$$

$x=\frac{7}{2}, y=-4$

Solution Set = $\{(\frac{7}{2},-4)\}$

By Cramer's Rule

$A=\begin{bmatrix}2 & 1\\6 & 5\end{bmatrix} B=\begin{bmatrix}3\\1\end{bmatrix}$

$$|A|=\begin{vmatrix}2 & 1\\6 & 5\end{vmatrix}$$

$=(2)(5)-(1)(6)$

$=10-6$

$=4$

Solution is possible because A is non-singular matrix.

$$|A_x|=\begin{vmatrix}3 & 1\\1 & 5\end{vmatrix}$$

$=(3)(5)-(1)(1)$

$=15-1$

$=14$

$$|A_y|=\begin{vmatrix}2 & 3\\6 & 1\end{vmatrix}$$

$=(2)(1)-(3)(6)$

$=2-18$

$=-16$

$x=\frac{|A_x|}{|A|}$

$x=\frac{14}{4}$

$x=\frac{7}{2}$

$y=\frac{|A_y|}{|A|}$

$y=\frac{16}{4}$

$y=-4$

Solution Set $=\left\{\left(\frac{7}{2},-4\right)\right\}$

(iii) $4x+2y=8$
$3x-y=-1$

By Matrices Inversion Method

$$\begin{bmatrix}4 & 2\\3 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}8\\-1\end{bmatrix}$$

Let $A=\begin{bmatrix}3 & -2\\3 & 2\end{bmatrix} X=\begin{bmatrix}x\\y\end{bmatrix} B=\begin{bmatrix}4\\6\end{bmatrix}$

$$|A|=\begin{vmatrix}4 & 2\\3 & -1\end{vmatrix}$$

$=(4)(-1)-(2)(3)$

$=-4-6$

$=-10$

Solution is possible because A is non singular matrix.

$$AdjA=\begin{bmatrix}-1 & -2\\-3 & 4\end{bmatrix}$$

As we know that

$AX=B$

$X=A^{-1}B$

$X=A^{-1}B$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-10}\begin{bmatrix}-1 & -2\\-3 & 4\end{bmatrix}\begin{bmatrix}8\\-1\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-10}\begin{bmatrix}-8+2\\-24+(-4)\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-10}\begin{bmatrix}-6\\-28\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{-6}{-10}\\\frac{-28}{-10}\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{3}{5}\\\frac{14}{5}\end{bmatrix}$$

$x=\frac{3}{5}, y=\frac{14}{5}$

Solution Set $=\left\{\left(\frac{3}{5},\frac{14}{5}\right)\right\}$

By Cramer's rule

$A=\begin{bmatrix}4 & 2\\3 & -1\end{bmatrix} B=\begin{bmatrix}8\\-1\end{bmatrix}$

$$|A|=\begin{vmatrix}4 & 2\\3 & -1\end{vmatrix}$$

$=(4)(-1)-(2)(3)$

$=-4-6$

$=-10$

Solution is possible because A is non singular matrix.

$$|A_x|=\begin{vmatrix}8 & 2\\-1 & -1\end{vmatrix}$$

$=(8)(-1)-(2)(-1)$

$=-8-(-2)$

$=-6$

$x=\frac{|A_x|}{|A|}$

$x=\frac{6}{10}$

$x=\frac{3}{5}$

$$|A_y|=\begin{bmatrix}4 & 8\\3 & -1\end{bmatrix}$$

$=(4)(-1)-(8)(3)$

$=-4-24$

$=-28$

$y=\frac{|A_y|}{|A|}$

$y=\frac{28}{10}$

$y=\frac{14}{5}$

Solution Set $=\left\{\left(\frac{3}{5},\frac{14}{5}\right)\right\}$

(iv) $3x-2y=-6$
$5x-2y=-10$

By Matrices Inversion Method

$$\begin{bmatrix}3 & -2\\5 & -2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-6\\-10\end{bmatrix}$$

Let $A=\begin{bmatrix}3 & -2\\3 & 2\end{bmatrix} X=\begin{bmatrix}x\\y\end{bmatrix} B=\begin{bmatrix}4\\6\end{bmatrix}$

$$|A|=\begin{vmatrix}3 & -2\\5 & -2\end{vmatrix}$$
$$=(3)(-2)-(-2)(5)$$
$$=-6-(-10)$$
$$=-6+10$$
$$=4$$
Solution is possible because A is non singular matrix.
$$AdjA=\begin{bmatrix}-2 & 2\\-5 & 3\end{bmatrix}$$
$$X=A^{-1}B$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}-2 & 2\\-5 & 3\end{bmatrix}\begin{bmatrix}-6\\-10\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}-2\times-6+2\times-10\\-5\times-63\times-10\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}12+(-20)\\30+(-30)\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}12-20\\30-30\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}-8\\0\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{-8}{4}\\\frac{0}{4}\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-2\\0\end{bmatrix}$$
$x=-2, y=0$

Solution Set $=\{(-2,0)\}$

By Cramer's rule
$$A=\begin{bmatrix}3 & -2\\5 & -2\end{bmatrix}, B=\begin{bmatrix}-6\\-10\end{bmatrix}$$
$$|A|=\begin{vmatrix}3 & -2\\5 & -2\end{vmatrix}$$
$$=(3)(-2)-(-2)(5)$$
$$=-6-(-10)$$
$$=-6+10$$
$$=4$$
Solution is possible because A is non singular matrix.
$$|A_x|=\begin{vmatrix}-6 & -2\\-10 & -2\end{vmatrix}$$
$$=(-6)(-2)-(-2)(-10)$$
$$=+12-(+20)$$
$$=12-20$$
$$=-8$$
$$|A_y|=\begin{vmatrix}3 & -6\\5 & -10\end{vmatrix}$$
$$=(3)(-10)-(-6)(5)$$
$$=-30-(-30)$$
$$=-30+30$$
$$=0$$
$$x=\frac{|A_x|}{|A|}$$
$$x=\frac{-8}{4}$$
$$x=-2$$
$$y=\frac{|A_y|}{|A|}$$
$$=\frac{0}{4}$$
$$y=0$$
Solution Set $=\{(-2,0)\}$

(v) $3x-2y=4$

$-6x+4y=7$

By Matrices Inversion Method
$$\begin{bmatrix}3 & -2\\-6 & 4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}4\\7\end{bmatrix}$$
$$A=\begin{bmatrix}3 & -2\\3 & 2\end{bmatrix} X=\begin{bmatrix}x\\y\end{bmatrix} B=\begin{bmatrix}4\\6\end{bmatrix}$$
$$|A|=\begin{vmatrix}3 & -2\\-6 & 4\end{vmatrix}$$
$$=(3)(4)-(-2)(-6)$$
$$=12-(+12)$$
$$=12-12$$
$$=0$$
Solution is not possible because A is singular matrix.

(vi) $4x+y=9$
$-3x-y=-5$

By Matrices Inversion Method

$$\begin{bmatrix}4 & 1\\ -3 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}9\\ -5\end{bmatrix}$$

Let $A=\begin{bmatrix}3 & -2\\ 3 & 2\end{bmatrix} X=\begin{bmatrix}x\\ y\end{bmatrix} B=\begin{bmatrix}4\\ 6\end{bmatrix}$

$$|A|=\begin{vmatrix}4 & 1\\ -3 & -1\end{vmatrix}$$

$$=(4)(-1)-(-1)(-3)$$

$$=-4+3$$

$$=-1$$

Solution is possible because $|A|$ is non singular

$$AdjA=\begin{vmatrix}-1 & -1\\ 3 & 4\end{vmatrix}$$

As we know that

$$X=A^{-1}B$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{-1}\begin{bmatrix}-1 & -1\\ 3 & 4\end{bmatrix}\begin{bmatrix}9\\ -5\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{-1}\begin{bmatrix}-9+5\\ 27+(-20)\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{-1}\begin{bmatrix}-4\\ 7\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}\frac{-4}{-1}\\ \frac{7}{-1}\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}4\\ -7\end{bmatrix}$$

$$x=4, y=-7$$

Solution Set$=\{(4,-7)\}$

By Cramer's rule

$$A=\begin{bmatrix}4 & 1\\ -3 & -1\end{bmatrix}, B=\begin{bmatrix}9\\ -5\end{bmatrix}$$

$$|A|=\begin{vmatrix}4 & 1\\ -3 & -1\end{vmatrix}$$

$$=(4)(-1)-(1)(-3)$$

$$=-4-(-3)$$

$$=-4+3$$

$$=-1$$

$$|A_x|=\begin{vmatrix}9 & 1\\ -5 & -1\end{vmatrix}$$

$$=(9)(-1)-(1)(-3)$$

$$=-9-(-5)$$

$$=-9+5$$

$$=-4$$

$$x=\frac{|A_x|}{|A|}$$

$$=\frac{-4}{-1}$$

$$x=4$$

$$|A_y|=\begin{bmatrix}4 & 9\\ -3 & -5\end{bmatrix}$$

$$=(4)(-5)-(9)(-3)$$

$$=-20-(-27)$$

$$=--20+27$$

$$=7$$

$$y=\frac{|A_y|}{|A|}$$

$$=\frac{7}{-1}$$

$$y=-7$$

Solution Set$=\{(4_7,-7)\}$

(vii) $2x-2y=4$
$-5x-2y=-10$

By Matrices Inversion Method

$$\begin{bmatrix}2 & -2\\ -5 & -2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}4\\ -10\end{bmatrix}$$

Let $A=\begin{bmatrix}3 & -2\\ 3 & 2\end{bmatrix} X=\begin{bmatrix}x\\ y\end{bmatrix} B=\begin{bmatrix}4\\ 6\end{bmatrix}$

$$|A|=\begin{vmatrix}2 & -2\\ -5 & -2\end{vmatrix}$$

$$=(2)(-2)-(-2)(-5)$$

$$=-4-(+10)$$

$$=-4-10$$

$$=-14$$

Solution is possible

$$AdjA = \begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}$$

As we know that

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|} \times AdjA \times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-14}\begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-14}\begin{bmatrix} -8+(-20) \\ 20+(-20) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-14}\begin{bmatrix} -8-20 \\ 20-20 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-14}\begin{bmatrix} -28 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-28}{-14} \\ \frac{0}{14} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$x = 2, y = 0$

Solution Set $= \{(2,0)\}$

By Cramer's rule

$$A = \begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix} B = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix}$$

$= (2)(-2) - (-2)(-5)$

$= -4 - (+10)$

$= -4 - 10$

$= -14$

Set is possible

$$|A_x| = \begin{bmatrix} 4 & -2 \\ -10 & -2 \end{bmatrix}$$

$= (4)(-2) - (-2)(-10)$

$= -8 - (+20)$

$= -8 - 20$

$= -28$

$$|A_y| = \begin{bmatrix} 2 & 4 \\ -5 & -10 \end{bmatrix}$$

$= (2)(-10) - (4)(-5)$

$= -20 - (-20)$

$= -20 + 20$

$= 0$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{-28}{-14}$$

$x = 2$

Solution Set $= \{(2,0)\}$

(viii) $3x - 4y = 4$

$x + 2y = 8$

By Matrices Inversion Method

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

Let $A = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$

$$|A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$$

$= (3)(2) - (-4)(1)$

$= 6 - (-4)$

$|A| = 6 + 4$

$= 10$

Solution is possible because A is non singular matrix.

$$AdjA = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$$

As we know that

$AX = B$

$X = A^{-1}B$

$$X = \frac{1}{|A|} \times AdjA \times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 2\times4+4\times8 \\ -1\times+3\times8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 8+32 \\ -4+24 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 40 \\ 20 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{40}{10} \\ \frac{20}{10} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$x = 4, y = 2$

Solution Set $= \{(4,2)\}$

By Cramer's rule

$$A = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} B = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$$

$= (3)(2) - (-4)(1)$

$= 6 - (-4)$

$= 6 + 4$

$= 10$

Solution is possible

$$|A_x| = \begin{bmatrix} 4 & -4 \\ 8 & 2 \end{bmatrix}$$

$$|A_x| = \begin{vmatrix} 4 & -4 \\ 8 & 2 \end{vmatrix}$$

$= (4)(2) - (-4)(8)$

$= 8 - (-32)$

$= 8 + 32$

$= 40$

$$|A_y| = \begin{bmatrix} 3 & 4 \\ 1 & 8 \end{bmatrix}$$

$$|A_y| = \begin{vmatrix} 3 & 4 \\ 1 & 8 \end{vmatrix}$$

$= (3)(8) - (4)(1)$

$= 24 - 4$

$= 20$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{40}{10}$$

$x = 4$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{20}{10}$$

$y = 2$

Solution Set $= \{(4,2)\}$

Q.2 The length of a rectangle is 4 times it width. The perimeter of the rectangle is 150cm. Find the dimensions of the rectangle.

Solution:

Let width of rectangle $= x$

Length of rectangle $= y$

According to 1st condition

$y = 4x$

$-4x + y = 0 \qquad \rightarrow ...(i)$

According to 2nd condition

2(length + Width)=Perimeter

$2(y + x) = 150$

$$y + x = \frac{150^{75}}{2}$$

$x + y = 75 \qquad \rightarrow ...(ii)$

$-4x + y = 0$

$x + y = 75$

Changing into matrix form

$$\begin{bmatrix} -4 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$X = A^{-1}B$

(By matrix inversion method)

Let $A = \begin{bmatrix} -4 & 1 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 0 \\ 75 \end{bmatrix}$

$$|A| = \begin{vmatrix} -4 & 1 \\ 1 & 1 \end{vmatrix}$$

$= (-4)(1) - (1)(1)$

$= -4 - 1$

$= -5$

$$AdjA = \begin{bmatrix} 1 & -1 \\ -1 & -4 \end{bmatrix}$$

As we know that

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|} \times AdjA \times B$$

$$= -\frac{1}{5} \begin{bmatrix} 1 & -1 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$= -\frac{1}{5} \begin{bmatrix} 0 - 75 \\ 0 - 300 \end{bmatrix}$$

$$= \frac{1}{-5} \begin{bmatrix} -75 \\ -300 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-75}{-5} \\ \frac{-300}{-5} \end{bmatrix}$$

$$= \begin{bmatrix} 15 \\ 60 \end{bmatrix}$$

$x = 15, y = 60$

Width of rectangle = x = 15cm

Length of rectangle = y = 60cm

By Cramer's rule

$$A = \begin{bmatrix} -4 & 1 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -4 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= (-4)(1) - (1)(1)$$

$$= -4 - 1$$

$$= -5$$

$$|A_x| = \begin{vmatrix} 0 & 1 \\ 75 & 1 \end{vmatrix}$$

$$= (0)(1) - (1)(75)$$

$$= 0 - 75$$

$$= -75$$

$$|A_y| = \begin{vmatrix} -4 & 0 \\ 1 & 75 \end{vmatrix}$$

$$= (-4)(75) - (0)(1)$$

$$= 0 - 300$$

$$= -300$$

$$x = \frac{|A_x|}{|A|}$$

$$= \frac{-75}{-5}$$

$$x = 15$$

$$y = \frac{|A_y|}{|A|}$$

$$= \frac{-300}{-5}$$

$$y = 60$$

Then

Width of rectangle = x = 15 cm

Length of rectangle = y = 60 cm

Q.3 Two sides of a rectangle differ by 3.5cm. Find the dimension of the rectangle if its perimeter is 67cm.

Solution:

Suppose Width of rectangle = x

Length of rectangle = y

According to 1st condition

$$y - x = 3.5$$

$$-x + y = 3.5 \qquad \rightarrow ...(i)$$

According to 2nd condition

$$2(L + B) = P$$

$$2(y + x) = 67$$

$$x + y = \frac{67}{2}$$

$$x + y = 33.5 \qquad \rightarrow (ii)$$

Changing into matrix form

$$\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3.5 \\ 33.5 \end{bmatrix}$$

(By matrix inversion method)

Let $A = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 3.5 \\ 33.5 \end{bmatrix}$

$$|A| = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= (-1)(1) - (1)(1)$$

$$= -1 - 1$$

$$= -2$$

$$AdjA = \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$$

As we know that

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|} \times AdjA \times B$$

$$= \frac{1}{-2} \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 3.5 \\ +33.5 \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} 1 \times 3.5 & 1 \times 33.5 \\ -1 \times 3.5 & 1 \times 33.5 \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} 3.5(-33.5) \\ -3.5(-33.5) \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} -30 \\ -37 \end{bmatrix}$$

$$=\begin{bmatrix}\frac{30}{2}\\ \frac{37}{2}\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}15\\ \frac{37}{2}\end{bmatrix}$$

$$x=15, y=\frac{37}{2}=18.5$$

By Cramer's rule

$$A=\begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix} B=\begin{bmatrix}3.5\\ 33.5\end{bmatrix}$$

$$|A|=\begin{vmatrix}-1 & 1\\ 1 & 1\end{vmatrix}$$

$$=(-1)(1)-(1)(1)$$

$$=-1-1$$

$$=-2$$

$$|A_x|=\begin{vmatrix}3.5 & 1\\ 33.5 & 1\end{vmatrix}$$

$$=(3.5)(1)-(1)(33.5)$$

$$=3.5-33.5$$

$$=-30$$

$$|A_y|=\begin{vmatrix}-1 & 3.5\\ 1 & 33.5\end{vmatrix}$$

$$=(-1)(33.5)-(3.5)(1)$$

$$=-33.5-3.5$$

$$=-37$$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{-30}{-2}$$

$$x=15$$

$$y=\frac{|A_y|}{|A|}$$

$$y=\frac{-37}{-2}$$

$$y=\frac{37}{2}=18.5$$

Width of rectangle =x=15cm
Length of rectangle =y=18.5cm

Q.4 The third angle of an isosceles Δ is $16°$ less than the sum of two equal angles. Find three angles of the triangle.

Solution:

Let each equal angles are x and third angle is y

According to condition $y=2x-16$

$$2x-y=16 \quad \text{(i)}$$

As we know that

$$x+x+y=180$$

$$2x+y=180 \quad \text{(ii)}$$

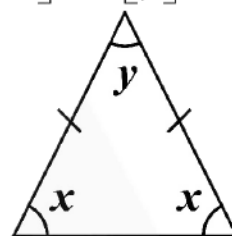
$$2x-y=16$$

$$2x+y=180$$

Changing into matrix form

$$\begin{bmatrix}2 & -1\\ 2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}16\\ 180\end{bmatrix}$$

$$X=A^{-1}B$$

Let $A=\begin{bmatrix}2 & -1\\ 2 & 1\end{bmatrix} X=\begin{bmatrix}x\\ y\end{bmatrix} B=\begin{bmatrix}16\\ 180\end{bmatrix}$

$X=A^{-1}B$

Where

$$A=\begin{bmatrix}2 & -1\\ 2 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}16\\ 180\end{bmatrix}$$

$$|A|=\begin{vmatrix}2 & -1\\ 2 & 1\end{vmatrix}$$

$$=2\times1-(-1)\times2$$

$$=2+2$$

$=4\neq0$ (None singular)

A^{-1} exist

$$AdjA=\begin{bmatrix}1 & 1\\ -2 & 2\end{bmatrix}$$

$$X=A^{-1}B$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}1 & 1\\ -2 & 2\end{bmatrix}\begin{bmatrix}16\\ 180\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}1\times16+1\times180\\ -2\times16+2\times180\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}16+180\\-32+360\end{bmatrix}$$

$$=\begin{bmatrix}196\\328\end{bmatrix}$$

$$=\begin{bmatrix}\frac{196}{4}\\\frac{328}{4}\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}49\\82\end{bmatrix}$$

$x = 49$

$y=82$

Cramer Rule

$$A=\begin{bmatrix}2 & -1\\2 & 1\end{bmatrix} B=\begin{bmatrix}16\\180\end{bmatrix}$$

$$|A|=\begin{vmatrix}2 & -1\\2 & 1\end{vmatrix}$$

$$=(2)(1)-(-1)(2)$$

$$=2-(-2)$$

$$=2+2$$

$$=4$$

$$|A_x|=\begin{bmatrix}16 & -1\\180 & 1\end{bmatrix}$$

$$=(16)(1)-(-1)(180)$$

$$=16+180$$

$$=196$$

$$|A_y|=\begin{bmatrix}2 & 16\\2 & 180\end{bmatrix}$$

$$=(2)(180)-(16)(2)$$

$$=360-32$$

$$=328$$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{196}{4}$$

$x = 49$

$$y=\frac{|A_y|}{|A|}$$

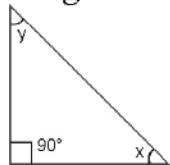
$$y=\frac{328}{4}$$

$y=82$

1st angle = x = 49° Ans
2nd angle= x = 49° Ans
3rd angle =y= 82° Ans

Q.5 One acute angle of a right triangle is $12°$ more than twice the other acute angle. Find the acute angles of the right triangle.

Solution:

Let one acute angle =x
And other acute angle =y
According to 1st condition

$x=2y+12$

$x-2y=12 \qquad \rightarrow(i)$

As we know

$x+y=90 \qquad \rightarrow(ii)$

By matrices inversion method
Changing into matrix form

$$\begin{bmatrix}1 & -2\\1 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}12\\90\end{bmatrix}$$

$X = A^{-1}B$

Let $A=\begin{bmatrix}1 & -2\\1 & 1\end{bmatrix} X=\begin{bmatrix}x\\y\end{bmatrix} B=\begin{bmatrix}12\\90\end{bmatrix}$

$$|A|=\begin{vmatrix}1 & -2\\1 & 1\end{vmatrix}$$

$$=(1)(1)-(-2)(1)$$

$$=1-(-2)$$

$= 3$ (Non singular)

$\therefore A^{-1}$ exists

$$AdjA=\begin{bmatrix}1 & 2\\-1 & 1\end{bmatrix}$$

As we know that

$X = A^{-1}B$ or

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$

$$=\frac{1}{3}\begin{bmatrix}1 & 2\\-1 & 1\end{bmatrix}\begin{bmatrix}12\\90\end{bmatrix}$$

$$=\frac{1}{3}\begin{bmatrix}12+180\\-12+90\end{bmatrix}$$

$$=\frac{1}{3}\begin{bmatrix}192\\78\end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{192}{3} \\ \frac{78}{3} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 64 \\ 26 \end{bmatrix}$$

$x = 64, y = 26$

Then

1st angle = x = 64°

2nd angle = y = 26°

By Cramer's rule

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 12 \\ 90 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix}$$

$= (1)(1) - (-2)(1)$

$= 1 - (-2)$

$= 1 + 2$

$= 3$

$$|A_x| = \begin{vmatrix} 12 & -2 \\ 90 & 1 \end{vmatrix}$$

$= (12)(1) - (-2)(90)$

$= 12 + 180$

$= 192$

$$|A_y| = \begin{vmatrix} 1 & 12 \\ 1 & 90 \end{vmatrix}$$

$= (90) - (12)$

$= 90 - 12$

$= 78$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{192}{3}$$

$x = 64$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{78}{3}$$

$y = 26$

1st angle = x = 64°

2nd angle = y = 26°

Q.6 Two cars that are 600 km apart are moving towards each other. Their speeds differ by 6 km per hour and the cars are 123 km apart after $4\frac{1}{2}$ hours. Find the speed of each car.

Solution:

Suppose speed of 1st car = x

Suppose speed of 2nd car = y

According to 1st condition

$x - y = 6 \qquad \rightarrow (\text{i})$

According to 2nd condition

Total distance = 600 km

Left distance = 123 km

Covered distance = total distance-left distance

Covered distance = 600-123

= 477 km

Total time = $4\frac{1}{2}$ hours =or $\frac{9}{2}$ hours

$$\text{Total Speed} = \frac{\text{Total Distance Covered}}{\text{Total Time Taken}}$$

$$x + y = \frac{477}{\frac{9}{2}} = 477 \div \frac{9}{2} = 477 \times \frac{2}{9}$$

$$x + y = \frac{\overset{53}{\cancel{477}} \times 2}{\cancel{9}}$$

$x + y = 106 \qquad \rightarrow (\text{ii})$

x - y = 6

x + y = 106

By matrices inversion method

Changing into matrix form

$$\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}6\\ 106\end{bmatrix}$$

$X = A^{-1}B$, where

Let $A=\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}6\\ 106\end{bmatrix}$

$$|A|=\begin{vmatrix}1 & -1\\ 1 & 1\end{vmatrix}$$

$$=(1)(1)-(-1)(1)$$

$$=1-(-1)$$

$$=1+1$$

$$=2$$

$$AdjA=\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{2}\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}6\\ 106\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}6+106.\\ -6+106\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}112\\ 100\end{bmatrix}$$

$$=\begin{bmatrix}\frac{112}{2}\\ \frac{100}{2}\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}56\\ 50\end{bmatrix}$$

$x=56, y=50$

Speed of 1st car = x= 56km/h

Speed of 2nd car = y= 50km/h

By Cramer's rule

$A=\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix} B=\begin{bmatrix}6\\ 106\end{bmatrix}$

$$|A|=\begin{vmatrix}1 & -1\\ 1 & 1\end{vmatrix}$$

$$=(1)(1)-(-1)(1)$$

$$=1-(-1)$$

$$=1+1$$

$$=2$$

$$|A_x|=\begin{vmatrix}6 & -1\\ 106 & 1\end{vmatrix}$$

$$=(6)(1)-(-1)(106)$$

$$=6-(-106)$$

$$=6+106$$

$$=112$$

$$|A_y|=\begin{vmatrix}1 & 6\\ 1 & 106\end{vmatrix}$$

$$=(106)(1)-(6)(1)$$

$$=106-6$$

$$=100$$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{112}{2}$$

$$x=56$$

$$y=\frac{|A_y|}{|A|}$$

$$y=\frac{100}{2}$$

$$y=50$$

Then

Speed of 1st car = x = 56km/h

Speed of 2nd car = y = 50km/h

Review Exercise 1

Q.1 Select the correct answer in each of the following.

(i) The order of matrix $\begin{bmatrix} 2 & 1 \end{bmatrix}$ **is….**

(a) 2-by-1 **(b)** 1-by-2
(c) 1-by-1 **(d)** 2-by-2

(ii) $\begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$ **is called …matrix.**

(a) Zero **(b)** Unit
(c) Scalar **(d)** Singular

(iii) Which is order of a square matrix?

(a) 2-by-2 **(b)** 1-by-2
(c) 2-by-1 **(d)** 3-by-2

(iv) Order of transpose of $\begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 3 & 2 \end{bmatrix}$ **is…**

(a) 3-by-2 **(b)** 2-by-3
(c) 1-by-3 **(d)** 3-by-1

(v) Adjoint of $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ **is…**

(a) $\begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix}$ **(d)** $\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$

(vi) Product of $\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ **is…**

(a) $[2x+y]$ **(b)** $[x-2y]$
(c) $[2x-y]$ **(d)** $[x+2y]$

(vii) If $\begin{vmatrix} 2 & 6 \\ 3 & x \end{vmatrix}$**=0, then** x **is equal to…**

(a) 9 **(b)** -6
(c) 6 **(d)** -9

(viii) If $X + \begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$**, then X is equal to…**

(a) $\begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$ **(b)** $\begin{bmatrix} 0 & 2 \\ 2 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$

ANSWER KEY

i	ii	iii	iv	v	vi	vii	viii
b	c	a	b	a	c	a	d

Q.2 Complete the follwoing:

(i) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is called ... matrix.

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is called ... matrix.

(iii) Additive inverse of $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$ is....

(iv) In matrix multiplication, in gereral, AB ...BA.

(v) Matrix A+B may be found if order of A and B is...

(vi) A matrix is called ... matrix if number of rows and columns are equal.

ANSWER KEY

i	ii	iii	iv	v	vi
Null	Unit	$\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$	$\neq$	Same	Square

Q.3 **If** $\begin{bmatrix} a+3 & 4 \\ 6 & b-1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$, **then find *a* and *b*.**

Solution: $\begin{bmatrix} a+3 & 4 \\ 6 & b-1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$

$a+3=-3$
$a=-3-3$
$a=-6$

$b-1=2$
$b=2+1$
$b=3$ **Ans**

Q.4 **If** $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}, B \begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$, **then find the following.**

(i) $2A+3B$

(ii) $-3A+2B$

(iii) $-3(A+2B)$

(iv) $\frac{2}{3}(2A-3B)$

Solution: (i)

$$2A+3B = 2\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 3\begin{bmatrix} 5 & 4 \\ -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 15 & -12 \\ -6 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+15 & 6-12 \\ 2-6 & 0-3 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & -6 \\ -4 & -3 \end{bmatrix}$$ **Ans**

Solution: (ii)

$$-3A+2B = -3\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 2\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -9 \\ -3 & 0 \end{bmatrix} + \begin{bmatrix} 10 & -8 \\ -4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -6+10 & -9-8 \\ -3-4 & 0-2 \end{bmatrix}$$

$$=\begin{bmatrix} 4 & -17 \\ -7 & -2 \end{bmatrix} \text{ Ans}$$

Solution: (iii)

$$-3(A+2B) = -3\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 2\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$$

$$= -3\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 10 & -8 \\ -4 & -2 \end{bmatrix}$$

$$= -3\begin{bmatrix} 2+10 & 3-8 \\ 1-4 & 0-2 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & -5 \\ -3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -36 & 15 \\ 9 & 6 \end{bmatrix} \text{ Ans}$$

Solution: (iv) $\frac{2}{3}(2A-3B)$

$$= \frac{2}{3}\left(2\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} - 3\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}\right)$$

$$= \frac{2}{3}\left(\begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 15 & -12 \\ -6 & -3 \end{bmatrix}\right)$$

$$= \frac{2}{3}\begin{bmatrix} 4-15 & 6-(-12) \\ 22(-6) & 0-(-3) \end{bmatrix}$$

$$= \frac{2}{3}\begin{bmatrix} -11 & 6+12 \\ 2+6 & 0+3 \end{bmatrix}$$

$$= \frac{2}{3}\begin{bmatrix} -11 & 18 \\ 8 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -11\times\frac{2}{3} & 18\times\frac{2}{3} \\ 8\times\frac{2}{3} & 3\times\frac{2}{3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-22}{3} & 12 \\ \frac{16}{3} & 2 \end{bmatrix} \text{ Ans}$$

Q.5 Find the value of X, if

$$\begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix} + X = \begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix}.$$

Solution: Given that

$$\begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix} + X = \begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix}$$

$$X = \begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4-2 & -2-1 \\ -1-3 & -2-(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 \\ -4 & -2+3 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} \text{ Ans}$$

Q.6 If $A = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}, B = \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$,

then prove that

(i) $AB \neq BA$

(ii) $A(BC) = (AB)C$

Solution: Given that

$$A = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}, B = \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$$

(i) $AB \neq BA$

L.H.S= $AB = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$

$$= \begin{bmatrix} 0\times(-3)+1\times5 & 0\times4+1\times(-2) \\ 2\times(-3)+(-3)\times5 & 2\times4+(-3)\times(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 0+5 & 0-2 \\ -6-15 & 8+6 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -2 \\ -21 & 14 \end{bmatrix} \quad \rightarrow (i)$$

R.H.S= $BA = \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}$

$$= \begin{bmatrix} -3(0)+4(2) & -3(1)+4(-3) \\ 5(0)+(-2)(2) & 5(1)+(-2)(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 0+8 & -3-12 \\ 0-4 & 5+6 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -15 \\ -4 & 11 \end{bmatrix} \quad \rightarrow (ii)$$

From (i) and (ii) , we get

$L.H.S \neq R.H.S$

$AB \neq BA$

Hence proved

(ii) $A(BC)=(AB)C$

Solution:

We cannot solve because matrix C is not given.

Q.7 **If** $A=\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$ **and** $B=\begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$, **then verify that**

(i) $(AB)^t = B^t A^t$

(ii) $(AB)^{-1} = B^{-1}A^{-1}$

Solution: Given that

$A=\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$ **and** $B=\begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$

(i) $(AB)^t = B^t A^t$

$AB=\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$

$=\begin{bmatrix} 3(2)+2(-3) & 3(4)+2(-5) \\ 1(2)+(-1)(-3) & 1(4)+(-1)(-5) \end{bmatrix}$

$=\begin{bmatrix} 6-6 & 12-10 \\ 2+3 & 4+5 \end{bmatrix}$

$=\begin{bmatrix} 0 & 2 \\ 5 & 9 \end{bmatrix}$

L.H.S$=(AB)^t=\begin{bmatrix} 0 & 2 \\ 5 & 9 \end{bmatrix}^t$

$=\begin{bmatrix} 0 & 5 \\ 2 & 9 \end{bmatrix} \quad \rightarrow (\text{i})$

$A^t=\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}^t$

$=\begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$

$B^t=\begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}^t=\begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix}$

R.H.S$=B^t A^t=\begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$

$=\begin{bmatrix} 2\times3+(-3)\times2 & 2(1)+(-3)(-1) \\ 4\times3+(-5)\times2 & 4(1)+(-5)(-1) \end{bmatrix}$

$=\begin{bmatrix} 6-6 & 2+3 \\ 12-10 & 4+5 \end{bmatrix}$

$=\begin{bmatrix} 0 & 5 \\ 2 & 9 \end{bmatrix} \quad \rightarrow (\text{ii})$

From equal (i) and (ii) we get

L.H.S=R.H.S

$(AB)^t = B^t A^t$

Hence proved

(ii) $(AB)^{-1} = B^{-1}A^{-1}$

$|AB|=\begin{vmatrix} 0 & 2 \\ 5 & 9 \end{vmatrix}$

$=0\times9-2\times5$

$=0-10$

$=-10$ (Non singular)

Inverse exists

$Adj(AB)=\begin{bmatrix} 9 & -2 \\ -5 & 0 \end{bmatrix}$

L.H.S$=(AB)^{-1}=\dfrac{1}{|AB|}Adj(AB)$

$=\dfrac{1}{-10}\begin{bmatrix} 9 & -2 \\ -5 & 0 \end{bmatrix}$

$=\begin{bmatrix} \dfrac{9}{10} & \dfrac{-2}{-10} \\ \dfrac{-5}{-10} & \dfrac{0}{-10} \end{bmatrix}$

$=\begin{bmatrix} \dfrac{-9}{10} & \dfrac{1}{5} \\ \dfrac{1}{2} & 0 \end{bmatrix} \quad \rightarrow (\text{i})$

$B=\begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$

$|B|=\begin{vmatrix} 2 & 4 \\ -3 & -5 \end{vmatrix}$

$=2(-5)-4\times(-3)$

$=-10+12$

$=2$ (non singular)

$\therefore B^{-1}$ exists

$$AdjB = \begin{bmatrix} -5 & -4 \\ 3 & 2 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} AdjB$$

$$= \frac{1}{2}\begin{bmatrix} -5 & -4 \\ 3 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 2 \\ 1 & -1 \end{vmatrix}$$

$$= 3(-1) - 2 \times 1$$

$$= -3 - 2$$

$= -5$ (non singular)

$\therefore A^{-1}$ exists

$$AdjA = \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times AdjA$$

$$= \frac{1}{-5}\begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

R.H.S$= B^{-1}A^{-1}$

$$= \left(\frac{1}{2}\begin{bmatrix} -5 & -4 \\ 3 & 2 \end{bmatrix}\right) \times \left(\frac{1}{-5}\begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}\right)$$

$$= \frac{1}{2}\left(-\frac{1}{5}\right)\begin{bmatrix} -5 & -4 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= -\frac{1}{10}\begin{bmatrix} -5(-1)+(-4)(-1) & -5(-2)+(-4)(3) \\ 3(-1)+2(-1) & 3(-2)+2(3) \end{bmatrix}$$

$$= -\frac{1}{10}\begin{bmatrix} 5+4 & 10-12 \\ -3-2 & -6+6 \end{bmatrix}$$

$$= -\frac{1}{10}\begin{bmatrix} 9 & -2 \\ -5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9}{-10} & \frac{-2}{-10} \\ \frac{-5}{-10} & \frac{0}{-10} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-9}{10} & \frac{1}{5} \\ \frac{1}{2} & 0 \end{bmatrix} \qquad \rightarrow (ii)$$

From equation (i) and (ii) we get

L.H.S =R.H.S

$$(AB)^{-1} = B^{-1}A^{-1}$$

Hence proved

Exercise 1.5

Q.1 Find the determinant of following matrices.

(i) $A=\begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$

Solution:

$A=\begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$

To write the determinant form

$|A|=\begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix}$

$= (-1)(0) - (2)(1)$
$= 0 - 2$
$= -2$

(ii) $B=\begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$

Solution:

$B=\begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$

To write in determinant form

$|B|=\begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix}$

$= (1)(-2) - (2)(3)$
$= -2-6$
$= -8$

(iii) $C=\begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$

Solution:

$C=\begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$

To write in determinant form

$|C|=\begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix}$

$= (3)(2) - (3)(2)$
$= 6-6$
$= 0$

(iv) $D=\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

Solution:

$D=\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

To write in determinant form

$|D|=\begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix}$

$= (3)(4) - (2)(1)$
$= 12-2$
$= 10$

Q.2 Find which of the following matrices are singular or non-singular?

(i) $A=\begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$

Solution:

$A=\begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$

To write in determinant form

$|A|=\begin{vmatrix} 3 & 6 \\ 2 & 4 \end{vmatrix}$

$|A| = (3)(4)-(2)(6)$

$|A| = 12-12$

$|A| = 0$

It is a singular matrix.

(ii) $B=\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

Solution:

$B=\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

To write in determinant form

$$|B|=\begin{vmatrix}4 & 1\\ 3 & 2\end{vmatrix}$$

$$|B|=(4)(2)-(3)(1)$$

$$|B|=8-3$$

$$|B|=5$$

It is non-singular matrix.

(iii) $C=\begin{bmatrix}7 & -9\\ 3 & 5\end{bmatrix}$

Solution:

$$C=\begin{bmatrix}7 & -9\\ 3 & 5\end{bmatrix}$$

To write in determinant form

$$|C|=\begin{vmatrix}7 & -9\\ 3 & 5\end{vmatrix}$$

$$|C|=(7)(5)-(3)(-9)$$

$$|C|=35+27$$

$$|C|=62$$

In not equal to zero so

It is non-singular matrix.

(iv) $D=\begin{bmatrix}5 & -10\\ -2 & 4\end{bmatrix}$

Solution:

$$D=\begin{bmatrix}5 & -10\\ -2 & 4\end{bmatrix}$$

To write in determinant form

$$|D|=\begin{vmatrix}5 & -10\\ -2 & 4\end{vmatrix}$$

$$|D|=(5)(4)-(-2)(-10)$$

$$|D|=20\text{-}20$$

$$|D|=0$$

It is singular matrix.

Q.3 Find the multiplicative inverse of each

(i) $A=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$

Solution:

$$A=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$$

To write in determinant form

$$|A|=\begin{vmatrix}-1 & 3\\ 2 & 0\end{vmatrix}$$

$$|A|=(-1)(0)-(2)(3)$$

$$|A|=0-6$$

$$|A|=-6\neq 0 \text{ (Non-Singular)}$$

A^{-1}exists

To write in Adj A

$$AdjA=\begin{bmatrix}0 & -3\\ -2 & -1\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\times AdjA$$

Putting the values

$$A^{-1}=\frac{1}{-6}\times\begin{bmatrix}0 & -3\\ -2 & -1\end{bmatrix}=\begin{bmatrix}0\times\frac{1}{-6} & -3\times\frac{1}{-6}\\ -2\times\frac{1}{-6} & -1\times\frac{1}{-6}\end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}\frac{0}{-6} & \frac{+3}{+6}\\ \frac{+2}{+6} & \frac{+1}{+6}\end{bmatrix}$$

$$=\begin{bmatrix}0 & \frac{1}{2}\\ \frac{1}{3} & \frac{1}{6}\end{bmatrix}$$

(ii) $B=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$

Solution:

$$B=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$$

To write in determinant form

$$|B|=\begin{vmatrix}1 & 2\\ -3 & -5\end{vmatrix}$$

$$|B|=(-1)(-5)-(-3)(2)$$

$$|B|=-5+6$$

$|B|=1\neq 0$ (Non-Singular)

B^{-1} exists

$$AdjB=\begin{bmatrix}-5 & -2\\ 3 & 1\end{bmatrix}$$

$$B^{-1}=\frac{1}{|B|}\times AdjB$$

Putting the values

$$B^{-1}=\frac{1}{1}\times\begin{bmatrix}-5 & -2\\ 3 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{2}\times-5 & \frac{1}{1}\times-2\\ \frac{1}{1}\times 3 & \frac{1}{1}\times 1\end{bmatrix}$$

$$=\begin{bmatrix}\frac{-5}{1} & \frac{-2}{1}\\ \frac{3}{1} & \frac{1}{1}\end{bmatrix}$$

$$=\begin{bmatrix}-5 & -2\\ 3 & 1\end{bmatrix}$$

(iii) $C=\begin{bmatrix}-2 & 6\\ 3 & -9\end{bmatrix}$

Solution:

To write in determinant form

$$|C|=\begin{vmatrix}-2 & 6\\ 3 & -9\end{vmatrix}$$

$$|C|=(-2)(-9)-(3)(6)$$

$$|C|=18-18$$

$|C|=0$ Singular

C^{-1} Does not exists.

(iv) $D=\begin{bmatrix}\frac{1}{2} & \frac{3}{4}\\ 1 & 2\end{bmatrix}$

Solution:

To write in determinant form

$$D=\begin{bmatrix}\frac{1}{2} & \frac{3}{4}\\ 1 & 2\end{bmatrix}$$

$$|D|=\begin{vmatrix}\frac{1}{2} & \frac{3}{4}\\ 1 & 2\end{vmatrix}=\frac{1}{2}\times 2-\frac{3}{4}\times 1$$

$$=1-\frac{3}{4}$$

$$=\frac{4-3}{4}$$

$|D|=\frac{1}{4}\neq 0$ (Non Singular)

D^{-1} exists

$$AdjD=\begin{bmatrix}2 & \frac{-3}{4}\\ -1 & \frac{1}{2}\end{bmatrix}$$

$$D^{-1}=\frac{1}{|\text{D}|}\times AdjD$$

By putting the values

$$=\frac{1}{\frac{1}{4}}\begin{bmatrix}2 & -\frac{3}{4}\\ -1 & \frac{1}{2}\end{bmatrix}$$

$$=1\div\frac{1}{4}\begin{bmatrix}2 & \frac{-3}{4}\\ -1 & \frac{1}{2}\end{bmatrix}$$

$$=1\times\frac{4}{1}\begin{bmatrix}2 & \frac{-3}{2}\\ -1 & \frac{1}{2}\end{bmatrix}$$

$$=4\begin{bmatrix}2 & -\frac{3}{4}\\ -1 & \frac{1}{2}\end{bmatrix}$$

$$=\begin{bmatrix}8 & -3\\ -4 & 2\end{bmatrix}$$

Q.4 **If** $A=\begin{bmatrix}1 & 2\\ 4 & 6\end{bmatrix}$ **and** $B=\begin{bmatrix}3 & -1\\ 2 & -2\end{bmatrix}$,

then

Then verify that

(i) A(AdjA)=(AdjA)A=(detA)I

Solution: A(AdjA)=(AdjA)A=(detA)I

$$A=\begin{bmatrix}1 & 2\\4 & 6\end{bmatrix}$$

$$\text{AdjA}=\begin{bmatrix}6 & -2\\-4 & 1\end{bmatrix}$$

$$\text{detA}=\begin{bmatrix}1 & 2\\4 & 6\end{bmatrix}$$

$$=1\times 6 \quad 2\times 4$$

$$=6-8$$

$$=-2$$

$$A(AdjA)=\begin{bmatrix}1 & 2\\4 & 6\end{bmatrix}\begin{bmatrix}6 & -2\\-4 & 1\end{bmatrix}$$

$$=\begin{bmatrix}6-8 & (-2)+2\\24-4 & -8+6\end{bmatrix}$$

$$\text{A (Adj A)}=\begin{bmatrix}-2 & 0\\0 & -2\end{bmatrix} \text{__________ (i)}$$

$$(AdjA)A=\begin{bmatrix}6 & -2\\-4 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\4 & 6\end{bmatrix}$$

$$(AdjA)A=\begin{bmatrix}(6)\times 12\times 4 & (6)\times 2+(-2)\times 6\\(-4)\times 1+(1)(4) & (-4)(2)+(1)(6)\end{bmatrix}$$

$$=\begin{bmatrix}6-8 & 12-12\\-4+4 & -8+6\end{bmatrix}$$

$$\text{(AdjA)A}=\begin{bmatrix}-2 & 0\\0 & -2\end{bmatrix} \text{__________ (ii)}$$

$$(\det A)\text{I}=-2\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}-2\times 1 & 0\times 2\\-2\times 0 & 1\times -2\end{bmatrix}$$

$$\text{(detA)I}=\begin{bmatrix}-2 & 0\\0 & -2\end{bmatrix} \text{__________ (iii)}$$

Hence proved

From eq (i), (ii) and (iii)

A(AdjA)=(AdjA)A=(detA)I

(ii) $BB^{-1} = I = B^{-1}B$

Solution: $BB^{-1} = I = B^{-1}B$

To write in determinant form

$$|B|=\begin{vmatrix}3 & -1\\2 & -2\end{vmatrix}$$

$$=-6-(-2)$$

$$=-6+2$$

$$=-4\neq 0 \text{ (None singular)}$$

$=B^{-1}$ exists.

To write in AdjB

$$AdjB=\begin{bmatrix}-2 & 1\\-2 & 3\end{bmatrix}$$

$$B^{-1}\frac{1}{|B|}Adj$$

$$B^{-1}=\frac{1}{-4}\begin{bmatrix}-2 & 1\\-2 & 3\end{bmatrix}$$

New

$$BB^{-1}=\begin{bmatrix}3 & -1\\2 & -2\end{bmatrix}\times\frac{1}{-4}\begin{bmatrix}-2 & 1\\-2 & 3\end{bmatrix}$$

$$=\frac{1}{-4}\begin{bmatrix}3 & -1\\2 & -2\end{bmatrix}\begin{bmatrix}-2 & 1\\-2 & 3\end{bmatrix}$$

$$=-\frac{1}{4}\begin{bmatrix}-6+2 & 3-3\\-4+4 & 2-6\end{bmatrix}$$

$$=\frac{1}{-4}\begin{bmatrix}-4 & 0\\0 & -4\end{bmatrix}$$

$$=\begin{bmatrix}\frac{4}{4} & 0\\0 & \frac{4}{4}\end{bmatrix}$$

$$BB^{-1}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=I$$

and

$$B^{-1}B=\frac{1}{-4}\begin{bmatrix}-2 & 1\\-2 & 3\end{bmatrix}\begin{bmatrix}3 & -1\\2 & -2\end{bmatrix}$$

$$=\frac{1}{-4}\begin{bmatrix}-6+2 & 2-2\\-6+6 & 2-6\end{bmatrix}$$

$$=\frac{1}{-4}\begin{bmatrix}-4 & 0\\0 & -4\end{bmatrix}$$

$$=\begin{bmatrix} \frac{-4}{-4} & 0 \\ 0 & \frac{-4}{-4} \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$B^{-1}B = I$

From (i) and (ii)

$BB^{-1}=I=B^{-1}B$

Hence proved

Q.5 Determine whether the given matrices are multiplicative inverses of each other.

(i) $\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$ and $\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$

Solution: $\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$ and $\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$

$$\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$$

$$=\begin{bmatrix} 21+(-20) & -15+15 \\ 28+(-28) & -20+21 \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The given matrices are multiplicative inverse of each other.

(ii) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

Solution: $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$$

$$=\begin{bmatrix} -3+4 & 2+(-2) \\ -6+6 & 4+(-3) \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Given matrices are multiplicative inverse of each other

Q.6

(i) $(AB)^{-1} = B^{-1}A^{-1}$

Solution: $(AB)^{-1} = B^{-1}A^{-1}$

$$A=\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}, B=\begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$$

$$AB=\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$$

$$=\begin{bmatrix} 4\times(-4)+0(1) & 4\times(-2)+0(-1) \\ -1\times(-4)+2(1) & -1\times(-2)+2(-1) \end{bmatrix}$$

$$=\begin{bmatrix} -16+0 & -8+0 \\ 4+2 & 2+(-2) \end{bmatrix}$$

$$=\begin{bmatrix} -16 & -8 \\ 6 & 0 \end{bmatrix}$$

To write in determinant form

$$|AB|=\begin{vmatrix} -16 & -8 \\ 6 & 0 \end{vmatrix}$$

$$|AB|=0-(-48)$$

$$|AB|=48$$

To write in Adj (AB)

$$\text{Adj (AB)}=\begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$$

$$(AB)^{-1}=\frac{1}{|AB|}\times AdjAB$$

$$=\frac{1}{48}\times\begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$$

$$=\begin{bmatrix} \frac{0}{48} & \frac{8}{48} \\ \frac{-6}{48} & \frac{-16}{48} \end{bmatrix}$$

$$=\begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix}$$

To solve R.H. S

To write in determinant form

$$|B|=\begin{vmatrix} -4 & -2 \\ 1 & -1 \end{vmatrix}$$

$$|B| = 4-(-2)$$
$$|B| = 4+2$$
$$|B| = 6$$

To write in Adj B

$$AdjB = \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \times AdjB$$

By putting value

$$B^{-1} = \frac{1}{6} \times \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$$

To write in determinant form

$$|A| = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix}$$

$$|A| = 8-(-0)$$
$$|A| = 8$$

To write in Adj A

$$AdjA = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times AdjA$$

$$= \frac{1}{8} \times \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

To solve R.H.S

$$B^{-1}A^{-1} = \frac{1}{6}\begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} \times \frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$= \frac{1}{6} \times \frac{1}{8}\begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$= \frac{1}{48}\begin{bmatrix} -2+2 & 0+8 \\ -2-4 & 0-16 \end{bmatrix}$$

$$= \frac{1}{48}\begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{0}{48} & \frac{8}{48} \\ \frac{-6}{48} & \frac{-16}{48} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 0 \times \frac{1}{48} & 8 \times \frac{1}{48} \\ -6 \times \frac{1}{48} & -16 \times \frac{1}{48} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix}$$

Hence proved

L.H.S = R.H.S

Unit 1: Matrices and Determinants

Overview

Matrix:

A rectangular array of real numbers enclosed within brackets is said to form matrix.

Rows of a Matrix:

In matrix, the entries presented in horizontal way are called rows.

i.e. $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 8 \\ 7 & 1 & 5 \end{bmatrix} \begin{matrix} \to R_1 \\ \to R_2 \\ \to R_3 \end{matrix}$.

Columns of a Matrix:

In matrix, all the entries presented in vertical way are called columns of matrix.

$$\begin{matrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 8 \\ 7 & 1 & 5 \end{bmatrix} \\ \downarrow \quad \downarrow \quad \downarrow \\ C_1 \quad C_2 \quad C_3 \end{matrix}$$

Order of a Matrix:

The number of rows and columns in a matrix specifies its order. If a matrix M has m rows and n columns then M is said to be of order, $m-by-n$.

i.e. $\begin{bmatrix} 0 & 8 & 0 \\ 0 & 4 & 8 \\ 7 & 1 & 5 \end{bmatrix}$ the order matrix is $3-by-3$

Equal Matrix's:

Let A and B be two matrices. Then A is said to be equal to B, and denoted by $A=B$, if and only if;

(i) The order of A = the order of B

(ii) Their corresponding entries are equal.

i.e. $A=\begin{bmatrix} 1 & 3 \\ -4 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 2+1 \\ -4 & 4-2 \end{bmatrix}$ are equal matrices.

Rectangular Matrix:

A matrix M is called rectangular if, the number of rows of M is not equal to the number of columns of M.

e.g., $B=\begin{bmatrix} a & b & c \\ d & e & d \end{bmatrix}$.

Square Matrix:

A matrix is called a square matrix if its number of rows is equal to its number of columns.

i.e, $A=\begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$

Null or Zero Matrix:

A matrix M is called a null or zero matrix if each of its entries is 0.

e.g. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

Transpose of a Matrix:

A matrix obtained by interchanging the rows into columns or columns into rows of a matrix is called transpose of that matrix.

Negative of a Matrix:

Let A be matrix. Then its negative, $-A$ is obtained by changing the signs of all the entries of A,

i.e. If $A=\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$, then $-A=\begin{bmatrix} -1 & 2 \\ -3 & -4 \end{bmatrix}$

Symmetric Matrix:

A square matrix is symmetric if it is equal to its transpose i.e., matrix A is symmetric if $A^t = A$.

Skew-Symmetric Matrix:

A square matrix A is said to be skew-symmetric if $A^t = -A$.

Diagonal Matrix:

A square matrix A is called a diagonal matrix if at least any one of the entries of its diagonal is not zero and non-diagonal entries must all be zero.

i.e. $A=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Scalar Matrix:

A diagonal matrix is called a scalar matrix, if all the diagonal entries are same and non-zero. For example $\begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$ where k is a constant $\neq 0,1$

Identity Matrix:

A diagonal matrix is called identity (unit) matrix if all diagonal entries are 1 and it is denoted by I.

e.g., $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ is a 3-by-3 identity matrix.

Addition of Matrices:

Let A and B be any two matrices with real number entries. The matrices A and B are conformable for addition, if they have the same order.

Subtraction of Matrices:

If A and B are two matrices of same order then subtraction of matrix B from matrix A is obtained by subtracting the entries of matrix B from the corresponding entries of matrix A and it is denoted by $A-B$.

Multiplication of Matrices:

Two matrices A and B conformable for multiplication, giving product AB if the number of columns of A is equal to the number of rows of B.

Determinant of a 2-by-2 Matrix:

Let $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ be a 2-by-2 square matrix. The determinant of A, denoted by **det A** or $|A|$ is defined as.

$$|A| = \det A = \det\begin{bmatrix}a & b\\ c & d\end{bmatrix} = \begin{vmatrix}a & b\\ c & d\end{vmatrix} = ad-bc = \lambda \in R$$

Singular Matrix:

A square matrix A is called singular if the determinant of A is equal to zero.

For example, $A=\begin{bmatrix}1 & 2\\ 0 & 0\end{bmatrix}$ is a singular matrix, since det $A = 1\times 0 - 0\times 2 = 0$.

Non-Singular Matrix:

A square matrix A is called non-singular if the determinant of A is not equal to zero.

For example $A=\begin{bmatrix}1 & 1\\ 0 & 2\end{bmatrix}$ is non-singular, since $\det A = 1\times 2 - 0\times 1 = 2 \neq 0$.

Adjoint of a Matrix:

Adjoint of a square matrix $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ is obtained by interchanging the diagonal entries and changing the sign of other entries. Adjoint of matrix A is denoted as Adj A.

i.e. Adj A $=\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$

Exercise 2.1

Q.1 Identity which of the following are rational and irrational numbers?

(i)	$\sqrt{3}$	Irrational number
(ii)	$\frac{1}{6}$	Rational number
(iii)	π	Irrational number
(iv)	$\frac{15}{2}$	Rational number
(v)	7.25	Rational number
(vi)	$\sqrt{29}$	Irrational number

Q.2 Convert the following fractions into decimal fractions.

(i) $\frac{17}{25}$

Solution: $\frac{17}{25}$

0.68
25) 170
−150
200
−200
0

$\frac{17}{25} = 0.68$ **Ans**

(ii) $\frac{19}{4}$

Solution: $\frac{19}{4}$

4.75
4) 19.000
16
30
28
20
20
0

$= \frac{19}{4}$

$= 4.75$ **Ans**

(iii) $\frac{57}{8}$

Solution: $\frac{57}{8}$

7.125
8) 57
−56
10
8
20
−16
40
40
0

$= \frac{57}{8}$

$= 7.125$ **Ans**

(iv) $\frac{205}{18}$

Solution: $\frac{205}{18}$

11.388
18) 205.000
25

$\frac{18}{70}$
-54
160
-144
160
-144
16

$\frac{208}{18}$
$= 11.3888$
$= 11.3889$ **Ans**

(v) $\frac{5}{8}$

Solution: $\frac{5}{8}$

$$\begin{array}{r} .625 \\ 8\overline{)5.000} \\ 48 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

$\frac{5}{8}$
$= 0.625$ **Ans**

(vi) $\frac{25}{38}$

Solution: $\frac{25}{38}$

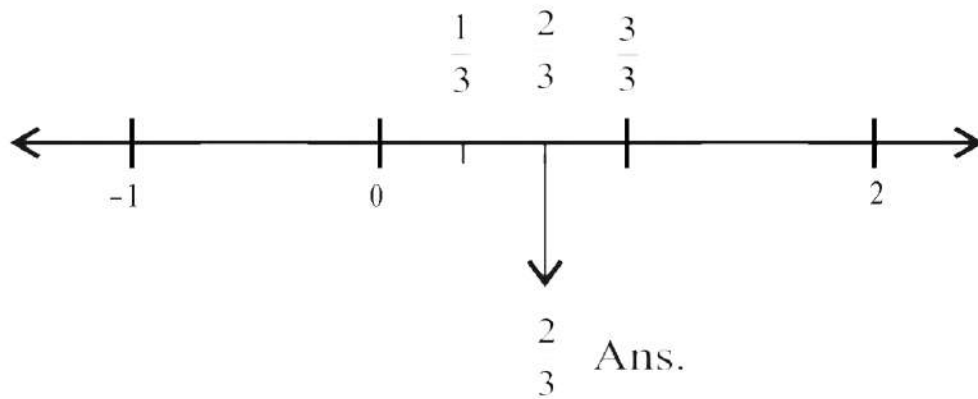
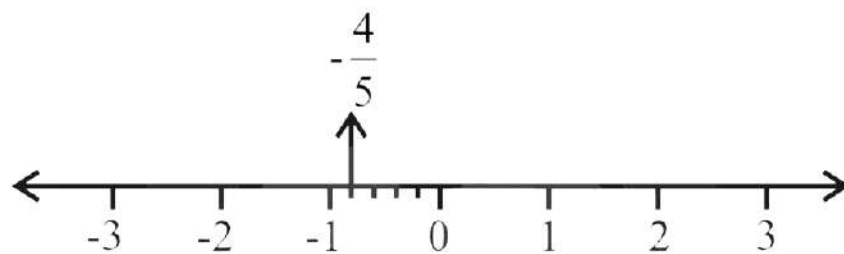
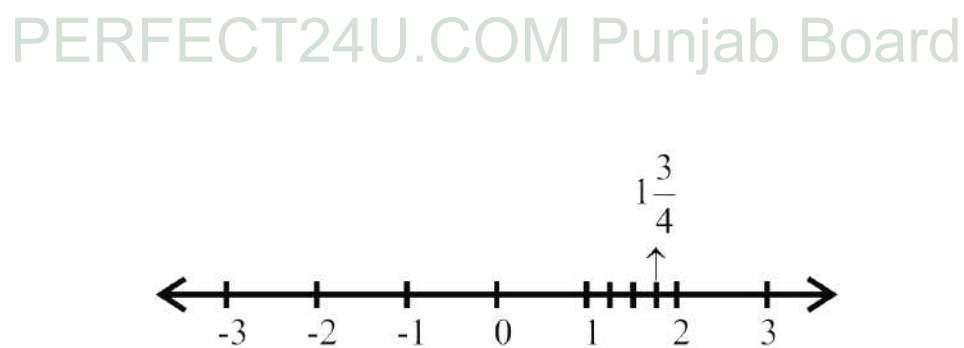
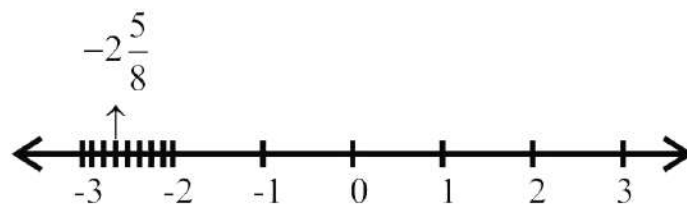
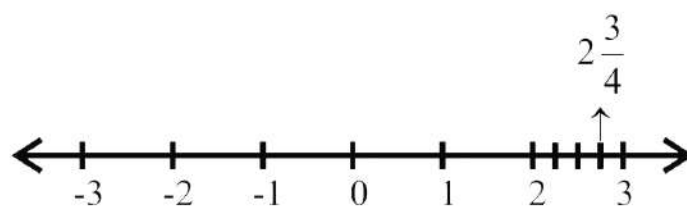
$$\begin{array}{r} 0.65789... \\ 38\overline{)250} \\ -228 \\ \hline 220 \\ -190 \\ \hline 300 \\ -266 \\ \hline 340 \\ -304 \\ \hline 360 \\ -342 \\ \hline 18 \end{array}$$

$\frac{25}{38}$
$= 0.65789$ **Ans**

Q.3 Which of the following statements are true and which are false?

(i)	$\frac{2}{3}$ is an irrational number.	False
(ii)	π is an irrational number.	True
(iii)	$\frac{1}{9}$ is a terminating fraction.	False
(iv)	$\frac{3}{4}$ is a terminating fraction.	True
(v)	$\frac{4}{5}$ is a recurring fraction.	False

Q.4 Represent the following numbers on the number line.

(i) $\frac{2}{3}$

(ii) $-\frac{4}{5}$

(iii) $1\frac{3}{4}$

(iv) $-2\frac{5}{8}$

(v) $2\frac{3}{4}$

(vi) $\sqrt{5}$

By Pythagoras theorem

$$(\text{Hypoteneus})^2 = (\text{Base})^2 + (\text{Perpencicular})^2$$

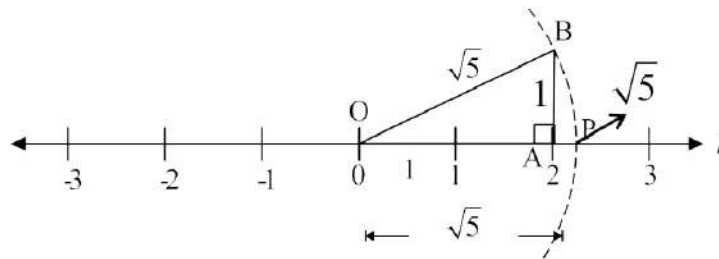
$$(\overline{OB})^2 = (2)^2 + (1)^2$$

$$(\overline{OB})^2 = 4 + 1$$

$$(\overline{OB})^2 = 5$$

Taking square root on both sides

$$\sqrt{(\overline{OB})^2} = \sqrt{5}$$

$$\overline{OB} = \sqrt{5}$$

Q.5 Give a rational number between $\frac{3}{4}$ and $\frac{5}{9}$

Solution:

Required No between

$$\frac{3}{4} \text{ and } \frac{5}{9}$$

$$= \left[\frac{3}{4} + \frac{5}{9}\right] \div 2$$

$$= \left[\frac{27 + 20}{36}\right] \div 2$$

$$= \left[\frac{47}{36}\right] \div 2$$

$$= \frac{47}{36} \times \frac{1}{2}$$

$$= \frac{47}{72} \textbf{ Ans}$$

Q.6 Express the following recurring decimals as the rational number $\frac{p}{q}$ where p, q are integer and $q \neq 0$.

(i) $0.\overline{5}$

Solution:

$$x = 0.\overline{5}$$

$$x = 0.555...$$

$$10 \times x = 10 \times 0.555...$$

$$10x = 5.555...$$

$$10x = 5 + 0.555...$$

$$10x = 5 + x$$

$$10x - x = 5$$

$$9x = 5$$

$$x = \frac{5}{9}$$

$$\therefore 0.\overline{5} = \frac{5}{9} \textbf{ Ans}$$

(ii) $0.\overline{13}$

Solutions:

Suppose

$x = 0.\overline{13}$

$x = 0.131313...$

$100^{x} x = 100 \times 1.131313...$

$100x = 13.1313...$

$100x = 13 + 0.1313...$

$100x = 13 + x$

$100x - x = 13$

$99x = 13$

$x = \frac{13}{99}$

$\therefore 0.\overline{13} = \frac{13}{99}$ **Ans**

(iii) $0.\overline{67}$

Solutions:

Suppose

$x = 0.\overline{67}$

$x = 0.676767...$

$100 \times x = 100 \times 0.676767...$

$100x = 67.6767...$

$100x = 67 + 0.6767...$

$100x = 67 + x$

$100x - x = 67$

$99x = 67$

$x = \frac{67}{99}$

$\therefore 0.\overline{67} = \frac{67}{99}$ **Ans**

Exercise 2.2

Q.1 Identify the property used in the following.

(i)	$a+b=b+a$	Commutative Property *w.r.t* addition
(ii)	$(ab)c=a(bc)$	Associative Property *w.r.t* multiplication
(iii)	$7\times1=7$	Multiplicative Identity
(iv)	$x>y$ or $x=y$ or $x<y$	Trichotomy
(v)	$ab=ba$	Commutative *w.r.t* multiplication
(vi)	$a+c=b+c=a+b$	Cancellation Property of addition
(vii)	$5+(-5)=0$	Additive Inverse
(viii)	$7\times\frac{1}{7}=1$	Multiplicative inverse
(ix)	$a>b\Rightarrow ac>bc\,(c>0)$	Multiplicative property

Q.2 Fill in the following blanks by stating the properties of real numbers used.

$3x+3(y-x)$

$=3x+3y-3x,\ldots$ Distributive property

$=3x-3x+3y,\ldots$ Commutative

$=0+3y,\ldots$ Additive Inverse

$=3y,\ldots$ Additive identity

Q.3 Give the name of property used in the following.

(i)	$\sqrt{24}+0=\sqrt{24}$	Additive Identity
(ii)	$-\frac{2}{3}\left[5+\frac{7}{2}\right]=\left[-\frac{2}{3}\right](5)+\left[-\frac{2}{3}\right]\left[\frac{7}{2}\right]$	Distributive Property
(iii)	$\pi+(-\pi)=0$	Additive Inverse
(iv)	$\sqrt{3}.\sqrt{3}$ is a real number.	Closure property w.r.t x.
(v)	$\left[-\frac{5}{8}\right]\left[-\frac{8}{5}\right]=1$	Multiplicative Inverse.

Exercise 2.3

Q.1 Write each radical expression in exponential notation and each exponential expression in radical notation. Do not simplify.

(i) $\sqrt[3]{-64}$

$=(-64)^{\frac{1}{3}}$

(ii) $2^{\frac{3}{5}}$

$=\sqrt[5]{2^3}$

(iii) $-7^{\frac{1}{3}}$

$-\sqrt[3]{7}$

(iv) $y^{-\frac{2}{3}}$

$=\sqrt[3]{y^{-2}}$

Q.2 Tell whether the following statements are true or false?

(i) $5^{\frac{1}{5}}=\sqrt{5}$ **False**

(ii) $2^{\frac{2}{3}}=\sqrt[3]{4}$ **True**

(iii) $\sqrt{49}=\sqrt{7}$ **False**

(iv) $\sqrt[3]{x^{27}}=x^3$ **False**

Q.3 Simplify the following radical expression.

(i) $\sqrt[3]{-125}$

Solution:

$=\sqrt[3]{-125}$

$=\sqrt[3]{-5\times-5\times-5}$

$=\sqrt[3]{(-5)^3}$

= - 5 **Ans**

(ii) $\sqrt[4]{32}$

Solutions:

$$= \sqrt[4]{32}$$
$$= \sqrt[4]{2 \times 2 \times 2 \times 2 \times 2}$$
$$= \sqrt[4]{2^4 \times 2}$$
$$= \sqrt[4]{2^4} \times \sqrt[4]{2}$$
$$= 2\sqrt[4]{2} \textbf{ Ans}$$

(iii) $\sqrt[5]{\frac{3}{32}}$

Solution:

$$= \sqrt[5]{\frac{3}{32}}$$
$$= \frac{\sqrt[5]{3}}{\sqrt[5]{32}}$$
$$= \frac{\sqrt[5]{3}}{\sqrt[5]{2 \times 2 \times 2 \times 2 \times 2}}$$
$$= \frac{\sqrt[5]{3}}{\sqrt[5]{(2)^5}}$$
$$= \frac{\sqrt[5]{3}}{2} \textbf{ Ans}$$

(iv) $\sqrt[3]{-\frac{8}{27}}$

Solution:

$$= \sqrt[3]{-\frac{8}{27}}$$
$$= \sqrt[3]{\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)}$$
$$= \sqrt[3]{\left(-\frac{2}{3}\right)^3}$$
$$= -\frac{2}{3} \textbf{ Ans}$$

Exercise 2.4

Q.1 Use laws of exponents to simplify.

(i) $\dfrac{(243)^{-\frac{2}{3}}(32)^{-\frac{1}{5}}}{\sqrt{(196)^{-1}}}$

Solution: $\dfrac{(243)^{-\frac{2}{3}}(32)^{-\frac{1}{5}}}{\sqrt{(196)^{-1}}}$

$$=\frac{(243)^{-\frac{2}{3}}(32)^{-\frac{1}{5}}}{\sqrt{(196)^{-1}}}$$

$$=\frac{(3^5)^{-\frac{2}{3}}\times(2^5)^{-\frac{1}{5}}}{\sqrt{\left[(14)^2\right]^{-1}}}$$

$$=\frac{(3)^{-\frac{10}{3}}\times 2^{-1}}{\sqrt{\left[(14)^{-1}\right]^2}}$$

$$=\frac{(3)^{-\frac{10}{3}}\times 2^{-1}}{(14)^{-1}}$$

$$=\frac{14}{(3)^{\frac{10}{3}}\times 2}$$

$$=\frac{7}{3^{\frac{10}{3}}}$$

$$=\frac{7}{\sqrt[3]{3^{10}}}$$

$$=\frac{7}{\sqrt[3]{\overline{3\times3\times3}\times\overline{3\times3\times3}\times\overline{3\times3\times3}\times3}}$$

$$=\frac{7}{\sqrt[3]{3^3\times3^3\times3^3\times3}}$$

$$=\frac{7}{\sqrt[3]{3^3}\times\sqrt[3]{3^3}\times\sqrt[3]{3^3}\times\sqrt[3]{3}}$$

$$=\frac{7}{3\times3\times3\times\sqrt[3]{3}}$$

$$=\frac{7}{27\sqrt[3]{3}}\ \textbf{Ans}$$

(ii) $(2x^5y^{-4})(-8x^{-3}y^2)$

Solution: $(2x^5y^{-4})(-8x^{-3}y^2)$

$$=-16x^{5-3}y^{-4+2}$$

$$=-16x^2y^{-2}$$

$$=\frac{-16x^2}{y^2}\ \textbf{Ans}$$

(iii) $\left[\dfrac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0}\right]^{-3}$

Solution: $\left[\dfrac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0}\right]^{-3}$

$$=\left[x^{-2-4}y^{-1+3}z^{-4-0}\right]^{-3}$$

$$=\left(x^{-6}y^{+2}z^{-4}\right)^{-3}$$

$$=\left(x^{-6}\right)^{-3}\left(y^2\right)^{-3}\left(y^{-4}\right)^{-3}$$

$$=x^{18}y^{-6}z^{12}$$

$$=\frac{x^{18}z^{12}}{y^6}\ \text{Ans}$$

(iv) $\dfrac{(81)^n.3^5-(3)^{4n-1}(243)}{(9^{2n})(3^3)}$

Solution: $\dfrac{(81)^n.3^5-(3)^{4n-1}(243)}{(9^{2n})(3^3)}$

$$=\frac{(3^4)^n.3^5-3^{4n}.3^{-1}.3^5}{(3^2)^{2n}.3^3}$$

$$=\frac{3^{4m}.3^5-3^{4n}.3^{-1+5}}{3^{4n}.3^3}$$

$$=\frac{3^{4n}.3^5-3^{4n}.3^4}{3^{4n}.3^3}$$

$$= \frac{3^{4n}.3^{4}(3-1)}{3^{4n}.3^{3}}$$

$$= 3^{4n-4n}.3^{4-3}.(2)$$

$$= 3^{0}.3^{1}.2$$

$$= 1 \times 3 \times 2$$

$= 6$ **Ans**

Q.2 Show that

$$\left[\frac{x^a}{x^b}\right]^{a+b} \times \left[\frac{x^b}{x^c}\right]^{b+c} \times \left[\frac{x^c}{x^a}\right]^{c+a} = 1$$

Proof:

L.H.S

$$= \left[\frac{x^a}{x^b}\right]^{a+b} \times \left[\frac{x^b}{x^c}\right]^{b+c} \times \left[\frac{x^c}{x^a}\right]^{c+a}$$

$$= \left(x^{a-b}\right)^{a+b} \times \left(x^{b-c}\right)^{b+c} \times \left(x^{c-a}\right)^{c+a}$$

$$= x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)}$$

$$= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$$

$$= x^{a^2-b^2+b^2-a^2+c^2-a^2}$$

$$= x^{\circ}$$

$$= 1$$

$1 =$ R.H.S **Ans**

Q.3 Simplify

(i) $$\frac{2^{\frac{1}{3}} \times (27)^{\frac{1}{3}} \times (60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}} \times (4)^{-\frac{1}{3}} \times (9)^{\frac{1}{4}}}$$

Solution: $$\frac{2^{\frac{1}{3}} \times (27)^{\frac{1}{3}} \times (60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}} \times (4)^{-\frac{1}{3}} \times (9)^{\frac{1}{4}}}$$

$$= \frac{2^{\frac{1}{3}} \times \left(3^{3}\right)^{\frac{1}{3}} \times (2 \times 2 \times 3 \times 5)^{\frac{1}{2}}}{(2 \times 2 \times 3 \times 3 \times 5)^{\frac{1}{2}} \times \left(2^{2}\right)^{-\frac{1}{3}} \times \left(3^{2}\right)^{\frac{1}{2}}}$$

$$= \frac{2^{\frac{1}{3}} \times 3\left(2^{2}\right)^{\frac{1}{2}} \times 3^{\frac{1}{2}} \times 5^{\frac{1}{2}}}{\left(2^{2}\right)^{\frac{1}{2}} \times \left(3^{2}\right)^{\frac{1}{2}} \times (5)^{\frac{1}{2}} \times 2^{-\frac{2}{3}} \times 3^{\frac{1}{2}}}$$

$$= \frac{2^{\frac{1}{3}} \times 3 \times 2 \times 3^{\frac{1}{2}} \times 5^{\frac{1}{2}}}{2 \times 3 \times 5^{\frac{1}{2}} \times 2^{-\frac{2}{3}} \times 3^{\frac{1}{2}}}$$

$$= 2^{\frac{1}{3}} \times 2^{+1} \times 2^{-1} \times 2^{\frac{+2}{3}} \times 3^{1} \times 3^{\frac{1}{2}} \times 3^{-1} \times 3^{\frac{-1}{2}} \times 5^{\frac{1}{2}} \times 5^{\frac{-1}{2}}$$

$$= 2^{\frac{1}{3}+1-1+\frac{2}{3}} \times 3^{1+\frac{1}{2}-1-\frac{1}{2}} \times 5^{\frac{1}{2}-\frac{1}{2}}$$

$$= 2^{\frac{1}{3}+\frac{2}{3}} \times 3^{0} \times 5^{0}$$

$$= 2^{\frac{1}{3}+\frac{2}{3}} \times 1 \times 1$$

$$= 2^{\frac{3}{3}}$$

$= 2$ **Ans**

(ii) $$\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{-\frac{1}{2}}}}$$

Solution: $$\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{-\frac{1}{2}}}}$$

$$= \sqrt{\frac{\left(6^{3}\right)^{\frac{2}{3}} \times \left(5^{2}\right)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{-\frac{1}{2}}}}$$

$$= \sqrt{\frac{6^{2} \times 5}{\left(\frac{100}{4}\right)^{\frac{1}{2}}}} \quad (25)$$

$$= \sqrt{\frac{6^{2} \times 5}{(25)^{\frac{1}{2}}}}$$

$$= \sqrt{\frac{6^{2} \times 5}{\left(5^{2}\right)^{\frac{1}{2}}}}$$

$$= \sqrt{\frac{6^{2} \times 5}{5}}$$

$$= \sqrt{6^{2} \times 5^{+1} \times 5^{-1}}$$

$$= \sqrt{6^{2} \times 5^{+1-1}}$$

$$= \sqrt{6^{2} \times 5^{0}}$$

$$= \sqrt{6^{2} \times 1}$$

$$= \sqrt{6^{2}}$$

$= 6$ **Ans**

(iii) $5^{2^3} \div \left(5^2\right)^3$

Solution: $5^{2^3} \div \left(5^2\right)^3$

$= 5^8 \div 5^6$

$= 5^{8-6}$

$= 5^2$

$= 25$ Ans

(iv) $\left(x^3\right)^2 \div x^{3^2}, x \neq 0$

Solution: $\left(x^3\right)^2 \div x^{3^2}, x \neq 0$

$= x^6 \div x^9$

$= x^{6-9}$

$= x^{-3}$

$= \dfrac{1}{x^3}$ **Ans**

Exercise 2.5

Q.1 Evaluate

(i) i^7

Solution:

$=i^7$
$=i^6.i$
$=\left(i^2\right)^3.i$
$=\left(-1\right)^3.i$
$=-1\times i$
$=-i$
$=-i$ **Ans**

(ii) i^{50}

Solution: i^{50}

$=\left(i^2\right)^{25}$
$=\left(-1\right)^{25}$
$=-1$ **Ans**

(iii) i^{12}

Solution:

i^{12}
$=\left(i^2\right)^6$
$=\left(-1\right)^6$
$=1$ Ans

(iv) $(-i)^8$

Solution:

$(-i)^8$
$=i^8$
$=\left(i^2\right)^4$
$=\left(-1\right)^4$
$=1$ **Ans**

(v) $(-i)^5$

Solution:

$(-i)^5$
$=-i^5$
$=-i^4.i$
$=-\left(i^2\right)^2.i$
$=-\left(-1\right)^2.i$
$=-(1)(i)$
$-i$ **Ans**

(vi) i^{27}

Solution: i^{27}

$=i^{26}.i$
$=\left(i^2\right)^{13}.i$
$=\left(-1\right)^{13}.i$
$=-1.i$
$=-i$ **Ans**

Q.2 Write the conjugate of the following numbers.

(i) $2+3i$
$=2-3i$

(ii) $3-5i$
$=3+5i$

(iii) $-i$
$=i$

(iv) $-3+4i$
$=-3-4i$

(v) $-4-i$
$=-4+i$

(vi) $i-3$
$=-i-3$

Q.3 Write the real and imaginary part of the following numbers.

(i) $1+i$
Real = 1
Imaginary = 1

(ii) $-1+2i$
Real = -1
Imaginary = 2

(iii) $-3i+2$

Real = 2

Imaginary = - 3

(iv) $-2-2i$

Real = -2

Imaginary = - 2

(v) $-3i$

Real = 0

Imaginary = - 3

(vi) $2+0i$

Real = 2

Imaginary = 0

Q.4 Find the value of x and y if

$x+iy+1=4-3i$

Solution: Given that

$x+iy+1=4-3i$

$x+iy=4-3i-1$

$x+iy=3-3i$

$x=3 \quad y=-3$

$x=3, y=-3$ **Ans**

Exercise 2.6

Q.1 Identify the following statement as true or false.

(i) $\sqrt{-3}\sqrt{-3}=3$ **False**

(ii) $i^{73}=-i$ **False**

(iii) $i^{10}=-1$ **True**

(iv) Complex conjugate of $\left(-6i+i^2\right)$ is $\left(-1+6i\right)$ **True**

(v) Difference of a complex number $z=a+bi$ and its conjugate is a real number. **False**

(vi) If $(a-1)-(b+3)i=5+8i$, then $a=6$ and $b=-11$. **True**

(vii) Product of a complex number and its conjugate is always a non-negative real number. **True**

Q.2 Express the each complex number in the standard form $a+bi$, where a and b are real number.

(i) $(2+3i)+(7-2i)$

Solution:

$=2+3i+7+2i$

$=2+7+3i-2i$

$=9+i$ **Ans**

(ii) $2(5+4i)-3(7+4i)$

Solution: $2(5+4i)-3(7+4i)$

$=10+8i-21-12i$

$=10-21+8i-12i$

$=-11-4i$ **Ans**

(iii) $(-3+5i)-(4+9i)$

Solution: $(-3+5i)-(4+9i)$

$=+3-5i-4-9i$

$=3-4-5i-9i$

$=-1-14i$ **Ans**

(iv) $2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}$

Solution: $2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}$

$=2(-1)+6i^2.i+3(i^2)^8-6(i^2)^9.i+4(i^2)^{12}.i$

$=-2+6(-1)i+3(-1)^8-6(-1).i+4(-1)^{12}.i$

$=-2-6i+3-6(-1)i+4(+1)i$

$=1-\cancel{6i}+\cancel{6i}+4i$

$=1+4i$ **Ans**

Q.3 Simplify and write your answer in the form $a+bi$

(i) $(-7+3i)(-3+2i)$

Solution: $(-7+3i)(-3+2i)$

$=-7(-3+2i)+3i(-3+2i)$

$=21-14i-9i+6i^2$

$=21-23i+6(-1)$

$=21-23i-6$

$=21-6-23i$

$=15-23i$ **Ans**

(ii) $\left(2-\sqrt{-4}\right)\left(3-\sqrt{-4}\right)$

Solution: $\left(2-\sqrt{-4}\right)\left(3-\sqrt{-4}\right)$

$$=\left(2-\sqrt{4\times -1}\right)\left(3-\sqrt{4\times -1}\right)$$
$$=\left(2-\sqrt{4i^2}\right)\left(3-\sqrt{4i^2}\right)$$
$$=(2-2i)(3-2i)$$
$$=2(3-2i)-2i(3-2i)$$
$$=6-4i-6i+4i^2$$
$$=6-10i+4(-1)$$
$$=6-10i-4$$
$$=2-10i \text{ Ans}$$

(iii) $\left(\sqrt{5}-3i\right)^2$

Solution: $\left(\sqrt{5}-3i\right)^2$

$$=\left(\sqrt{5}\right)^2+(3i)^2-2\left(\sqrt{5}\right)(3i)$$
$$=5+9i^2-6\sqrt{5}i$$
$$=5+9(-1)-6\sqrt{5}i$$
$$=5-9-6\sqrt{5}i$$
$$=-4-6\sqrt{5}i \text{ Ans}$$

(iv) $(2-3i)\left(\overline{3-2i}\right)$

Solution: $(2-3i)\left(\overline{3-2i}\right)$

$$=(2-3i)(3+2i)$$
$$=2(3+2i)-3i(3+2i)$$
$$=6+4i-9i-6i^2$$
$$=6-5i-6(-1)$$
$$=6-5i+6$$
$$=6+6-5i$$
$$=12-5i \text{ Ans}$$

Q.4 Simplify and write your answer in the form $a+bi$.

(i) $\dfrac{-2}{1+i}$

Solution: $\dfrac{-2}{1+i}$

$$=\frac{-2}{1+i}\times\frac{1-i}{1-i}$$
$$=\frac{-2(1-i)}{(1)^2-(i)^2}$$
$$=\frac{-2+2i}{1-i^2}$$
$$=\frac{-2+2i}{1-(-1)}$$
$$=\frac{-2+2i}{1+1}$$
$$=\frac{-2+2i}{2}$$
$$=-\frac{2}{2}+\frac{2i}{2}$$
$$=-1+i \text{ Ans}$$

(ii) $\dfrac{2+3i}{4-i}$

Solution: $\dfrac{2+3i}{4-i}$

$$=\frac{2+3i}{4-i}\times\frac{4+i}{4+i}$$
$$=\frac{(2+3i)(4+i)}{(4)^2-(i)^2}$$
$$=\frac{2(4+i)+3i(4+i)}{16-(-1)}$$
$$=\frac{8+2i+12i+3i^2}{16+1}$$
$$=\frac{8+4i+3(-1)}{17}$$
$$=\frac{8+14i-3}{17}$$
$$=\frac{8-3+14i}{17}$$
$$=\frac{5+14i}{17}$$
$$=\frac{5}{17}+\frac{14}{17}i \text{ Ans}$$

(iii) $\dfrac{9-7i}{3+i}$

Solution: $\dfrac{9-7i}{3+i}$

$=\dfrac{9-7i}{3+i}\times\dfrac{3-i}{3-i}$

$=\dfrac{(9-7i)(3-i)}{(3)^2-(i)^2}$

$=\dfrac{9(3-i)-7i(3-i)}{9-(-1)}$

$=\dfrac{27-9i-21i+7i^2}{9+1}$

$\dfrac{27-30i+7(-1)}{10}$

$=\dfrac{27-30i-7}{10}$

$=\dfrac{27-7-30i}{10}$

$=\dfrac{20-30i}{10}$

$=\dfrac{20}{10}-\dfrac{30i}{10}$

$=2-3i$ **Ans**

(iv) $\dfrac{2-6i}{3+i}-\dfrac{4+i}{3+i}$

Solution: $\dfrac{2-6i}{3+i}-\dfrac{4+i}{3+i}$

$=\dfrac{2-6i-(4+i)}{3+i}$

$=\dfrac{2-6i-4-i}{3+i}$

$=\dfrac{2-4-6i-i}{3+i}$

$=\dfrac{-2-7i}{3+i}$

$=\dfrac{-2-7i}{3+i}\times\dfrac{3-i}{3-i}$

$=\dfrac{-2-7i}{3+i}\times\dfrac{3-i}{3-i}$

$=\dfrac{-2(3-i)-7i(3-i)}{(3)^2-(i)^2}$

$=\dfrac{-6+2i-21i+7i^2}{9-(-1)}$

$=\dfrac{-6-19i+7(-1)}{9+1}$

$=\dfrac{-6-19i-7}{10}$

$=\dfrac{-6-7-19i}{10}$

$=\dfrac{-13-19i}{10}$

$=\dfrac{-13}{10}-\dfrac{19i}{10}$ **Ans**

(v) $\left[\dfrac{1+i}{1-i}\right]^2$

Solution: $\left[\dfrac{1+i}{1-i}\right]^2$

$=\dfrac{(1+i)^2}{(1-i)^2}$

$=\dfrac{(1)^2+(i)^2+2ab}{(1)^2+(i)^2-2ab}$

$=\dfrac{(1)^2+(i)^2+2(1)(i)}{(1)^2+(i)^2-2(1)(i)}$

$=\dfrac{1+(-1)+2i}{+1+(-1)-2i}$

$=\dfrac{\cancel{1}-\cancel{1}+2i}{\cancel{1}-\cancel{1}-2i}$

$=\dfrac{\cancel{2i}}{-\cancel{2i}}=-1$

$=-1$

$=-1+0i$ **Ans**

(vi) $\dfrac{1}{(2+3i)(1-i)}$

Solution: $\dfrac{1}{(2+3i)(1-i)}$

$=\dfrac{1}{2(1-i)+3i(1-i)}$

$=\dfrac{1}{2-2i+3i-3i^2}$

$$= \frac{1}{2+i-3(-1)}$$

$$= \frac{1}{2+i+3}$$

$$= \frac{1}{2+3+i}$$

$$= \frac{1}{5+i}$$

$$= \frac{1}{5+i} \times \frac{5-i}{5-i}$$

$$= \frac{1(5-i)}{(5)^2-(i)^2}$$

$$= \frac{5-i}{25-(-1)}$$

$$= \frac{5-i}{25+1}$$

$$= \frac{5-i}{26}$$

$$= \frac{5}{26} - \frac{1i}{26}$$ **Ans**

Q.5 Calculate $(a)\bar{z}(b)z+\bar{z}(c)z-\bar{z}(d)z\bar{z}$ **for each of the following.**

(i) $z = -i$

Solution: $z = -i$

(a) $\bar{z} = +i$

(b) $z + \bar{z} = -i + i$

$= 0$

(c) $z - \bar{z} = (-i) - (i)$

$= -2i$

(d) $z\bar{z} = (-i)(i)$

$= -i^2$

$= -(-1)$

$= 1$ **Ans**

(ii) $z = 2+i$

Solution: $z = 2+i$

$z + 2i$

(a) $\bar{z} = 2-i$

(b) $z + \bar{z} = (2+i)+(2-i)$

$= 2 + \not{i} + 2 - \not{i}$

$= 2+2$

$= 4$

(c) $z - \bar{z} = (2+i)-(2-i)$

$= \not{2} + i - \not{2} + i$

$= i + i$

$= 2i$

(d) $z\bar{z} = (2+i)(2-i)$

$= (2)^2 - (i)^2$

$= 4 - i^2$

$= 4 - (-1)$

$= 4+1$

$= 5$ **Ans**

(iii) $z = \frac{1+i}{1-i}$

Solution: $z = \frac{1+i}{1-i}$

$$z = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$$

$$= \frac{1(1+i)+i(1+i)}{(1-i)(1+i)}$$

$$= \frac{1+i+i+(-1)}{(1)^2-(i)^2}$$

$$= \frac{1+2i+(-1)}{1-(-1)}$$

$$= \frac{\not{1}+2i-\not{1}}{1+1}$$

$$= \frac{\not{2}i}{\not{2}}$$

$= i$

$z = i$

(a) $\bar{z} = -i$

(b) $z + \bar{z} = i + (-i)$

$= \not{i} - \not{i}$

$= 0$

(c) $z - \bar{z} = i - (-i)$

$= i + i$

$= 2i$

(d) $z\bar{z} = (i)(-i)$

$= -i^2$

$= -(-1)$

$= +1$ **Ans**

(iv) $z = \frac{4-3i}{2+4i}$

Solution: $z = \frac{4-3i}{2+4i}$

$z = \frac{4-3i}{2+4i} \times \frac{2-4i}{2-4i}$

$= \frac{4(2-4i)-3i(2-4i)}{(2+4i)(2-4i)}$

$= \frac{8-16i-6i+12i^2}{(2)^2-(4i)^2}$

$= \frac{8-22i+12(-1)}{4-16i^2}$

$= \frac{8-22i-12}{4-16(-1)}$

$= \frac{8-12-22i}{4+16}$

$= \frac{-4-22i}{20}$

$= \frac{-4}{20} - \frac{22}{20}i$

$= -\frac{1}{5} - \frac{11}{10}i$

(a) $\bar{z} = \frac{-1}{5} + \frac{11}{10}i$

(b)

$z + \bar{z} = \left(-\frac{1}{5} - \frac{11}{10}i\right) + \left(-\frac{1}{5} + \frac{11}{10}i\right)$

$= -\frac{1}{5} - \frac{11}{10}i - \frac{1}{5} + \frac{11}{10}i$

$= -\frac{1}{5} - \frac{1}{5}$

$= \frac{-1-1}{5}$

$= -\frac{2}{5}$

(c)

$z - \bar{z} = \left(-\frac{1}{5} - \frac{11}{10}i\right) - \left(-\frac{1}{5} + \frac{11}{10}i\right)$

$= -\frac{1}{5} - \frac{11}{10}i + \frac{1}{5} - \frac{11}{10}i$

$= -\frac{11}{10}i - \frac{11}{10}i = \frac{-11i-11i}{10}$

$= -\frac{22^{11}i}{10^{5}}$

$= -\frac{11}{5}i$

(d) $z\bar{z} = \left(-\frac{1}{5} - \frac{11}{10}i\right)\left(-\frac{1}{5} + \frac{11}{10}i\right)$

$= \left(-\frac{1}{5}\right)^2 - \left(\frac{11}{10}i\right)^2$

$= \frac{1}{25} - \frac{121}{100}i^2$

$= \frac{1}{25} - \frac{121}{100}(-1)$

$= \frac{1}{25} + \frac{121}{100}$

$= \frac{4+121}{100}$

$= \frac{125}{100}$

$= \frac{5}{4}$ **Ans**

Q.6 If $z = 2+3i$ and show that.

(i) $\overline{z+w} = \bar{z} + \bar{w}$

Solution: $\overline{z+w} = \bar{z} + \bar{w}$

$z + w = 2+3i+5-4i$

$= 2+5+3i-4i$

$= 7-i$

L.H.S $= \overline{z+w}$

$= \overline{7-i}$

$= 7+i$ ... (i)

R. H. S $= \bar{z} + \bar{w}$

$= \left(\overline{2+3i}\right) + \left(\overline{5-4i}\right)$

$= 2-3i+5+4i$

$= 2+5-3i+4i$

$= 7 + i$ ...

(ii)

From (i) and (ii) we get

L.H.S=R.H.S

$\overline{z+w} = \bar{z} + \bar{w}$

Hence proved

(ii) $\overline{z-w} = \bar{z} - \bar{w}$

Solution: $\overline{z-w} = \bar{z} - \bar{w}$

$z - w = (2+3i) - (5-4i)$

$= 2 + 3i - 5 + 4i$

$= 2 - 5 + 3i + 4i$

$= -3 + 7i$

L.H.S $= \overline{z-w}$

$= \overline{-3+7i}$

$= -3 - 7i$ ...(i)

R.H.S $= \bar{z} - \bar{w}$

$= (\overline{2+3i}) - (\overline{5-4i})$

$= 2 + 3i - (5+4i)$

$= 2 - 3i - 5 - 4i$

$= -3 - 7i$

From (i) and (ii) we get

L.H.S=R.H.S

$\overline{z-w} = \overline{z-w}$

Hence proved

(iii) $\overline{zw} = \bar{z}\,\bar{w}$

Solutions: $\overline{zw} = \bar{z}\,\bar{w}$

$zw = (2+3i)(5+4i)$

$= 2(5-4i) + 3i(5-4i)$

$= 10 - 8i + 15i - 12i^2$

$= 10 + 7i - 12(-1)$

$= 10 + 7i + 12$

$= 22 + 7i$

L.H.S $= \overline{zw}$

$= \overline{22+7i}$

$= 22 - 7i$

R.H.S $= \overline{zw}$

$= (\overline{2+3i})(\overline{5-4i})$

$= (2-3i)(5+4i)$

$= 2(5+4i) - 3i(5+4i)$

$= 10 + 8i - 15i - 12i^2$

$= 10 - 7i - 12(-1)$

$= 10 - 7i + 12$

$= 22 - 7i$

From (i) and (ii) we get

L.H.S=R.H.S

$\overline{zw} = \overline{z}\overline{w}$

Hence proved

(iv) $\overline{\left[\frac{z}{w}\right]} = \frac{\bar{z}}{\bar{w}}$, where $\neq 0$

Solutions: $\overline{\left[\frac{z}{w}\right]} = \frac{\bar{z}}{\bar{w}}$

$\frac{z}{w} = \frac{2+3i}{5-4i} \times \frac{5+4i}{5+4i}$

$= \frac{2(5+4i) + 3i(5+4i)}{(5-4i)(5+4i)}$

$= \frac{10 + 8i + 15i + 12i^2}{(5)^2 - (4i)^2}$

$= \frac{10 + 23i + 12(-1)}{25 - 16i^2}$

$= \frac{10 + 23i - 12}{25 - (6(-1))}$

$= \frac{10 + 23i - 12}{25 + 16}$

$= \frac{-2 + 23i}{41}$

L.H.S$= \overline{\left(\frac{z}{w}\right)}$

$= \overline{\left(\frac{-2+23i}{41}\right)}$

$= \frac{-2}{41} - \frac{23}{41}i$ ... (i)

R.H.S $= \frac{\bar{z}}{\bar{w}}$

$= \frac{(\overline{2+3i})}{(\overline{5-4i})}$

$$= \frac{2-3i}{5+4i}$$

$$= \frac{2-3i}{5+4i} \times \frac{5-4i}{5-4i}$$

$$= \frac{2(5-4i)-3i(5-4i)}{(5+4i)(5-4i)}$$

$$= \frac{10-8i-15i+12i^2}{(5)^2-(4i)^2}$$

$$= \frac{10-23i+12(-1)}{25-16i^2}$$

$$= \frac{10-23i+12(-1)}{25-16(-1)}$$

$$= \frac{10-23i-12}{25+16}$$

$$= \frac{-2-23i}{41}$$

$$= \frac{-2}{41} - \frac{23}{41}i \qquad \ldots \text{(ii)}$$

From (i) and (ii) we get

L.H.S=R.H.S

Hence Proved

$$\overline{\left[\frac{z}{w}\right]} = \frac{\overline{z}}{\overline{w}}$$

(v) $\frac{1}{2}(z+\overline{z})$ **is the real part of** z **.**

Solution: $\frac{1}{2}(z+\overline{z})$

$$= \frac{1}{2}\left[(2+3i)+\left(\overline{2+3i}\right)\right]$$

$$= \frac{1}{2}\left[(2+3i)+(2-3i)\right]$$

$$= \frac{1}{2}\left[2+\cancel{3i}+2-\cancel{3i}\right]$$

$$= \frac{1}{2}[2+2]$$

$$= \frac{1}{\cancel{2}}\left[\cancel{4}^{2}\right]$$

$$= 2 = \text{Re}(z)$$

$\frac{1}{2}(z+\overline{z})$ is the real part of z . **Ans**

(vi) $\frac{1}{2}(z-\overline{z})$ **is the imaginary part of** z **.**

Solution: $\frac{1}{2}(z-\overline{z})$

$$\frac{1}{2}(z-\overline{z}) =$$

$$= \frac{1}{2}\left[(2+3i)-\left(\overline{2+3i}\right)\right]$$

$$= \frac{1}{2}\left[(2+3i)-(2-3i)\right]$$

$$= \frac{1}{2}\left[\cancel{2}+3i-\cancel{2}+3i\right]$$

$$= \frac{1}{\cancel{2}}\left[{}^{3}\cancel{6}i\right]$$

$$= 3i$$

$$= \text{Imaginary}(z)$$

$\frac{1}{2}(z-\overline{z})$ is the imaginary part of z . **Ans**

Q.7 Solve the following equations for real x **and** y **.**

(i) $(2-3i)(x+yi) = 4+i$

Solution: $(2-3i)(x+yi) = 4+i$

$$x+yi = \frac{4+i}{2-3i}$$

$$x+yi = \frac{4+i}{2-3i} \times \frac{2+3i}{2+3i}$$

$$= \frac{4(2+3i)+i(2+3i)}{(2-3i)(2+3i)}$$

$$= \frac{8+12i+2i+3i^2}{(2)^2-(3i)^2}$$

$$= \frac{8+14i+3(-1)}{4-9i^2}$$

$$= \frac{8+14i-3}{4-9(-1)}$$

$$= \frac{8-3+14i}{4+9}$$

$$=\frac{5+14i}{13}$$

$$x+yi=\frac{5}{13}+\frac{14}{13}i$$

$x=\frac{5}{13}, y=\frac{14}{13}$ **Ans**

(ii) $(3-2i)(x+yi)=2(x-2yi)+2i-1$

Solution:

$$(3-2i)(x+yi)=2(x-2yi)+2i-1$$

$$3(x+yi)-2i(x+yi)=2x-4yi+2i-1$$

$$3x+3yi-2xi-2yi^2=(2x-1)+i(2-4y)$$

$$3x+(3x-2x)i-2y(-1)=(2x-1)+i(2-4y)$$

$$3x+(3y-2x)i+2y=(2x-1)+i(2-4)$$

$$(3x+2y)+(3y-2x)i=(2x-1)+(2-4y)i$$

Comparing the real and imaginary parts.

$3x+2y=2x-1$,

$3y-2x=2-4y$

$3x-2x+2y=-1$,

$3y-2x=2-4y$

$x+2y=-1$,

$$-2x+3y+4y=2$$

$$-2x+7y=2$$

$x+2y=-1$ ________ (i)

$-2x+7y=2$ ________ (ii)

Multiply equation (i) with (2)

$$2(x+2y)=-1\times 2$$

$2x+4y=-2$ ________ (iii)

$\cancel{2x}+4y=\cancel{-2}$

$\cancel{-2x}+7y=\cancel{2}$

$$11y=0$$

$$y=\frac{0}{11}$$

$$y=0$$

Putting $y=0$ in equation (i)

$$x+2y=-1$$

$$x+2(0)=-1$$

$$x+0=-1$$

$$x=-1+0$$

$x=-1$ **Ans**

(iii) $(3+4i)^2-2(x-yi)=x+yi$

Solution: $(3+4i)^2-2(x-yi)=x+yi$

$$(3+4i)^2-2(x-yi)=x+yi$$

$$9+24i+16i^2-2x+2yi=x+yi$$

$$9+24i+16(-1)-2x+2yi=x+yi$$

$$9+24i-16-2x+2yi=x+yi$$

$$9+24i-16-2x=x+2yi-yi=0$$

$$9+24i-16-3x+yi=0$$

$$-3x+yi=-9-24i+16$$

$$-3x+yi=16-9-24i$$

$$-3x+yi=7-24i$$

Comparing the real and imaginary parts.

$-3x = 7$

$y = -24$

$x = \frac{-7}{3}$

$y = -24$ **Ans**

Review Exercise 2

Q.1 Multiple choice questions. Choose the correct answer.

(i) $\left(27x^{-1}\right)^{-\frac{2}{3}}$ ___________

(a) $\frac{\sqrt[3]{x^2}}{9}$ **(b)** $\frac{\sqrt{x^3}}{9}$

(c) $\frac{\sqrt[3]{x^2}}{8}$ **(d)** $\frac{\sqrt{x^3}}{8}$

(ii) **Write $\sqrt[7]{x}$ in the exponential form** ___________

(a) x **(b)** x^7

(c) $x^{\frac{1}{7}}$ **(d)** $x^{\frac{7}{2}}$

(iii) **Write $4^{\frac{2}{3}}$ with radical sing** ___________

(a) $\sqrt[3]{4^2}$ **(b)** $\sqrt[2]{4^3}$

(c) $\sqrt[2]{4^3}$ **(d)** $\sqrt{4^6}$

(iv) **In $\sqrt[3]{35}$ the radicand is;**

(a) 3 **(b)** $\frac{1}{3}$

(c) 35 **(d)** None

(v) $\left(\frac{25}{16}\right)^{-\frac{1}{2}} =$ ___________

(a) $\frac{5}{4}$ **(b)** $\frac{4}{5}$

(c) $-\frac{5}{4}$ **(d)** $-\frac{4}{5}$

(vi) **The conjugate of $5+4i$ is** ___________

(a) $-5+4i$ **(b)** $-5-4i$

(c) $5-4i$ **(d)** $5+4i$

(vii) **The value of i^9 is;**

(a) 1 **(b)** -1

(c) i **(d)** $-i$

(viii) **Every real number is** __________

(a) Positive integer
(b) A rational number
(c) A negative integer
(d) A complex number

(ix) **Real point of** $2ab\left(i+i^2\right)$ **is** __________

(a) $2ab$
(b) $-2ab$
(c) $2abi$
(d) $-2abi$

(x) **Imaginary part of** $-i(3i+2)$ **is** __________

(a) -2
(b) 2
(c) 3
(d) -3

(xi) **Which of the following sets have the closure property w.r.t addition** __________

(a) $\{0\}$
(b) $\{0,1\}$
(c) $\{0,1\}$
(d) $\left\{1,\sqrt{2},\frac{1}{2}\right\}$

(xii) **Name the property of real number used in** $\left[-\frac{\sqrt{5}}{2}\right]\times 1=-\frac{\sqrt{5}}{2}$ __________

(a) Additive identity
(b) Additive inverse
(c) Multiplicative identity
(d) Multiplicative inverse

(xiii) **If** $x,y,z\in R, z<0,$ **then** $x<y\Rightarrow >$ ...

(a) $xz<yz$
(b) $xz>yz$
(c) $xz=yz$
(d) None of these

(xiv) **IF** $a,b\in\ \in\ R,$ **only one of** $a=b$ **or** $a<b$ or $a>b$ **hold is called** __________

(a) Trichotomy property
(b) Transitive property
(c) Additive property
(d) Multiplicative property

(xv) **A non-terminating, non-recurring decimal represents …**

(a) A natural number
(b) A rational number
(c) An irrational number
(d) A prime number

ANSWER KEY

i	a	vi	c	xi	a
ii	c	vii	c	xii	c
iii	a	viii	d	xiii	b
iv	c	ix	b	xiv	a
v	b	x	a	xv	c

Q.2 True or False? Identity

(i) Division is not an associative operation. **True**
(ii) Every whole number is a natural number. **False**
(iii) Multiplicative inverse of 0.02 is 50. **True**
(iv) π is rational number. **False**
(v) Every integer is a rational number. **True**
(vi) Subtraction is a commutative operation. **False**
(vii) Every real number is a rational number. **False**
(viii) Decimal representation of a rational number is either terminating or recurring. **True**
(ix) $1.\bar{8}=1+\frac{8}{9}$ **True**

Q.3 Simplify the following

(i) $\sqrt[4]{81y^{-12}x^{-8}}$

Solution:

$$=\left(3^4y^{12}x^{-8}\right)^{\frac{1}{4}}$$

$$=3^{4\times\frac{1}{4}}y^{-12\times\frac{1}{4}}x^{-8\times\frac{1}{4}}$$

$$=3y^{-3}x^{-2}$$

$$\sqrt[4]{81y^{-12}x^{-8}}=\frac{3}{y^3x^2}\ \textbf{Ans}$$

(ii) $\sqrt{25x^{10}y^{8m}}$

Solution:

$$=\sqrt{25x^{10n}y^{8m}}$$

$$=\left(5^2x^{10n}y^{8m}\right)^{\frac{1}{2}}$$

$$=5^{2\times\frac{1}{2}}.x^{10n\times\frac{1}{2}}.y^{8m\times\frac{1}{2}}$$

$$\sqrt{25x^{10}y^{8m}}=5x^{5n}.y^{4m}\ \textbf{Ans}$$

(iii) $\left[\frac{x^3y^4z^5}{x^{-2}y^{-1}z^{-5}}\right]^{\frac{1}{5}}$

Solution:

$$=\left(x^{3+2}.y^{4+1}.z^{5+5}\right)^{\frac{1}{5}}$$

$$=\left(x^5y^5z^{10}\right)^{\frac{1}{5}}$$

$$=x^{5\times\frac{1}{5}}\times y^{5\times\frac{1}{5}}\times z^{10\times\frac{1}{5}}$$

$$\left[\frac{x^3y^4z^5}{x^{-2}y^{-1}z^{-5}}\right]^{\frac{1}{5}}=x.y.z^2\ \textbf{Ans}$$

(iv) $\left(\frac{32x^{-6}y^{-4}z}{625x^4yz^{-4}}\right)^{\frac{2}{5}}$

Solution:

$$=\left(\frac{2^5x^{-6}y^{-4}z}{5^4x^4yz^{-4}}\right)^{\frac{2}{5}}$$

$$=\left[\frac{2^5z^{1+4}}{5^4x^{4+6}\times y^{1+4}}\right]^{\frac{2}{5}}$$

$$=\left[\frac{2^5z^5}{5^4x^{10}y^5}\right]^{\frac{2}{5}}$$

$$=\frac{2^{5\times\frac{2}{5}}\times z^{5\times\frac{2}{5}}}{5^{4\times\frac{2}{5}}\times x^{10\times\frac{2}{5}}\times y^{5\times\frac{2}{5}}}$$

$$=\frac{2^2\times z^2}{5^{\frac{8}{5}}\times x^4\times y^2}$$

$$=\frac{4z^2}{5^{\frac{5}{5}+\frac{3}{5}}\times x^4y^2}$$

$$=\frac{4z^2}{5^{1+\frac{3}{5}}\times x^4y^2}$$

$$\left(\frac{32x^{-6}y^{-4}z}{625x^4yz^{-4}}\right)^{\frac{2}{5}}=\frac{4z^2}{5\times5^{\frac{3}{5}}x^4y^2}\ \textbf{Ans}$$

Q.4 Simplify $\sqrt{\dfrac{(216)^{\frac{2}{3}}\times(25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}}$

Solution: $\sqrt{\dfrac{(216)^{\frac{2}{3}}\times(25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}}$

$$=\sqrt{\dfrac{(6^3)^{\frac{2}{3}}\times(5^2)^{\frac{1}{2}}}{\left(\dfrac{4}{100}\right)^{-\frac{3}{2}}}}$$

$$=\sqrt{\dfrac{6^2\times5}{\left(\dfrac{100}{4}\right)^{\frac{3}{2}}}}$$

$$=\sqrt{\dfrac{6^2\times5}{(5^2)^{\frac{3}{2}}}}$$

$$=\sqrt{\dfrac{6^2\times5}{(5)^3}}$$

$$=\sqrt{\dfrac{6^2}{5^3\times5^{-1}}}$$

$$=\sqrt{\dfrac{6^2}{5^{3-1}}}$$

$$=\sqrt{\dfrac{6^2}{5^2}}$$

$$=\sqrt{\left(\dfrac{6}{5}\right)^2}$$

$$=\left(\dfrac{6}{5}\right)^{2\times\frac{1}{2}}$$

$$=\dfrac{6}{5}$$ **Ans**

Q.5 $\left(\dfrac{a^p}{a^q}\right)^{p+q}\times\left(\dfrac{a^q}{a^r}\right)^{q+r}\div5\left(a^q a^r\right)^{p-r}$

Solution:

$$=\dfrac{\left(a^{p-q}\right)^{p+q}\left(a^{q-r}\right)^{q+r}}{5\left(a^{p+r}\right)^{p-r}}$$

$$=\dfrac{a^{(p-q)(p+q)}a^{(q-r)(q+r)}}{5a^{(p+r)(p-r)}}$$

$$=\dfrac{a^{p^2-q^2}a^{q^2-r^2}}{5a^{p^2-r^2}}$$

$$=\dfrac{a^{p^2-q^2+q^2-r^2}}{5a^{p^2-r^2}}$$

$$=\dfrac{a^{p^2-r^2-p^2+r^2}}{5}$$

$$=\dfrac{a^0}{5}$$

$$\left(\dfrac{a^p}{a^q}\right)^{p+q}\times\left(\dfrac{a^q}{a^r}\right)^{q+r}\div5\left(a^q a^r\right)^{p-r}$$

$$=\dfrac{1}{5}$$ **Ans**

Q.6 Simplify $\left(\dfrac{a^{2l}}{a^{l+m}}\right)\left(\dfrac{a^{2m}}{a^{m+n}}\right)\left(\dfrac{a^{2n}}{a^{n+2}}\right)$

Solution:

$$=a^{2l-l-m}a^{2m-m-n}a^{2n-n-n}$$

$$=a^{l-m}a^{m-n}a^{n-l}$$

$$=a^{l-m+m-n+n-l}$$

$$=a^0$$

$$\left(\dfrac{a^{2l}}{a^{l+m}}\right)\left(\dfrac{a^{2m}}{a^{m+n}}\right)\left(\dfrac{a^{2n}}{a^{n+2}}\right)=1$$ **Ans**

Q.7 Simplify $\sqrt[3]{\dfrac{a^l}{a^m}}\times\sqrt[3]{\dfrac{a^m}{a^n}}\times\sqrt[3]{\dfrac{a^n}{a^r}}$

Solution:

$$=\sqrt[3]{a^{l-m}}\sqrt[3]{a^{m-n}}\sqrt[3]{a^{n-l}}$$

$$=\left(a^{l-m}\right)^{\frac{1}{3}}\times\left(a^{m-n}\right)^{\frac{1}{3}}\times\left(a^{n-l}\right)^{\frac{1}{3}}$$

$$=a^{\frac{l-m}{3}}\times a^{\frac{m-n}{3}}\times a^{\frac{n-l}{3}}$$

$$=a^{\frac{l-m}{3}+\frac{m-n}{3}+\frac{n-l}{3}}$$

$$=a^{\frac{l-m+m-n+n-l}{3}}$$

$$=a^{\frac{0}{3}}$$

$$=a^0$$

$$\sqrt[3]{\dfrac{a^l}{a^m}}\times\sqrt[3]{\dfrac{a^m}{a^n}}\times\sqrt[3]{\dfrac{a^n}{a^r}}=1$$ **Ans**

Unit 2: Real and Complex Numbers

Overview

Natural Numbers:

The numbers $1, 2, 3, \ldots$ which we use for counting certain objects are called natural numbers or positive integers. The set natural numbers is denoted by N.

i.e. $N = \{1, 2, 3, \ldots\}$

Whole Numbers:

If we include 0 in the set of natural number, the resulting set is the set of whole numbers, denoted by W,

i.e. $W = \{0, 1, 2, 3, \ldots\}$

Integers:

The set of integers consist of positive integers, 0 and negative integers and is denoted by Z i.e. $Z\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Rational Numbers:

All numbers of the form $\frac{p}{q}$ where p, q are integers and q is not zero are called rational numbers. The set of rational numbers is denoted by Q,

i.e. $Q = \left\{\frac{p}{q} \mid p, q \in Z \wedge q \neq 0, (p, q) = 1\right\}$

Irrational Numbers:

The numbers which cannot be expressed as quotient of integers are called irrational numbers. The set of irrational numbers is denoted by Q',

i.e. $Q' = \left\{x \mid x \neq \frac{p}{q}, p, q \in Z \wedge q \neq 0\right\}$

The union of the set of rational numbers and irrational numbers is known as the set of real numbers. It is denoted by R,

i.e. $R = Q \cup Q'$

Types of Rational Numbers:

(i) Terminating Decimal Fractions

The decimal fraction in which there are finite number of digits in its decimal part is called a terminating decimal fraction. For example $\frac{2}{5} = 0.4$ and $\frac{3}{8} = 0.375$.

(ii) Recurring and Non-terminating Decimal Fractions:

The decimal fraction (non-terminating) in which some digits are repeated again and again in the same order in its decimal part is called recurring decimal fraction.

For example $\frac{2}{9} = 0.2222...$ and $\frac{4}{11} = 0.363636...$

Concept of Radicals and Radicands:

In the radical $\sqrt[n]{a}$, the symbol $\sqrt{\ }$ is called the radical sign, n is called the index of the radical and the real number a under the radical sign is called the radicand or base.

Base and Exponent:

In the exponential notation of (read as a to the nth power) we call 'a' as the base and 'n' as the exponent or the power t which the base is raised.

Definition of a Complex Number:

A number of the form $z = a + bi$ where a and b are real numbers and $i - \sqrt{-1}$, is called a complex number and is represented by z i.e., $z = a + ib$

Conjugate of a Complex Number:

If we change i to $-i$ in $z = a + bi$, we obtain another complex number $a - bi$ called the complex conjugate of z and is denoted by $\overline{z}$ (read z bar).

Exercise 3.1

Q.1 Express each of the following numbers in scientific notations.

(i) 5700
$= 5.7 \times 10^3$ **Ans**

(ii) 49,800,000
$= 4.98 \times 10^7$ **Ans**

(iii) 96000000
$= 9.6 \times 10^7$ **Ans**

(iv) 416.9
$= 4.169 \times 10^2$ **Ans**

(v) 83000
$= 8.3 \times 10^4$ **Ans**

(vi) 0.00643
$= 6.43 \times 10^{-3}$ **Ans**

(vii) 0.0074
$= 7.4 \times 10^{-3}$ **Ans**

(viii) 60,000,000

$= 6 \times 10^7$ Ans

(ix) 0.00000000395
$= 3.95 \times 10^{-9}$ **Ans**

(x) $\dfrac{275000}{0.0025}$

$= \dfrac{2.75 \times 10^5}{2.5 \times 10^{-3}}$ **Ans**

Q.2 Express the following number in ordinary notation.

(i) 6×10^{-4}
$= 0.0006$ **Ans**

(ii) 5.06×10^{10}
$= 50600000000$ **Ans**

(iii) 9.018×10^{-6}
$= 0.000009018$ Ans

(iv) 7.865×10^8
$= 786500000$ **Ans**

Exercise 3.2

Q.1 Find the common logarithms of each of the following numbers.

(i) 232.92

Solution: 232.92

Suppose $x = 232.92$

Taking log

$\log x = \log 232.92$

$Ch = 2$

Mantissa = 0.3672

$\log x = 2.3672$ **Ans**

(ii) 29.326

Solution: 29.326

Suppose $x = 29.326$

Taking log

$\log x = \log 29.326$

$Ch = 1$

Mantissa = 0.4672

$\log x = 1.4672$ **Ans**

(iii) 0.00032

Solution: 0.00032

Suppose $x = 0.00032$

Taking log

$\log x = \log 0.00032$

$Ch = \bar{4}$

Mantissa = 0.5051

$\log x = \bar{4}.5051$ **Ans**

(iv) 0.3206

Solution: 0.3206

Suppose $x = 0.3206$

Taking log:

$\log x = \log 0.3206$

$Ch = \bar{1}$

Mantissa = 0.5059

$\log x = \bar{1}.5059$ **Ans**

Q.2 If $\log 31.09 = 1.4926$, find the value of the following.

If

$\log 31.09 = 1.4926$

Then

(i) $\log 3.109 = 0.4926$

(ii) $\log 310.9 = 2.4926$

(iii) $\log 0.003109 = \bar{3}.4926$

(iv) $0.3109 = \bar{1}.4926$

Solution:

(i) $\log 3.109$

Characteristics = 0

Mantissa = 0.4926

$\log 3.109 = 0.4926$ **Ans**

(ii) $\log 310.9$

Characteristics = 2

Mantissa =0.4926

$\log 310.9 = 2.4926$ **Ans**

(iii) $\log 0.003109$

Characteristics = $\bar{3}$

Mantissa = 0.4926

$\log 0.003109 = \bar{3}.4926$ **Ans**

(iv) $\log 0.3109$

Characteristics = $\bar{1}$

Mantissa = 0.4926

$\log 0.3109 = \bar{1}.4926$ **Ans**

Q.3 Find the numbers whose common logarithms are

(i) 3.5621

Solution:

$\log x = 3.5621$

Ch=3(If ch is positive, then plus for reference point)

Mantissa = 0.5621

$x = \text{antilog } 3.5621$

$x = 3649.0$ **Ans**

(ii) $\bar{1}.7427$

Solution:

$\log x = \bar{1}.7427$

$\text{Ch} = \bar{1}$

Mantissa = 0.7427

$x = \text{anti}\log \bar{1}.7427$

$x = 0.5530$ **Ans**

Q.4 What replacement for the unknown in each of the following will make the true statements?

(i) $\log_3 81 = L$

Solution: $\log_3 81 = L$

Writing in exponential form.

$3^L = 81$

$3^L = 3^4$

$\because$ Bases are equal so

$L = 4$ **Ans**

(ii) $\log_a 6 = 0.5$

Solution: $\log_a 6 = 0.5$

$a^{0.5} = 6$

$a^{\frac{1}{2}} = 6$

$\sqrt{a} = 6$ Taking square on both sides

$\sqrt{(a)}^2 = (6)^2$

$a = 36$ **Ans**

(iii) $\log_5 n = 2$

Write in exponential form

$5^2 = n$

$25 = n$

Or $n = 25$ **Ans**

(iv) $10^P = 40$

Solution: $10^P = 40$

Changing into logarithmic form

$P = \log_{10} 40$

$= \log 40$

$= 1.6021$ **Ans**

Q.5 Evaluate.

(i) $\log_2 \frac{1}{128}$

Solution: $\log_2 \frac{1}{128}$

Suppose $\log_2 \frac{1}{128} = x$

Writing in exponential form.

$2^x = \frac{1}{128}$

$2^x = \frac{1}{2^7}$

$2^x = 2^{-7}$

$\because$ Bases are equal so

$x = -7$ **Ans**

(ii) log 512 to the base $2\sqrt{2}$

Solution: $\log_{2\sqrt{2}} 512 = x$

Writing in exponential form

$\left(2\sqrt{2}\right)^x = 512$

$\left(2^1 . 2^{\frac{1}{2}}\right)^x = 2^9$

$\left(2^{\frac{3}{2}}\right)^x = 2^9$

$2^{\frac{3}{2}x} = 2^9$

$\because$ Bases are equal so

$\frac{3}{2}x = 9$

$x = \frac{9 \times 2}{3}$

$x = \frac{\cancel{18}^{6}}{\cancel{3}}$

$x = 6$ **Ans**

Q.6 Find the value of x from the following statements.

(i) $\log_2 x = 5$

Solution: $\log_2 x = 5$

Write in exponential form.

$2^5 = x$

$32 = x$ **Ans**

(ii) $\log_{81} 9 = x$

Solution: $\log_{81} 9 = x$

Writing in the exponential form.

$81^x = 9$

$(9^2)^x = 9$

$9^{2x} = 9$

$2x = 1$

$x = \frac{1}{2}$ **Ans**

(iii) $\log_{64} 8 = \frac{x}{2}$

Solution: $\log_{64} 8 = \frac{x}{2}$

Writing in exponential form.

$64^{\frac{x}{2}} = 8$

$(8^2)^{\frac{x}{2}} = 8$

$8^x = 8$

$x = 1$ **Ans**

(iv) $\log_x 64 = 2$

Solution: $\log_x 64 = 2$

Writing in exponential form

$x^2 = 64$

$x^2 = 8^2$

$x = 8$ **Ans**

(v) $\log_3 x = 4$

Solution: $\log_3 x = 4$

$3^4 = x$

$81 = x$

Or $x = 81$ **Ans**

Exercise 3.3

Q.1 Write the following into sum or difference $\log(A\times B)$

(i) $\log(A\times B)$

Solution: $\log(A\times B)$

$$\log A\times B=\log A+\log B \text{ **Ans**}$$

(ii) $\log\frac{15.2}{30.5}$

Solution: $\log\frac{15.2}{30.5}$

$$\log\frac{15.2}{30.5}=\log 15.2-\log 30.5 \text{ **Ans**}$$

(iii) $\log\frac{21\times 5}{8}$

Solution: $\log\frac{21\times 5}{8}$

$$\log\frac{21\times 5}{8}=\log(21\times 5)-\log 8$$
$$=\log 21+\log 5-\log 8 \text{ **Ans**}$$

(iv) $\log\sqrt[3]{\frac{7}{15}}$

Solution: $\log\sqrt[3]{\frac{7}{15}}$

$$\log\sqrt[3]{\frac{7}{15}}=\log\left(\frac{7}{15}\right)^{\frac{1}{3}}$$
$$=\frac{1}{3}\log\left(\frac{7}{15}\right)$$
$$=\frac{1}{3}(\log 7-\log 15)$$
$$=\frac{1}{3}\log 7-\frac{1}{3}\log 15 \text{ **Ans**}$$

(v) $\log\frac{(22)^{\frac{1}{3}}}{5^3}$

Solution: $\log\frac{(22)^{\frac{1}{3}}}{5^3}$

$$\log\frac{(22)^{\frac{1}{3}}}{5^3}=\log 22^{\frac{1}{3}}-\log 5^3$$
$$=\frac{1}{3}\log 22-3\log 5 \text{ **Ans**}$$

(vi) $\log\frac{25\times 97}{29}$

Solution: $\log\frac{25\times 97}{29}$

$$\log\frac{25\times 47}{29}=\log(25\times 47)-\log 29$$
$$=\log 25+\log 47-\log 29 \text{ **Ans**}$$

Q.2 Express $\log x-2\log x+3\log(x+1)-\log(x^2-1)$ **as a single logarithm.**

Solution:

$$\log x-2\log x+3\log(x+1)-\log(x^2-1)$$
$$=\log x-\log x^2+\log(x+1)^3-\log(x^2-1)$$
$$=\log\left(\frac{x}{x^2}\right)+\log\frac{(x+1)^3}{x^2-1}$$
$$=\log\left(\frac{x}{x^2}\times\frac{(x+1)^3}{x^2-1}\right)$$
$$=\log\left(\frac{x(x+1)^3}{x^2(x^2-1)}\right)$$
$$=\log\frac{x(x+1)^2\,(x+1)}{x\times x(x-1)\,(x+1)}$$
$$=\log\frac{(x+1)^2}{x(x-1)} \text{ **Ans**}$$

Q.3 Write the following in the form of a single logarithm.

(i) $\log 21 + \log 5$

Solution: $\log 21 + \log 5$

$= \log(21 \times 5)$ **Ans**

(ii) $\log 25 - 2\log 3$

Solution: $\log 25 - 2\log 3$

$= \log 25 - 2\log 3$

$= \log 25 - \log 3^2$

$= \log \frac{25}{3^2}$ **Ans**

(iii) $2\log x - 3\log y$

Solution: $2\log x - 3\log y$

$= 2\log x - 3\log y$

$= \log x^2 - \log y^3$

$= \log \frac{x^2}{y^3}$ **Ans**

(iv) $\log 5 + \log 6 - \log 2$

Solution: $\log 5 + \log 6 - \log 2$

$= \log 5 + \log 6 - \log 2$

$= \log(5 \times 6) - \log 2$

$= \log \frac{5 \times 6}{2}$ **Ans**

Q.4 Calculate the following.

(i) $\log_3 2 \times \log_2 81$

Solution: $\log_3 2 \times \log_2 81$

$= \frac{\log 2}{\log 3} \times \frac{\log 81}{\log 2}$

$= \frac{\log 81}{\log 3}$

$= \frac{\log 3^4}{\log 3}$

$= \frac{4\log 3}{\log 3}$

$= 4$ **Ans**

(ii) $\log_3 \times \log_3 25$

Solution: $\log_3 \times \log_3 25$

$= \frac{\log 3}{\log 5} \times \frac{\log 25}{\log 3}$

$= \frac{\log 25}{\log 5}$

$= \frac{\log 5^2}{\log 5}$

$= \frac{2\log 5}{\log 5}$

$= 2$ **Ans**

Q.5 If $\log 2 = 0.3010, \log 3 = 0.4771$ and $\log 5 = 0.6990$, then find the values of the following.

(i) $\log 32$

$= \log 2^5$

$\because$ using 3^{rd} law of logarithm

$= 5\log 2$

By putting the value of $\log 2$

$= 5(0.3010)$

$= 1.5050$ **Ans**

(ii) $\log 24$

Solution: log 24

$= \log(2^3 \times 3)$

$= \log 2^3 + \log 3$

$= 3\log 2 + \log 3$

By putting the value of $\log 2$ and $\log 3$

$= 3(0.3010) + 0.4771$

$= 0.9030 + 0.4771$

$= 1.3801$ **Ans**

(iii) $\log \sqrt{3\frac{1}{3}}$

Solution: $\log \sqrt{3\frac{1}{3}}$

$= \log\left(\frac{10}{3}\right)^{\frac{1}{2}}$

$$=\frac{1}{2}\log\left[\frac{2\times 5}{3}\right]$$

$$=\frac{1}{2}(\log 2+\log 5-\log 3)$$

By putting the values of $\log 2, \log 3$ and $\log 5$

$$=\frac{1}{2}(0.3010+0.69900-0.4771)$$

$$=\frac{1}{2}(1-0.4771)$$

$$=\frac{1}{2}(0.5229)$$

$=0.26145$ **Ans**

(iv) $\log\frac{8}{3}$

Solution: $\log\frac{8}{3}$

$$=\log\frac{2^3}{3}$$

$$=\log 2^3-\log 3$$

$$=3\log 2-\log 3$$

By putting the values of $\log 2$ and $\log 3$

$$=3(0.3010)-0.4771$$

$$=0.9030-0.4771$$

$=0.4259$ **Ans**

(v) $\log 30$

Solution: $\log 30$

$$=\log(5\times 2\times 3)$$

$\because$ using first law of logarithm

$$=\log 5+\log 2+\log 3$$

By putting the values of $\log 2, \log 3$ $\log 5$

$$=(0.6990)+(0.3010)+(0.4771)$$

$=1.4771$ **Ans**

Exercise 3.4

Q.1 Use log tables to find the value of

(i) 0.8176×13.64

Solution: 0.8176×13.64

Suppose

$x = 0.8176 \times 13.64$

Taking log on both sides

$\log x = \log(0.8176 \times 13.64)$

According to first law of logarithm

$\log x = \log 0.8176 + \log 13.64$

$= \bar{1}.9125 + 1.1348$

$\log x = -1 + 0.9125 + 1.1348$

$\log x = 1.0473$

To find antilog

$x = \text{antilog } 1.0473$

Ch $= 1$

$x = 1.115$

Reference point

$x = 11.15$ **Ans**

(ii) $(789.5)^{\frac{1}{8}}$

Solution: $(789.5)^{\frac{1}{8}}$

Let $x = (789.5)^{\frac{1}{8}}$

Taking log on both sides

$\log x = \log(789.5)^{\frac{1}{8}}$

According to third law

$\log x = \frac{1}{8}\log(789.5)$

$\log x = \frac{1}{8}(2.8974)$

$= \frac{2.8974}{8}$

$\log x = 0.3622$

To find antilog

$x = \text{antilog } 0.3622$

Characteristics = 0

$x = 2.302$

Reference point

$x = 2.302$ **Ans**

(iii) $\frac{0.678 \times 9.01}{0.0234}$

Solution: $\frac{0.678 \times 9.01}{0.0234}$

Suppose

$x = \frac{0.678 \times 9.01}{0.0234}$

Taking log on both sides

$\log x = \log \frac{0.678 \times 9.01}{0.0234}$

According to 1st and 2nd law of log

$\log x = \log 0.678 + \log 9.01 - \log 0.0234$

$\log x = \bar{1}.8312 + 0.9547 - \bar{2}.3692$

$= -1 + 0.8312 + 0.9547 - (-2 + 0.3692)$

$= 2.4167$

To find antilog

$x = \text{antilog } 2.4167$

Characteristics = 2

$x = 2.610$

$x = 261.0$ **Ans**

(iv) $\sqrt[5]{2.709} \times \sqrt[7]{1.239}$

Solution: $\sqrt[5]{2.709} \times \sqrt[7]{1.239}$

$(2.709)^{\frac{1}{5}} \times (1.239)^{\frac{1}{7}}$

Suppose:

$x = (2.709)^{\frac{1}{5}} \times (1.239)^{\frac{1}{7}}$

Taking log on both side

$\log x = \log\left[(2.709)^{\frac{1}{5}} \times (1.239)^{\frac{1}{7}}\right]$

According to law of logarithm

$\log x = \log(2.709)^{\frac{1}{5}} + \log(1.239)^{\frac{1}{7}}$

According to third law of logarithm

$\log x = \frac{1}{5}\log(2.709) + \frac{1}{7}\log(1.239)$

$\log x = \frac{1}{5}\log(2.709) + \frac{1}{7}\log(1.239)$

$=\frac{1}{5}0.4328+\frac{1}{7}0.0931$

$=\frac{0.4328}{5}+\frac{0.0931}{7}$

$0.0866+0.0133$

$=0.0999$

To find antilog

x = antilog 0.999

Characteristics = 0

$x=1.259$

Reference point

$x=1.259$ **Ans**

(v) $\frac{1.23\times0.6975}{0.0075\times1278}$

Solution: $\frac{1.23\times0.6975}{0.0075\times1278}$

Suppose

$x=\frac{1.23\times0.6975}{0.0075\times1278}$

$\log x=\log\frac{1.23\times0.6975}{0.0075\times1278}$

$=\log(1.23\times0.6975)-\log(0.0075\times1278)$

$=\log1.23+\log0.6975-(\log0.0075+\log1278)$

$=\log1.23+\log0.6975-\log0.0075-\log1278$

$=0.0899+\bar{1}.8435-\bar{3}.8751-3.1065$

$=0.8999+(-1+0.8435)-(-3+0.8751)+3.1065$

$=-1.0482$

$\log x=-2+2-1.0482$

$\log x=02+0.9515$

$\log x=\bar{2}.9518$

To find antilog

x = antilog $\bar{2}.9518$

Ch = $\bar{2}$

$x=8950$

$=0.08950$ **Ans**

(vi) $\sqrt[3]{\frac{0.7214\times20.37}{60.8}}$

Solution: $\sqrt[3]{\frac{0.7214\times20.37}{60.8}}$

Let $x=\left[\frac{0.7214\times20.37}{60.8}\right]^{\frac{1}{3}}$

Taking log on both sides

$\log x=\log\left(\frac{0.7214\times20.37}{60.8}\right)^{\frac{1}{3}}$

3rd of logarithm

$\log x=\frac{1}{3}\log\left[\frac{0.7214\times20.37}{60.8}\right]$

According to first and 2nd law

$\log x=\frac{1}{3}[\log0.7214+\log37-\log60.8]$

$\log x=\frac{1}{3}[\bar{1}.8582+1.3089-1.7839]$

$\frac{1}{3}[-1+0.8582+1.3089-1.7839]$

$=\frac{1}{3}(-0.6168)$

$=-0.2056$

$\log x$ is in negative, so

$\log x=-1+1-0.2056$

$=-1+79144$

$=\bar{1}.7944$

To find antilog

x = antilog $\bar{1}.7944$

Ch = $\bar{1}$

$x=6229$

Reference point

0.6229 **Ans**

(vii) $\frac{83\times\sqrt[3]{92}}{127\times\sqrt[5]{246}}$

Solution: $\frac{83\times\sqrt[3]{92}}{127\times\sqrt[5]{246}}$

Suppose: $x=\frac{83\times\sqrt[3]{92}}{127\times\sqrt[5]{246}}$

$$x=\frac{83\times(92)^{\frac{1}{3}}}{127\times(246)^{\frac{1}{5}}}$$

Taking on both side

$$\log x=\log\frac{83\times(92)^{\frac{1}{3}}}{127\times(246)^{\frac{1}{5}}}$$

According to 1st and 2nd law of log

$$\log x=\log 83+\log(92)^{\frac{1}{3}}-\log 127-\log(246)^{\frac{1}{5}}$$

According to third law of log

$$\log x=\log 83+\frac{1}{3}\log 92-\log 27-\frac{1}{5}\log 246$$

$$\log x=(1.9191)+\frac{1}{3}(1.9638)-(2.1038)$$

$$-\frac{1}{5}(2.3909)$$

$= 1.9191 + 0.65460 - 2.1038 - 0.47818$
$=1.9191+0.6546-2.1038-0.47818$
$=-0.0083$

$\log x$ is in negative, so

$\log x=-1+1-0.0083$
$=-1+0.9917$
$=\bar{1}.9917$

To find antilog

$x=$ antilog $\bar{1}.9917$

Ch = $\bar{1}$

$x=9.811$

Reference point

$x=0.9811$ **Ans**

(viii) $\dfrac{(438)^3\sqrt{0.056}}{(388)^4}$

Solution: $\dfrac{(438)^3\sqrt{0.056}}{(388)^4}$

Suppose: $x=\dfrac{(438)^3\sqrt{0.056}}{(388)^4}$

$$x=\frac{(438)^3(0.056)^{\frac{1}{2}}}{(388)^4}$$

$$x=\frac{(438)^3(0.056)^{\frac{1}{2}}}{(388)^4}$$

Taking log on both side

$$\log x=\log\left(\frac{(438)^3(0.056)^{\frac{1}{2}}}{(388)^4}\right)$$

According to 1st and 2nd law

$$\log x=\log(438)^3+\log(0.056)^{\frac{1}{2}}-\log(388)^4$$

According to third law

$$\log x=3\log(438)+\frac{1}{2}\log(0.056)-4\log(38)$$

$$\log x=3(2.6415)+\frac{1}{2}(\bar{2}.7482)-4(2.5888)$$

$$=7.9245+\frac{1}{2}(-2+0.7482)-10.3552$$

$$=7.9245+\frac{1}{2}(-1.2518)-10.3552$$

$=7.9245-0.6259-10.3552$
$=-3.0566$

log is in negative, so

$\log x=-4+4-3.0566$
$=-4+0.9434$

To find antilog

$x=$antilog $\bar{4}.9434$

Ch = $\bar{4}$

$x=8778$

Reference point

$=0.0008778$ **Ans**

Q.2 A gas is expanding according to the law $pv^n = C$. Find C when p = 80, v = 3.1 and $n=\frac{5}{4}$.

Solution: Given that $pv^n = C$

Taking log on both sides

Log (pv^n)= log C
Log P+log v^n= log C
Log C =log P+log v^n
Log C =log P+n log v

Putting P=80, v=3.1 and $n=\frac{5}{4}$

$\text{Log } C = \log 80 + \frac{5}{4} \log 3.1$

$= 1.9031 + \frac{5}{4}(0.4914)$

=1.9031+0.6143

Log C=2.5174

Taking antilog both sides

C=Antilog (2.5174)

C=329.2 **Ans:**

Q.3 The formula $p = 90\ (5)^{-q/10}$ applies to the demand of a product, where q is the number of units and p is the price of one unit. How many units will be demanded if the price is Rs 18.00?

Solution: Given that $p = 90(5)^{\frac{-q}{10}}$

Taking log on both sides

$Log\ p = \log\left(90(5)^{\frac{-q}{10}}\right)$

$Log\ p = \log 90 + \log 5^{\frac{-q}{10}}$

$\text{Log } P = \log 90 - \frac{q}{10} \log 5$

$\text{Log } 18 = \log 90 - \frac{q}{10} \log 5$

(P =18)

$1.2553 = 1.9542 - \frac{q}{10} \times 0.6990$

$1.2553 - 1.9542 = -\frac{q}{10} \times 0.6990$

-0.6989 x 10 = - q x 0.6990

-6.989=-q x 0.6996

6.989=q x 0.6996

$\frac{6.989}{0.6990} = q$

q =10 approximately

Hence 10 units will be demanded

Q.4 If $A = \pi r^2$, find A, when $\pi = \frac{22}{7}$ and $r = 15$.

Solution: Given that $A = \pi r^2$

Taking log on both sides

Log A=log πr^2

Log A=log π +log r^2

Log A=log π +2 log r

Putting $\pi = \frac{22}{7}$ and r =15

$\text{Log } A = \log \frac{22}{7} + 2 \log 15$

=log 22-log 7+2 log 15

=1.3424-0.8451+2(1.1761)

=0.4973+2.3522

Log A=2.8495

Taking antilog on both sides

A=antilog 2.8495

A=707.1 **Ans**

Q.5 If $V = \frac{1}{3}\pi r^2 h$, find V, when $\pi = \frac{22}{7}$, $r = 2.5$ and h=4.2.

Solution: Given that $V = \frac{1}{3}\pi r^2 h$

Taking log on both sides

$Log V = \log \frac{1}{3}\pi r^2 h$

$= \log \frac{1}{3} + \log \pi r^2 h$

=log 1-log 3+ log πr^2+ log h

= 0-0.4771+ log π +log r^2+ log h

$= -0.4771 + \log \frac{22}{7} + 2 \log r + \log h$

$\left(\pi = \frac{22}{7}, r = 2.5 \text{ and } h = 4.2\right)$

$= -0.4771 + \log 22 - \log 7 + 2\log 2.5 + \log 4.2$

$= -0.4771 + 1.3424 - 0.8450 + 2 \times 0.3979 + 0.6232$

$= -0.4771 + 1.3424 - 0.8450 + 0.7959 + 0.6232$

Log V=1.4394

Taking antilog on both sides

V=antilog 1.4394

V=27.50 **Ans**

Review Exercise 3

Q.1 Multiple choice Questions. Choose of the correct answer.

(i) If $a^x = n$, then…

(a) $a = \log_x n$ **(b)** $x = \log_n a$

(c) $x = \log_a n$ **(d)** $a = \log_n x$

(ii) The relation $y = \log_z x$ implies…

(a) $x^y = z$ **(b)** $z^y = x$

(c) $x^z = y$ **(d)** $y^z = x$

(iii) The logarithm of unity to any base is…

(a) 1 **(b)** 10

(c) e **(d)** 0

(iv) The logarithm of any number to itself as base is…

(a) 1 **(b)** 0

(c) e **(d)** 10

(v) Log e=…,where e $\approx$ 2.718

(a) 0 **(b)** 0.4343

(c) ∞ **(d)** 1

(vi) The value of $\log\left(\frac{p}{q}\right)$ is…

(a) $\log p - \log q$ **(b)** $\frac{\text{logp}}{\log q}$

(c) $\log p + \log q$ **(d)** $\log q - \log p$

(vii) Log p-log q is same as …

(a) $\log\left(\frac{q}{p}\right)$ **(b)** $\log(p-q)$

(c) $\frac{\log p}{\log q}$ **(d)** $\log q - \log p$

(viii) Log(m^n) can be written as…

(a) $(\log m)^n$ **(b)** $m \log n$

(c) $n \log m$ **(d)** $\log(mn)$

(ix) $\log_b a \times \log_c b$ **can be written as…**

(a) $\log_a c$ **(b)** $\log_c a$

(c) $\log_a b$ **(d)** $\log_b c$

(x) **$\text{Log}_y x$ will be equal to…**

(a) $\dfrac{\log_z x}{\log_y z}$ **(b)** $\dfrac{\log_x z}{\log_y z}$

(c) $\dfrac{\log_z x}{\log_z y}$ **(d)** $\dfrac{\log_z y}{\log_z x}$

ANSWER KEY

i	ii	iii	iv	v	vi	vii	viii	ix	x
c	b	d	a	b	a	d	c	b	c

Q.2 Complete the following:

(i) For common logarithm, the base is…

(ii) The integral part of the common logarithm of a number is called the …

(iii) The decimal part of the common logarithm of a number is called the …

(iv) If x = log y, then y is called the… of x.

(v) If the characteristic of the logarithm of a number have...zero(s) immediately after the decimal point.

(vi) If the characteristic of the logarithm of a number is 1, that number will have digits in its integral part.

ANSWER KEY

i	ii	iii	iv	v	vi
10	**Characteristic**	**Mantissa**	**Antilogarithm**	**One**	**2**

Q.3 Find the value of x in the following.

(i) $\log_3 x = 5$

Solution: $\log_3 x = 5$

Write in exponential form.

$3^5 = x$

$243 = x$ **Ans**

(ii) $\log_4 256 = x$

Solution: $\log_4 256 = x$

Write in exponential form

$4^x = 256$

$4^x = 4^4$

$x = 4$

$x = 4$ **Ans**

(iii) $\log_{625} 5 = \frac{1}{4}x$

Solution: $\log_{625} 5 = \frac{1}{4}x$

Write in exponential form

$(625)^{\frac{1}{4}x} = 5$

$(625)^{\frac{x}{4}} = 5$

$\left(5^4\right)^{\frac{x}{4}} = 5$

$5^{\frac{4x}{4}} = 5$
$5^x = 5^1$
$x = 1$ **Ans**

(iv) $\log_{64} x = -\frac{2}{3}$

Solution: $\log_{64} x = -\frac{2}{3}$

Write in exponential form

$(64)^{\frac{-2}{3}} = x$

$(4^3)^{-\frac{2}{3}} = x$

$4^{-\frac{6}{3}} = x$

$4^{-2} = x$

$\frac{1}{4^2} = x$

$\frac{1}{16} = x$ **Ans**

Q.4 Find the value of x in the following.

(i) $\log x = 2.4543$

Solution: $\log x = 2.4543$

$\log x = 2.4543$

$x = \text{antilog } 2.4543$

$\text{Ch} = 2$

$x = 284.6$ **Ans**

(ii) $\log x = 0.1821$

Solution: $\log x = 0.1821$

$\log x = 0.1821$

$x = \text{antilog } 0.1821$

$\text{Ch} = 0$

$x = 1.521$ **Ans**

(iii) $\log x = 0.0044$

Solution: $\log x = 0.0044$

$\log x = 0.0044$

$x = \text{antilog } 0.0044$

$\text{Ch} = 0$

$x = 1.010$ **Ans**

(iv) $\log x = \bar{1}.6238$

Solution: $\log x = \bar{1}.6238$

$\log x = \bar{1}.6238$

$x = \text{antilog } \bar{1}6333$

$\text{Ch} = \bar{1}$

$x = 0.4206$ **Ans**

Q.5 If $\log 2 = 0.3010$, $\log 3 = 0.4771$, and $\log 5 = 0.6990$ then find the values of the following.

(i) $\log 45$

Solution: $\log 45$

$= \log(9 \times 5)$

$= \log(3^2 \times 5)$

$= \log 3^2 + \log 5$

$= 2\log 3 + \log 5$

$= 2(0.4771) + 0.6990$

$= 0.9542 + 0.6990$

$= 1.6532$ **Ans**

(ii) $\log \frac{16}{15}$

Solution: $\log \frac{16}{15}$

$= \log \frac{2^4}{3 \times 5}$

$= \log 2^4 - \log(3 \times 5)$

$= 4\log 2 - (\log 3 + \log 5)$

$= \log 2^4 - \log 3 - \log 5$

$= 4\log 2 - \log 3 - \log 5$

$= 4(0.3010) - 0.4771 - 0.6990$

$= 1.2040 - 0.4771 - 0.6990$

$= 0.0279$ **Ans**

(iii) $\log 0.048$

Solution: $\log 0.048$

$= \log \frac{48}{1000}$

$= \log \frac{2\times2\times2\times2\times3}{2\times2\times2\times5\times5\times5}$

$= \log \frac{2^4\times3}{2^3\times5^3}$

$= \log 2^4 + \log 3 - \log 2^3 - \log 5^3$

$= 4\log 2 + \log 3 - 3\log 2 - 3\log 5$

$= 4(0.3010) + 0.4771 - 3(0.3010) - 3(0.6990)$

$= 1.2040 + 0.4771 - 0.9030 - 2.0970$

$= -1.3189$

$= -1 - 0.3189$

$= -1 - 1 + 1 - 0.3189$

$= -2 + 0.6811$

$= \bar{2}.6811$ **Ans**

Q.6 Simplify the following.

(i) $\sqrt[3]{25.47}$

Solution: $\sqrt[3]{25.47}$

Let $x = \sqrt[3]{25.74}$

$= (25.47)^{\frac{1}{3}}$

Taking log on both sides

$\log x = \log(25.47)^{\frac{1}{3}}$

$= \frac{1}{3}\log 25.47$

$= \frac{1}{3}(1.4060)$

$\log x = 0.4687$

$x = \text{anti}\log 0.4687$

Ch $= 0$

$x = 2.943$ **Ans**

(ii) $\sqrt[5]{342.2}$

Solution: $\sqrt[5]{342.2}$

Let

$x = \sqrt[5]{342.2}$

$x = (242.)^{\frac{1}{5}}$

Taking log on both sides

$\log x = (342.2)^{\frac{1}{5}}$

$\log x = \frac{1}{5}\log 342.2$

$= \frac{1}{5}(2.5343)$

$\log x = 0.5069$

$\log x = \text{antilog}\, 0.5069$

Ch $= 0$

$x = 3.213$ **Ans**

(iii) $\frac{(8.97)^3\times(3.95)^2}{\sqrt[3]{15.37}}$

Solution: $\frac{(8.97)^3\times(3.95)^2}{\sqrt[3]{15.37}}$

Let $x = \frac{(8.97)^3\times(3.95)^2}{\sqrt[3]{15.37}}$

Taking log on both sides

$\log x = \log\frac{(8.97)^3\times(3.95)^2}{\sqrt[3]{15.37}}$

$= \log(8.97)^3 + \log(3.95)^2 - \log(15.37)^{\frac{1}{3}}$

$= 3\log 8.97 + 2\log 3.95 - \frac{1}{3}\log 15.37$

$= 3(0.9528) + 2(0.5966) - \frac{1}{3}(1.1867)$

$= 2.8584 + 1.1932 - 0.3956$

$\log x = 3.656$

$x = \text{antilog}\, 3.656$

Ch $= 3$

$x = 4529$ **Ans**

Unit 3: Logarithms

Overview

Scientific Notation:

A number written in the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer, is called the scientific notation.

Logarithm of a Real Number:

If $a^x = y$ they x is called the logarithm of y to the base 'a' and is written as $\log_a y = x$, where $a > 0, a \neq 1$ and $y > 0$

Characteristic of logarithm of the Number:

An integral part which is positive for a number greater than 1 and negative for a number less than 1, is called the characteristic of logarithm of the number.

Mantissa of the logarithm of the Number:

A decimal part which is always positive, is called the mantissa of the logarithm of the number.

Antilogarithm:

The number whose logarithm is given is called antilogarithm.

Exercise 4.1

Q.1 Identify whether the following algebraic expressions are polynomials (Yes or No).

(i) $3x^2+\frac{1}{x}-5$

No (Because of $\frac{1}{x}$) Ans.

(ii) $3x^3-4x^2-x\sqrt{x}+3$

No (Because $\sqrt{x}$ or $(x)^{\frac{1}{2}}$) Ans.

(iii) $x^2-3x+\sqrt{2}$

Yes (Because no variable has power in fraction). **Ans**

(iv) $\frac{3x}{2x-1}+8$

No (Because of $\frac{1}{2x-1}$) **Ans**

Q.2 State whether each of the following expressions is a rational expression or not.

(i) $\frac{3\sqrt{x}}{3\sqrt{x}+5}$

Irrational **Ans**

(ii) $\frac{x^3-2x^3+\sqrt{3}}{2+3x-x^2}$

Rational **Ans**

(iii) $\frac{x^2+6x+9}{x^2-9}$

Rational **Ans**

(iv) $\frac{2\sqrt{x}+3}{2\sqrt{x}-3}$

Irrational **Ans**

Q.3 Reduce the following expression to the lowest form.

(i) $\frac{120x^2y^3z^5}{30x^3yz^2}$

Solution: $\frac{{}^4\cancel{120}x^2y^3z^5}{\cancel{30}x^3yz^2}$

$=\frac{120x^2y^3z^5}{30x^3yz^2}$

$=4x^{2-3}y^{3-1}z^{5-2}$

$=4x^{-1}y^2z^3$

$=\frac{4y^2z^3}{x}$ **Ans**

(ii) $\frac{8a(x+1)}{2(x^2-1)}$

Solution: $\frac{8a(x+1)}{2(x^2-1)}$

$=\frac{{}^4\cancel{8}a(x+1)}{\cancel{2}(x^2-1)}$

$=\frac{4a\cancel{(x+1)}}{(x-1)\cancel{(x+1)}}$

$=\frac{4a}{x-1}$ **Ans**

(iii) $\frac{(x+y)^2-4xy}{(x-y)^2}$

Solution: $\frac{(x+y)^2-4xy}{(x-y)^2}$

$\therefore (x+y)^2=x^2+y^2+2xy$

$\therefore (x-y)^2=x^2+y^2-2xy$

$=\frac{x^2+y^2+2xy-4xy}{x^2+y^2-2xy}$

$=\frac{x^2+y^2-2xy}{x^2+y^2-2xy}$

$$= \frac{\cancel{(x-y)^2}}{\cancel{(x-y)^2}}$$

$= 1$ **Ans**

(iv) $\dfrac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)}$

Solution: $\dfrac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)}$

$$(a^3+b^3) = (a-b)(a^2+ab+b^2)$$

$$= \frac{\cancel{(x^3-y^3)}(x^2-2xy+y^2)}{\cancel{(x^3-y^3)}}$$

$$= x^2-2xy+y^2$$

$$\therefore (x-y)^2 = x^2-2xy+y^2$$

$= (x-y)^2$ **Ans**

(v) $\dfrac{(x+2)(x^2-1)}{(x+1)(x^2-4)}$

Solution: $\dfrac{(x+2)(x^2-1)}{(x+1)(x^2-4)}$

$$= \frac{(x+2)\left[(x)^2-(1)^2\right]}{(x+1)\left[(x)^2-(2)^2\right]}$$

$$= \frac{\cancel{(x+2)}(x-1)\cancel{(x+1)}}{\cancel{(x+1)}(x-2)\cancel{(x+2)}}$$

$= \dfrac{(x-1)}{(x-2)}$ **Ans**

(vi) $\dfrac{x^2-4x+4}{2x^2-8}$

Solution: $\dfrac{x^2-4x+4}{2x^2-8}$

$$\therefore (a-b)^2 = a^2-2ab+b^2$$

$$\therefore a^2-b^2 = (a+b)(a-b)$$

$$= \frac{(x)^2-2(x)(2)+(2)^2}{2(x^2-4)}$$

$$= \frac{(x-2)^2}{2\left[(x)^2-(2)^2\right]}$$

$$= \frac{(x-2)^2}{2(x+2)(x-2)}$$

$$= \frac{(x-2)\cancel{(x-2)}}{2(x+2)\cancel{(x-2)}}$$

$= \dfrac{x-2}{2(x+2)}$ **Ans**

(vii) $\dfrac{64x^5-64x}{(8x^2+8)(2x+2)}$

Solution: $\dfrac{64x^5-64x}{(8x^2+8)(2x+2)}$

$$= \frac{64x(x^4-1)}{8(x^2+1).2(x+1)}$$

$$= \frac{64\left[(x^2)^2-(1)^2\right]}{16(x^2+1)(x+1)}$$

$$= \frac{{}^{4}\cancel{64}(x^2-1)\cancel{(x^2+1)}}{\cancel{16}\cancel{(x^2+1)}(x+1)}$$

$$= \frac{4x(x-1)\cancel{(x+1)}}{\cancel{(x+1)}}$$

$= 4x(x-1)$ **Ans**

(viii) $\dfrac{9x^2-(x^2-4)^2}{4+3x-x^2}$

Solution: $\dfrac{9x^2-(x^2-4)^2}{4+3x-x^2}$

$$=\frac{(3x)^2-(x^2-4)^2}{4+3x-x^2}$$

$$=\frac{(3x+x^2-4)(3x-x^2+4)}{4+3x-x^2}$$

$$=\frac{(x^2+3x-4)\cancel{(-x^2+3x+4)}}{\cancel{(-x^2+3x+4)}}$$

$$=x^2+3x-4 \textbf{ Ans}$$

Q.4 Evaluate

(a) $\frac{x^3y-2z}{xz}$ **for**

(i) $x=3, y=-1, z=-2$

(ii) $x=-1, y=-9, z=4$

Solution for 1st part

When $x=3,\ y=-1,\ z=-2$

$$\frac{x^3y-2z}{xz}=$$

$$=\frac{(3)^3(-1)-2(-2)}{(3)(-2)}$$

$$=\frac{27(-1)+4}{-6}$$

$$=\frac{-27+4}{-6}$$

$$=\frac{-23}{-6}$$

$$=\frac{23}{6} \textbf{ Ans}$$

Solution for 2nd Part.

When $x=-1,\ y=-9,\ z=4$

$$\frac{x^3y-2z}{xz}=$$

$$=\frac{(-1)^3(-9)-2(4)}{(-1)(4)}$$

$$=\frac{-1(-9)-8}{-4}$$

$$=\frac{9-8}{-4}$$

$$=\frac{1}{-4}$$

$$=-\frac{1}{4} \textbf{ Ans}$$

(b) $\frac{x^2y^2-5z^4}{xyz}$ for $x=4, y=-2$ and $z=-1$

Solution: $\frac{x^2y^3-5z^4}{xyz}$

$$=\frac{(4)^2(-2)^3-5(-1)^4}{(4)(-2)(-1)}$$

$$=\frac{16(-8)-5(1)}{8}$$

$$=\frac{16(-8)-5(1)}{8}$$

$$=\frac{-128-5}{8}$$

$$=-\frac{133}{8}$$

$$=-16\frac{5}{8} \textbf{ Ans}$$

Q.5 Perform the indicated operation and simplify.

(i) $\frac{15}{2x-3y}-\frac{4}{3y-2x}$

Solution: $\frac{15}{2x-3y}-\frac{4}{3y-2x}$

$$=\frac{15}{2x-3y}-\frac{4}{-2x+3y}$$

$$=\frac{15}{2x-3y}-\frac{4}{-(2x-3y)}$$

$$=\frac{15}{2x-3y}+\frac{4}{2x-3y}$$

$$=\frac{19}{2x-3y} \textbf{ Ans}$$

(ii) $\frac{1+2x}{1-2x}-\frac{1-2x}{1+2x}$

Solution: $\frac{1+2x}{1-2x}-\frac{1-2x}{1+2x}$

$$=\frac{(1+2x)^2-(1-2x)^2}{(1-2x)(1+2x)}$$

$$=\frac{(1)^2+(2x)^2+2(2x)(1)-\left[(1)^2+(2x)^2-2(2x)(1)\right]}{(1)^2-(2x)^2}$$

$$=\frac{1+4x^2+4x-\left[1+4x^2-4x\right]}{1-4x^2}$$

$$=\frac{1+4x^2+4x-1-4x^2+4x}{1-4x^2}$$

$$=\frac{4x+4x}{1-4x^2}$$

$$=\frac{8x}{1-4x^2}$$ **Ans**

(iii) $\frac{x^2-25}{x^2-36}-\frac{x+5}{x+6}$

Solution: $\frac{x^2-25}{x^2-36}-\frac{x+5}{x+6}$

$$=\frac{(x)^2-(5)^2}{(x)^2-(6)^2}-\frac{x+5}{x+6}$$

$$=\frac{(x+5)(x-5)}{(x+6)(x-6)}-\frac{x+5}{x+6}$$

$$=\frac{(x+5)(x-5)-(x-6)(x+5)}{(x+6)(x-6)}$$

$$=\frac{(x+5)\left[(x-5)-(x-6)\right]}{x^2-6^2}$$

$$=\frac{(x+5)(x-5-x+6)}{x^2-36}$$

$$=\frac{(x+5)(1)}{x^2-36}$$

$$=\frac{x+5}{x^2-36}$$ **Ans**

(iv) $\frac{x}{x-y}-\frac{y}{x+y}-\frac{2xy}{x^2-y^2}$

Solution: $\frac{x}{x-y}-\frac{y}{x+y}-\frac{2xy}{x^2-y^2}$

$$=\frac{x(x+y)-y(x-y)}{(x-y)(x+y)}-\frac{2xy}{x^2-y^2}$$

$$=\frac{x^2+\cancel{xy}-\cancel{xy}+y^2}{(x)^2-(y)^2}-\frac{2xy}{x^2-y^2}$$

$$=\frac{x^2+y^2}{x^2-y^2}-\frac{2xy}{x^2-y^2}$$

$$=\frac{x^2+y^2-2xy}{x^2-y^2}$$

$$=\frac{(x-y)^2}{x^2-y^2}$$

$$=\frac{(x-y)\cancel{(x-y)}}{(x+y)\cancel{(x-y)}}$$

$$=\frac{x-y}{x+y}$$ **Ans**

(v) $\frac{x-2}{x^2+6x+9}-\frac{x+2}{2x^2-18}$

Solution: $\frac{x-2}{x^2+6x+9}-\frac{x+2}{2x^2-18}$

$$=\frac{x-2}{(x)^2+2(3)(x)+3^2}-\frac{x+2}{2(x^2-9)}$$

$$=\frac{x-2}{(x+3)^2}-\frac{x+2}{2\left[(x)^2-(3)^2\right]}$$

$$=\frac{x-2}{(x+3)^2}-\frac{x+2}{2(x-3)(x+3)}$$

$$=\frac{x-2}{(x+3)(x+3)}-\frac{x+2}{2(x+3)(x-3)}$$

$$=\frac{2(x-3)(x-2)-(x+3)(x+2)}{2(x+3)(x+3)(x-3)}$$

$$=\frac{2(x^2-2x-3x+6)-(x^2+2x+3x+6)}{2(x+3)(x+3)(x-3)}$$

$$=\frac{2(x^2-5x+6)-(x^2+5x+6)}{2(x+3)(x+3)(x-3)}$$

$$= \frac{2x^2 - 10x + 12 - x^2 - 5x - 6}{2(x+3)^2(x-3)}$$

$$= \frac{x^2 - 15x + 6}{2(x+3)^2(x-3)}$$ **Ans**

(vi) $\frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$

Solution: $\frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$

$$= \frac{(x+1)-(x-1)}{(x-1)(x+1)} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{\cancel{x}+1-\cancel{x}+1}{x^2-1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{2}{x^2-1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{2(x^2+1)-2(x^2-1)}{(x^2-1)(x^2+1)} - \frac{4}{x^4-1}$$

$$= \frac{\cancel{2x^2}+2-\cancel{2x^2}+2}{(x^2)^2-(1)^2} - \frac{4}{x^4-1}$$

$$= \frac{4}{x^4-1} - \frac{4}{x^4-1}$$

$$= \frac{4-4}{x^4-1}$$

$$= \frac{0}{x^4-1}$$

$= 0$ **Ans**

Q.6 Perform the indicated operation and simplify.

(i) $(x^2-49).\frac{5x+2}{x+7}$

Solution: $(x^2-49).\frac{5x+2}{x+7}$

$$= \left[(x)^2-(7)^2\right].\frac{5x+2}{x+7}$$

$$= (x+7)(x-7)\frac{(5x+2)}{(x+7)}$$

$= (x-7)(5x+2)$ **Ans**

(ii) $\frac{4x-12}{x^2-9} \div \frac{18-2x^2}{x^2+6x+9}$

Solution: $\frac{4x-12}{x^2-9} \div \frac{18-2x^2}{x^2+2(x)(3)+(3)^2}$

$$= \frac{4(x-3)}{(x^2)-(3)^2} \div \frac{2(9-x^2)}{(x+3)^2}$$

$$= \frac{4\cancel{(x-3)}}{\cancel{(x-3)}(x+3)} \times \frac{(x+3)^2}{2(9-x^2)}$$

$$= \frac{4}{x+3} \times \frac{(x+3)^2}{2(3+x)(3-x)}$$

$$= \frac{{}^2\cancel{4}\,x\,\cancel{(x+3)^2}}{\cancel{2}\,\cancel{(x+3)^2}(3-x)}$$

$$= \frac{2}{3-x}$$ **Ans**

(iii) $\frac{x^6-y^6}{x^2-y^2} \div (x^4+x^2y^2+y^4)$

Solution: $\frac{x^6-y^6}{x^2-y^2} \div (x^4+x^2y^2+y^4)$

$$= \frac{(x^2)^3-(y^2)^3}{x^2-y^2} \div (x^4+x^2y^2+y^4)$$

$$= \frac{\cancel{(x^2-y^2)}\left[(x^2)^2+x^2y^2+(y^2)^2\right]}{\cancel{(x^2-y^2)}} \div (x^4+x^2y^2+y^4)$$

$$= \cancel{(x^4+x^2y^2+y^4)} \times \frac{1}{\cancel{(x^4+x^2y^2+y^4)}}$$

$= 1$ **Ans**

(iv) $\frac{x^2-1}{x^2+2x+1}.\frac{x+5}{1-x}$

Solution: $\frac{x^2-1}{x^2+2x+1}.\frac{x+5}{1-x}$

$$= \frac{(x+1)(x-1)}{(x^2+2(x)(1)+(1)^2)} \times \frac{x+5}{-(x-1)}$$

$$= \frac{(x+1)\cancel{(x-1)}}{(x+1)^2} \times \frac{(x+5)}{-\cancel{(x-1)}}$$

$$= -\frac{\cancel{(x+1)}(x+5)}{\cancel{(x+1)}(x+1)}$$

$$= -\frac{(x+5)}{x+1}$$ **Ans**

(v) $\frac{x^2+xy}{y(x+y)} \cdot \frac{x^2+xy}{y(x+y)} \div \frac{x^2-x}{xy-2y}$

Solution: $\frac{x^2+xy}{y(x+y)} \cdot \frac{x^2+xy}{y(x+y)} \div \frac{x^2-x}{xy-2y}$

$$= \frac{x\cancel{(x+y)}}{y\cancel{(x+y)}} \cdot \frac{x\cancel{(x+y)}}{y\cancel{(x+y)}} \div \frac{x(x-1)}{y(x-2)}$$

$$= \frac{x.\cancel{x}}{y.\cancel{y}} \times \frac{\cancel{y}(x-2)}{\cancel{x}(x-1)}$$

$$= \frac{x(x-2)}{y(x-1)}$$ **Ans**

Exercise 4.2

Q.1 Solve

(i) If $a+b=10$ and $a-b=6$, then find the value of (a^2+b^2)

Solution:

$2(a^2+b^2)=(a+b)^2+(a-b)^2$

$2(a^2+b^2)=(10)^2+(6)^2$

$2(a^2+b^2)=100+36$

$2(a^2+b^2)=136$

$(a^2+b^2)=\frac{136}{2}$ (136 cancelled to 68)

$(a^2+b^2)=68$ **Ans**

(ii) If $a+b=5, a-b=\sqrt{17}$, then find the value of ab.

Solution:

$4ab=(a+b)^2-(a-b)^2$

$4ab=(5)^2-(\sqrt{17})^2$

$4ab=25-17$

$4ab=8$

$ab=\frac{8}{4}$

$ab=2$

$ab=2$ **Ans**

Q.2 If $a^2+b^2+c^2=45$ and $a+b+c=-1$, then find the value of $ab+bc+ca$.

Solution: $a^2+b^2+c^2=45$

$a+b+c=-1$

$ab+bc+ca$ **=?**

We know that

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

$(-1)^2=45+2(ab+bc+ca)$

$1=45+2(ab+bc+ca)$

$1-45=2(ab+bc+ca)$

$-44=2(ab+bc+ca)$

$\frac{-44}{2}=(ab+bc+ca)$ (44 cancelled to 22)

$(ab+bc+ca)=-22$ **Ans**

Q.3 If $m+n+p=10$ and $mn+np+np=27$, find the value of $m^2+n^2+p^2$

Solution: $m+n+p=10$

$mn+np+np=27,$

$m^2+n^2+p^2$ **=?**

We know that

$(m+n+p)^2=m^2+n^2+p^2+2mn+2np+2mp$

$(10)^2=m^2+n^2+p^2+2(mn+np+mp)$

$100=m^2+n^2+p^2+2(27)$

$100=m^2+n^2+p^2+54$

$100-54=m^2+n^2+p^2$

$m^2+n^2+p^2=46$ **Ans**

Q.4 If $x^2+y^2+z^2=78$ and $xy+yz+zx=59$, find the value of $x+y+z$.

Solution: $x^2+y^2+z^2=78$

$xy+yz+zx=59,$

$x+y+z$ **=?**

We know that

$(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$

$(x+y+z)^2=78+2(xy+yz+zx)$

$(x+y+z)^2=78+2(59)$

$(x+y+z)^2=78+118$

$(x+y+z)^2=196$

Taking square root at both sides

$\sqrt{(x+y+z)^2} = \pm\sqrt{196}$

$x+y+z = \pm 14$ **Ans**

Q.5 **If** $x+y+z=12$ **and** $x^2+y^2=64$, **find the value of** $xy+yz+zx$.

Solution: $x+y+z=12$

$x^2+y^2=64$

$xy+yz+zx$ **=?**

We know that

$(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

$(x+y+z)^2 = x^2+y^2+z^2+2(xy+yz+zx)$

$(12)^2 = 64+2(xy+yz+zx)$

$144-64 = 2(xy+yz+zx)$

$80 = 2(xy+yz+zx)$

$\frac{\overset{40}{\cancel{80}}}{\cancel{2}} = (xy+yz+zx)$

$40 = xy+yz+zx$

$xy+yz+zx = 40$ **Ans**

Q.6 **If** $x+y=7$ **and** $xy=12$, **then find the value of** x^3+y^3

Solution: $x+y=7$

$xy=12$

x^3+y^3 **=?**

We know that

$(x+y)^3 = x^3+y^3+3xy(x+y)$

$(7)^3 = x^3+y^3+3(12)(7)$

$343 = x^3+y^3+252$

$343-252 = x^3+y^3$

$91 = x^3+y^3$

$x^3+y^3 = 91$ **Ans**

Q.7 **If** $3x+4y=11$ **and** $xy=12$, **then find the value of** $27x^3+64y^3$.

Solution: $3x+4y=11$

$xy=12$

$27x^3+64y^3$ **=?**

$(x+y)^3 = x^3+y^3+3xy(x+y)$

$(3x+4y)^3 = (3x)^3+(4y)^3+3(3x)(4y)(3x+4y)$

$(3x+4y)^3 = 27x^3+64y^3+36xy(3x+4y)$

$(11)^3 = 27x^3+64y^3+36(12)(11)$

$1331 = 27x^3+64y^3+4752$

$1331-4752 = 27x^3+64y^3$

$-3421 = 27x^3+64y^3$

$27x^3+64y^3 = -3421$ Ans

Q.8 **If** $x-y=4$ **and** $xy=21$, **then find the value of** x^3-y^3

Solution: $x-y=4$

$xy=21$

x^3-y^3 **=?**

We know that

$(x-y)^3 = x^3-y^3-3xy(x-y)$

$(4)^3 = x^3-y^3-3(21)(4)$

$64 = x^3-y^3-252$

$64+252 = x^3-y^3$

$316 = x^3-y^3$

$x^3-y^3 = 316$ **Ans**

Q.9 **If** $5x-6y=13$ **and** $xy=6$, **then find the value of b**$125x^3-216y^3$

Solution: $5x-6y=13$

$xy=6$

$125x^3-216y^3$**=?**

We know that

$(x-y)^3 = x^3-y^3-3xy(x-y)$

$(5x-6y)^3 = (5x)^3-(6y)^3-3(5x)(6y)(5x-6y)$

$(5x-6y)^3 = 125x^3-216y^3-90xy(5x-6y)$

$(13)^3 = 125x^3-216y^3-90(6)(13)$

$2197 = 125x^3-216y^3-7020$

$2197+7020 = 125x^3-216y^3$

$9217 = 125x^3 - 216y^3$

$125x^3 - 216y^3 = 9217$ **Ans**

Q.10 **If** $x + \frac{1}{x} = 3$ **then find the value of** $x^3 + \frac{1}{x^3}$

Solution: $x + \frac{1}{x} = 3$

$x^3 + \frac{1}{x^3} = ?$

We know that

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$$

$$(3)^3 = x^3 + \frac{1}{x^3} + 3(3)$$

$$27 = x^3 + \frac{1}{x^3} + 9$$

$$27 - 9 = x^3 + \frac{1}{x^3}$$

$$18 = x^3 + \frac{1}{x^3}$$

$x^3 + \frac{1}{x^3} = 18$ **Ans**

Q.11 **If** $x - \frac{1}{x} = 7$, **then find the value of** $x^3 - \frac{1}{x^3}$

Solution: $x - \frac{1}{x} = 7$

$x^3 - \frac{1}{x^3} = ?$

We know that

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$$

$$(7)^3 = x^3 - \frac{1}{x^3} - 3(7)$$

$$343 = x^3 - \frac{1}{x^3} - 21$$

$$343 + 21 = x^3 - \frac{1}{x^3}$$

$$364 = x^3 - \frac{1}{x^3}$$

$x^3 - \frac{1}{x^3} = 364$ **Ans**

Q.12 **If** $\left[3x + \frac{1}{3x}\right] = 5$, **then find the value of** $\left[27x^3 + \frac{1}{27x^3}\right]$

Solution: $\left[3x + \frac{1}{3x}\right] = 5$

$\left[27x^3 + \frac{1}{27x^3}\right] = ?$

We know that

$$\left(3x + \frac{1}{3x}\right)^3 = (3x)^3 + \left(\frac{1}{3x}\right)^3 + 3(3x)\left(\frac{1}{3x}\right)\left(3x + \frac{1}{3x}\right)$$

$$(5)^3 = 27x^3 + \frac{1}{27x^3} + 3\left(3x + \frac{1}{3x}\right)$$

$$125 = 27x^3 + \frac{1}{27x^3} + 3(5)$$

$$125 = 27x^3 + \frac{1}{27x^3} + 15$$

$$125 - 15 = 27x^3 + \frac{1}{27x^3}$$

$$110 = 27x^3 + \frac{1}{27x^3}$$

$27x^3 + \frac{1}{27x^3} = 110$ **Ans**

Q.13 **If** $\left(5x - \frac{1}{5x}\right) = 6$, **then find the value of** $\left(125x^3 - \frac{1}{125x^3}\right)$

Solution: $\left(5x - \frac{1}{5x}\right) = 6$

$\left(125x^3 - \frac{1}{125x^3}\right) = ?$

We know that

$$\left(5x-\frac{1}{5x}\right)^3=(5x)^3-\left(\frac{1}{5x}\right)^3-3(5x)\left(\frac{1}{5x}\right)\left(5x-\frac{1}{5x}\right)$$

$$(6)^3=125x^3-\frac{1}{125x^3}-3(6)$$

$$216=125x^3-\frac{1}{125x^3}-18$$

$$216+18=125x^3-\frac{1}{125x^3}$$

$$234=125x^3-\frac{1}{125x^3}$$

$$125x^3-\frac{1}{125x^3}=234 \textbf{ Ans}$$

Q.14 Factorize

(i) x^3-y^3-x+y

Solution: x^3-y^3-x+y

$$=(x)^3-(y)^3-1(x-y)$$

$$=(x-y)(x^2+xy+y^2)-1(x-y)$$

$$=(x-y)(x^2+xy+y^2-1) \textbf{ Ans}$$

(ii) $8x^3-\frac{1}{27y^3}$

Solution: $8x^3-\frac{1}{27y^3}$

$$=(2x)^3-\left(\frac{1}{3y}\right)^3$$

$$=\left[2x-\frac{1}{3y}\right]\left[(2x)^2+(2x)\left(\frac{1}{3y}\right)+\left(\frac{1}{3y}\right)^2\right]$$

$$=\left(2x-\frac{1}{3y}\right)\left(4x^2+\frac{2x}{3y}+\frac{1}{9y^2}\right) \textbf{ Ans}$$

Q.15 Find the products, using formula.

(i) $(x^2+y^2)(x^4-x^2y^2+y^4)$

Solution: $(x^2+y^2)(x^4-x^2y^2+y^4)$

$$=(x^2+y^2)\left[(x^2)^2-(x^2)(y^2)+(y^2)^2\right]$$

$$\left[(x^2)^3+(y^2)^3\right]$$

$$=x^6+y^6 \textbf{ Ans}$$

(ii) $(x^3-y^3)(x^6+x^3y^3+y^6)$

Solution: $(x^3-y^3)(x^6+x^3y^3+y^6)$

$$(x^3-y^3)\left[(x^3)^2+(x^3)(y^3)+(y^3)^2\right]$$

$$=(x^3)^3-(y^3)^3$$

$$=x^9-y^9 \textbf{ Ans}$$

(iii) $(x-y)(x+y)(x^2+y^2)(x^2+xy+y^2)$
$(x^2+xy+y^2)(x^4-x^2y^2+y^4)$

Solution:

$$(x-y)(x+y)(x^2+y^2)(x^2+xy+y^2)$$

$$(x^2+xy+y^2)(x^4-x^2y^2+y^4)$$

$$=\left[(x-y)(x^2+xy+y^2)\right]\left[(x+y)(x^2-xy+y^2)\right]$$

$$\left[(x^2+y^2)(x^4-x^2y^2+y^4)\right]$$

$$=\left[(x^3-y^3)(x^3+y^3)\right]\left[(x^2)^3+(y^2)^3\right]$$

$$=\left[(x^3)^2-(y^3)^2\right]\left[(x^6+y^6)\right]$$

$$=\left[(x^6-y^6)(x^6+y^6)\right]$$

$$=\left[(x^6)^2-(y^6)^2\right]$$

$$=x^{12}-y^{12} \textbf{ Ans}$$

(iv) $(2x^2-1)(2x^2+1)(4x^4+2x^2+1)(4x^4-2x^2+1)$

Solution:

$$(2x^2-1)(2x^2+1)(4x^4+2x^2+1)(4x^4-2x^2+1)$$

$$=\left[(2x^2-1)(4x^4+2x^2+1)\right]\left[(2x^2+1)(4x^4-2x^2+1)\right]$$

$$=\left[(2x^2)^3-(1)^3\right]\left[(2x^2)^3+(1)^3\right]$$

$$=(8x^6-1)(8x^6+1)$$

$$=(8x^6)^2-(1)^2$$

$$=64x^{12}-1 \ \textbf{Ans}$$

Exercise 4.3

Q.1 Express each of the following surd in the simplest form:

(i) $\sqrt{180}$

Solution: $\sqrt{180}$

$$=(180)^{\frac{1}{2}}$$
$$=(2\times2\times3\times3\times5)^{\frac{1}{2}}$$
$$=(2^2\times3^2\times5)^{\frac{1}{2}}$$
$$=2^{2\times\frac{1}{2}}\times3^{2\times\frac{1}{2}}\times5^{\frac{1}{2}}$$
$$=2\times3\times\sqrt{5}$$
$$=6\sqrt{5}$$ **Ans**

(ii) $3\sqrt{162}$

Solution: $3\sqrt{162}$

$$3\left(\sqrt{81\times2}\right)$$
$$=3\left(\sqrt{9^2}\times\sqrt{2}\right)$$
$$=3\times9\left(\sqrt{2}\right)$$
$$=27\sqrt{2}$$ **Ans**

(iii) $\frac{3}{4}\sqrt[3]{128}$

Solution: $\frac{3}{4}\sqrt[3]{128}$

$$=\frac{3}{4}\left(\sqrt[3]{64\times2}\right)$$
$$=\frac{3}{4}\left(\sqrt[3]{4^3\times2}\right)$$
$$=\frac{3}{4}\left[\sqrt[3]{4^3}\times\sqrt[3]{2}\right]$$
$$=\frac{3}{4}\times4\times\sqrt[3]{2}$$
$$=3\times\sqrt[3]{2}$$
$$=3\sqrt[3]{2}$$ **Ans**

(iv) $\sqrt[5]{96x^6y^7z^8}$

Solution: $\sqrt[5]{96x^6y^7z^8}$

$$=\sqrt{32\times3\times x^5y^5z^5\times x^1y^2z^3}$$
$$=\sqrt[5]{2^5\times3\times x^5y^5z^5\times xy^2z^3}$$
$$=\sqrt[5]{2^5x^5y^5z^5}\times\sqrt[5]{3xy^2z^3}$$
$$=\sqrt[5]{2^5}\times\sqrt[5]{x^5}\times\sqrt[5]{y^5}\times\sqrt[5]{z^5}\times\sqrt[5]{3xy^2z^3}$$
$$=2xyz\sqrt[5]{3xy^2z^3}$$ **Ans**

Q.2 Simplify

(i) $\frac{\sqrt{18}}{\sqrt{3}\sqrt{2}}$

Solution: $\frac{\sqrt{18}}{\sqrt{3}\sqrt{2}}$

$$=\frac{\sqrt{9\times2}}{\sqrt{3}\times\sqrt{2}}$$
$$=\frac{\sqrt{3^2}\times\cancel{\sqrt{2}}}{\sqrt{3}\times\cancel{\sqrt{2}}}$$
$$=\frac{3}{\sqrt{3}}$$
$$=\frac{3}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$
$$=\frac{3\sqrt{3}}{\left(\sqrt{3}\right)^2}$$
$$=\frac{\cancel{3}\sqrt{3}}{\cancel{3}}$$
$$=\sqrt{3}$$ **Ans**

(ii) $\frac{\sqrt{21}\sqrt{9}}{\sqrt{63}}$

Solution: $\frac{\sqrt{21}\sqrt{9}}{\sqrt{63}}$

$=\frac{\sqrt{21}\sqrt{3^2}}{\sqrt{9\times 7}}$

$=\frac{\sqrt{21}\times 3}{\sqrt{3^2}\times\sqrt{7}}$

$=\frac{\sqrt{21}\times 3}{3\sqrt{7}}$

$=\frac{\cancel{3}\sqrt{21}}{\cancel{3}\sqrt{7}}$

$=\frac{\sqrt{21}}{\sqrt{7}}$

$=\frac{\sqrt{7\times 3}}{\sqrt{7}}$

$=\frac{\cancel{\sqrt{7}}\times\sqrt{3}}{\cancel{\sqrt{7}}}$

$=\sqrt{3}$ **Ans**

(iii) $=\sqrt[5]{243x^5y^{10}z^{15}}$

Solution: $=\sqrt[5]{243x^5y^{10}z^{15}}$

$=\sqrt[5]{3^5x^5\left(y^2\right)^5\left(z^3\right)^5}$

$=\sqrt[3]{3^5}\times\sqrt[5]{x^5}\times\sqrt[5]{\left(y^2\right)^5}\times\sqrt[3]{\left(z^3\right)^5}$

$=3\times x\times y^2\times z^3$

$=3xy^2z^3$ **Ans**

(iv) $\frac{4}{5}\sqrt[3]{125}$

Solution: $\frac{4}{5}\sqrt[3]{125}$

$=\frac{4}{5}\sqrt[3]{5\times 5\times 5}$

$=\frac{4}{5}\sqrt[3]{5^3}$

$=\frac{4}{\cancel{5}}\times\cancel{5}$

$=4$ **Ans**

(v) $\sqrt{21}\times\sqrt{7}\times\sqrt{3}$

Solution: $\sqrt{21}\times\sqrt{7}\times\sqrt{3}$

$=\sqrt{7\times 3}\times\sqrt{7}\times\sqrt{3}$

$=\sqrt{7\times 3\times 7\times 3}$

$=\sqrt{7\times 7\times 3\times 3}$

$=\sqrt{7^2}\times\sqrt{3^2}$

$=7\times 3$

$=21$ **Ans**

Q.3 Simplify by combining similar terms.

(i) $\sqrt{45}-3\sqrt{20}+4\sqrt{5}$

Solution: $\sqrt{45}-3\sqrt{20}+4\sqrt{5}$

$=\sqrt{9\times 5}-3\sqrt{5\times 4}+4\sqrt{5}$

$=\sqrt{3^2}\times\sqrt{5}-3\sqrt{2^2}\times\sqrt{5}+4\sqrt{5}$

$=3\sqrt{5}-3\times 2\sqrt{5}+4\sqrt{5}$

$=3\sqrt{5}-6\sqrt{5}+4\sqrt{5}$

$=\sqrt{5}\left(3-6+4\right)$

$=\sqrt{5}\left(3-2\right)$

$=\sqrt{5}\left(1\right)$

$=\sqrt{5}$ **Ans**

(ii) $4\sqrt{12}+5\sqrt{27}-3\sqrt{75}+\sqrt{300}$

Solution: $4\sqrt{12}+5\sqrt{27}-3\sqrt{75}+\sqrt{300}$

$=4\sqrt{4\times 3}+5\sqrt{9\times 3}-3\sqrt{25\times 3}+\sqrt{100\times 3}$

$=4\times 2\sqrt{3}+5\times 3\sqrt{3}-3\times 5\sqrt{3}+10\sqrt{3}$

$=8\sqrt{3}+15\sqrt{3}-15\sqrt{3}+10\sqrt{3}$

$=8\sqrt{3}+15\sqrt{3}-15\sqrt{3}+10\sqrt{3}$

$= \sqrt{3}\left(8 + \cancel{15} - \cancel{15} + 10\right)$

$= \sqrt{3}\left(8 + 10\right)$

$= \sqrt{3}\left(18\right)$

$= 18\sqrt{3}$ **Ans**

(iii) $\sqrt{3}\left(2\sqrt{3} + 3\sqrt{3}\right)$

Solution: $\sqrt{3}\left(2\sqrt{3} + 3\sqrt{3}\right)$

$= \sqrt{3} \times \sqrt{3}\left(2 + 3\right)$

$= \left(\sqrt{3}\right)^2 \times \left(5\right)$

$= 3\left(5\right)$

$= 15$ Ans

(iv) $2\left(6\sqrt{5} - 3\sqrt{5}\right)$

Solution: $2\left(6\sqrt{5} - 3\sqrt{5}\right)$

$= 2 \times \sqrt{5}\left(6 - 3\right)$

$= 2 \times \sqrt{5}\left(3\right)$

$= 6\sqrt{5}$ Ans

Q.4 Simplify

(i) $\left(3 + \sqrt{3}\right)\left(3 - \sqrt{3}\right)$

Solution: $\left(3 + \sqrt{3}\right)\left(3 - \sqrt{3}\right)$

$= \left(3\right)^2 - \left(\sqrt{3}\right)^2$

$= 9 - 3$

$= 6$ **Ans**

(ii) $\left(\sqrt{5} + \sqrt{3}\right)^2$

Solution: $\left(\sqrt{5} + \sqrt{3}\right)^2$

$= \left(\sqrt{5}\right)^2 + 2\left(\sqrt{5}\right)\left(\sqrt{3}\right) + \left(\sqrt{3}\right)^2$

$= 5 + 2\sqrt{5 \times 3} + 3$

$= 8 + 2\sqrt{15}$ **Ans**

(iii) $\left(\sqrt{5} + \sqrt{3}\right)\left(\sqrt{5} - \sqrt{3}\right)$

Solution: $\left(\sqrt{5} + \sqrt{3}\right)\left(\sqrt{5} - \sqrt{3}\right)$

$= \left(\sqrt{5}\right)^2 - \left(\sqrt{3}\right)^2$

$= 5 - 3$

$= 2$ **Ans**

(iv) $\left(\sqrt{2} + \frac{1}{\sqrt{3}}\right)\left(\sqrt{2} - \frac{1}{\sqrt{3}}\right)$

Solution: $\left(\sqrt{2} + \frac{1}{\sqrt{3}}\right)\left(\sqrt{2} - \frac{1}{\sqrt{3}}\right)$

$= \left(\sqrt{2}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2$

$= 2 - \frac{(1)^2}{\left(\sqrt{3}\right)^2}$

$= 2 - \frac{1}{3}$

$= \frac{6 - 1}{3}$

$= \frac{5}{3}$ **Ans**

(v) $\left(\sqrt{x} + \sqrt{y}\right)\left(\sqrt{x} - \sqrt{y}\right)\left(x + y\right)\left(x^2 + y^2\right)$

Solution:

$\left(\sqrt{x} + \sqrt{y}\right)\left(\sqrt{x} - \sqrt{y}\right)\left(x + y\right)\left(x^2 + y^2\right)$

$= \left[\left(\sqrt{x}\right)^2 - \left(\sqrt{y}\right)^2\right]\left(x + y\right)\left(x^2 + y^2\right)$

$= \left(x - y\right)\left(x + y\right)\left(x^2 + y^2\right)$

$= \left[\left(x\right)^2 - \left(y\right)^2\right]\left(x^2 + y^2\right)$

$= \left(x^2 - y^2\right)\left(x^2 + y^2\right)$

$$=\left[\left(x^2\right)^2-\left(y^2\right)^2\right]$$

$$=x^4-y^4$$ **Ans**

Exercise 4.4

Q.1 Rationalize the denominator of the following

(i) $\frac{3}{4\sqrt{3}}$

Solution: $\frac{3}{4\sqrt{3}}$

$$=\frac{3}{4\sqrt{3}}$$

$$=\frac{3}{4\sqrt{3}}\times\frac{4\sqrt{3}}{4\sqrt{3}}$$

$$=\frac{3\left(4\sqrt{3}\right)}{\left(4\sqrt{3}\right)^2}$$

$$=\frac{12\sqrt{3}}{16\left(\sqrt{3}\right)^2}$$

$$=\frac{12\sqrt{3}}{16\times3}$$

$$=\frac{\cancel{12}\sqrt{3}}{\cancel{48}}$$

$$=\frac{\sqrt{3}}{4}\ \textbf{Ans}$$

(ii) $\frac{14}{\sqrt{98}}$

Solution: $\frac{14}{\sqrt{98}}$

$$=\frac{14}{\sqrt{98}}$$

$$=\frac{14}{\sqrt{98}}\times\frac{\sqrt{98}}{\sqrt{98}}$$

$$=\frac{14\left(\sqrt{98}\right)}{\left(\sqrt{98}\right)^2}$$

$$=\frac{14\left(\sqrt{7\times7\times2}\right)}{98}$$

$$=\frac{14\times7\times\sqrt{2}}{98}$$

$$=\frac{\cancel{98}\times\sqrt{2}}{\cancel{98}}$$

$$=\sqrt{2}\ \textbf{Ans}$$

(iii) $\frac{6}{\sqrt{8}\sqrt{27}}$

Solution: $\frac{6}{\sqrt{8}\sqrt{27}}$

$$=\frac{6}{\sqrt{8}\sqrt{27}}$$

$$=\frac{6}{\sqrt{8}\sqrt{27}}\times\frac{\sqrt{8}\sqrt{27}}{\sqrt{8}\sqrt{27}}$$

$$=\frac{6\left(\sqrt{8}\sqrt{27}\right)}{\left(\sqrt{8}\right)^2\left(\sqrt{27}\right)^2}$$

$$=\frac{6\left(\sqrt{4\times2}\right)\left(\sqrt{9\times3}\right)}{8\times27}$$

$$=\frac{6\times2\sqrt{2}\times3\sqrt{3}}{216}$$

$$=\frac{6\times3\times2\left(\sqrt{2\times3}\right)}{216}$$

$$=\frac{\cancel{36}\sqrt{6}}{\cancel{216}^{6}}$$

$$=\frac{\sqrt{6}}{6}\ \textbf{Ans}$$

(iv) $\frac{1}{3+2\sqrt{5}}$

Solution: $\frac{1}{3+2\sqrt{5}}$

$$=\frac{1}{3+2\sqrt{5}}$$

$$=\frac{1}{3+2\sqrt{5}}\times\frac{3-2\sqrt{5}}{3-2\sqrt{5}}$$

$$=\frac{3-2\sqrt{5}}{(3)^2-\left(2\sqrt{5}^2\right)}$$

$$=\frac{3-2\sqrt{5}}{9-4.5}$$

$$=\frac{3-2\sqrt{5}}{9-20}$$

$$=\frac{3-2\sqrt{5}}{-11}$$ **Ans**

(v) $\frac{15}{\sqrt{31}-4}$

Solution: $\frac{15}{\sqrt{31}-4}$

$$=\frac{15}{\sqrt{31}-4}$$

$$=\frac{15}{\sqrt{31}-4}\times\frac{\sqrt{31}+4}{\sqrt{31}+4}$$

$$=\frac{15\left(\sqrt{31}+4\right)}{\left(\sqrt{31}\right)^2-(4)^4}$$

$$=\frac{15\left(\sqrt{31}+4\right)}{31-16}$$

$$=\frac{\cancel{15}\left(\sqrt{31}+4\right)}{\cancel{15}}$$

$=\sqrt{31}+4$ **Ans**

(vi) $\frac{2}{\sqrt{5}-\sqrt{3}}$

Solution: $\frac{2}{\sqrt{5}-\sqrt{3}}$

$$\frac{2}{\sqrt{5}-\sqrt{3}}$$

$$=\frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}\times\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{\left(\cancel{\sqrt{5}}\right)^2-\left(\sqrt{3}\right)^2}$$

$$=\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3}$$

$$=\frac{\cancel{2}\left(\sqrt{5}+\sqrt{3}\right)}{\cancel{2}}$$

$=\sqrt{5}+\sqrt{3}$ **Ans**

(vii) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

Solution: $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

$$=\frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$=\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$=\frac{\left(\sqrt{3}-1\right)\left(\sqrt{3}-1\right)}{\left(\sqrt{3}\right)^2-(1)^2}$$

$$=\frac{\left(\sqrt{3}-1\right)^2}{3-1}$$

$$=\frac{\left(\sqrt{3}\right)^2-2\left(\sqrt{3}\right)(1)+(1)^2}{2}$$

$$=\frac{3-2\sqrt{3}+1}{2}$$

$$=\frac{4-2\sqrt{3}}{2}$$

$$= \frac{\cancel{2}\left(2-\sqrt{3}\right)}{\cancel{2}}$$

$= 2-\sqrt{3}$ **Ans**

(viii) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

Solution: $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$= \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{\left(\sqrt{5}\right)^2+2\left(\sqrt{5}\right)\left(\sqrt{3}\right)+\left(\sqrt{3}\right)^2}{5-3}$$

$$= \frac{5+2\sqrt{15}+3}{2}$$

$$= \frac{8+2\sqrt{15}}{2}$$

$$= \frac{\cancel{2}\left(4+\sqrt{15}\right)}{\cancel{2}}$$

$= 4+\sqrt{15}$ **Ans**

Q.2 **find the conjugate of** $x+\sqrt{y}$

(i) $3+\sqrt{7}$

Solution

Conjugate $3-\sqrt{7}$

(ii) $4-\sqrt{5}$

Solution

Conjugate $4+\sqrt{5}$

(iii) $2+\sqrt{3}$

Solution

Conjugate $2-\sqrt{3}$

(iv) $2+\sqrt{5}$

Solution

Conjugate $2-\sqrt{5}$

(v) $5+\sqrt{7}$

Solution

Conjugate $5-\sqrt{7}$

(vi) $4-\sqrt{15}$

Solution

Conjugate $4+\sqrt{15}$

(vii) $7-\sqrt{6}$

Solution

Conjugate $7+\sqrt{6}$

(viii) $9+\sqrt{2}$

Solution

Conjugate $9-\sqrt{2}$

Q.3

(i) **If** $x=2-\sqrt{3}$, **find** $\frac{1}{x}$

Solution: Given that $x=2-\sqrt{3}$

$$\frac{1}{x} = \frac{1}{2-\sqrt{3}}$$

$$= \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$= \frac{2+\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{2+\sqrt{3}}{4-3}$$

$$= \frac{2+\sqrt{3}}{1}$$

$\frac{1}{x} = 2+\sqrt{3}$ **Ans**

(ii) **If** $x=4-\sqrt{17}$, **find** $\frac{1}{x}$

Solution: Given that $x=4-\sqrt{17}$

$$\frac{1}{x} = \frac{1}{4-\sqrt{17}}$$

$$= \frac{1}{4-\sqrt{17}} \times \frac{4+\sqrt{17}}{4+\sqrt{17}}$$

$$= \frac{4+\sqrt{17}}{(4)^2-\left(\sqrt{17}\right)^2}$$

$$= \frac{4+\sqrt{17}}{16-17}$$

$$= \frac{4+17}{-1}$$

$$= -1\left(4+\sqrt{17}\right)$$

$$\frac{1}{x} = -4-\sqrt{17} \textbf{ Ans}$$

(iii) **If** $x=\sqrt{3}+2$, **find** $x+\frac{1}{x}$

Solution: Given that $x=\sqrt{3}+2$

$$\frac{1}{x} = \frac{1}{\sqrt{3}+2}$$

$$= \frac{1}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2}$$

$$= \frac{\sqrt{3}-2}{\left(\sqrt{3}\right)^2-(2)^2}$$

$$= \frac{\sqrt{3}-2}{3-4}$$

$$= \frac{\sqrt{3}-2}{-1}$$

$$= -\left(\sqrt{3}-2\right)$$

$$= -\sqrt{3}+2$$

$$x+\frac{1}{x} = \left(\sqrt{3}+2\right)+\left(-\sqrt{3}+2\right)$$

$$= \sqrt{3}+2-\sqrt{3}+2$$

$$= 2+2$$

$$x+\frac{1}{x} = 4 \textbf{ Ans}$$

Q.4 Simplify

(i) $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

Solution: $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

$$= \frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$$

$$= \frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{\left(1+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2} + \frac{\left(1-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{\left(\sqrt{5}-\sqrt{3}\right)+\sqrt{2}\left(\sqrt{5}-\sqrt{3}\right)}{5-3}$$

$$+ \frac{1\left(\sqrt{5}+\sqrt{3}\right)-\sqrt{2}\left(\sqrt{5}+\sqrt{3}\right)}{5-3}$$

$$= \frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}}{2} + \frac{\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{2}$$

$$= \frac{\sqrt{5}}{2} - \cancel{\frac{\sqrt{3}}{2}} + \cancel{\frac{\sqrt{10}}{2}} - \frac{\sqrt{6}}{2} + \frac{\sqrt{5}}{2} + \cancel{\frac{\sqrt{3}}{2}} - \cancel{\frac{\sqrt{10}}{2}} - \frac{\sqrt{6}}{2}$$

$$= \frac{\cancel{2}\sqrt{5}}{\cancel{2}} - \frac{\cancel{2}\sqrt{6}}{\cancel{2}}$$

$$= \sqrt{5}-\sqrt{6} \textbf{ Ans}$$

(ii) $\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2+\sqrt{5}}$

Solution: $\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2+\sqrt{5}}$

$$= \frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2+\sqrt{5}}$$

$$= \left(\frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}\right) + \left(\frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)$$

$$+ \left(\frac{1}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}\right)$$

$$=\left(\frac{2-\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}\right)+\left(\frac{2\times\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}\right)$$

$$+\left(\frac{2-\sqrt{5}}{(2)^2-\left(\sqrt{5}\right)^2}\right)$$

$$=\left(\frac{2-\sqrt{3}}{4-3}\right)+\left(\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3}\right)+\left(\frac{2-\sqrt{5}}{4-5}\right)$$

$$=\left(\frac{2-\sqrt{3}}{1}\right)+\left(\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{2}\right)+\left(\frac{2-\sqrt{5}}{-1}\right)$$

$$=2-\sqrt{3}+\sqrt{5}+\sqrt{3}-2+\sqrt{5}$$

$$=\cancel{2}-\cancel{2}-\cancel{\sqrt{3}}+\cancel{\sqrt{3}}+\sqrt{5}+\sqrt{5}$$

$$=\sqrt{5}+\sqrt{5}$$

$=2\sqrt{5}$ **Ans**

(iii) $\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}$

Solution: $\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}$

$$=\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}$$

$$=\left(\frac{2}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)+\left(\frac{1}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)$$

$$-\left(\frac{3}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{7}}\right)$$

$$=\left(\frac{2\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}\right)+\left(\frac{\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}\right)-\left(\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}\right)$$

$$=\left(\frac{2\left(\sqrt{5}-\sqrt{3}\right)}{5-3}+\frac{\sqrt{3}-\sqrt{2}}{3-2}\right)-\left(\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{5-2}\right)$$

$$=\left(\frac{2\left(\sqrt{5}-\sqrt{3}\right)}{2}\right)+\left(\frac{\sqrt{3}-\sqrt{2}}{1}\right)-\left(\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{3}\right)$$

$$=\cancel{\sqrt{5}}-\cancel{\sqrt{3}}+\cancel{\sqrt{3}}-\cancel{\sqrt{2}}-\cancel{\sqrt{5}}+\cancel{\sqrt{2}}$$

$=0$ **Ans**

Q.5 **If $x=2+\sqrt{3}$, then find the value of $x-\frac{1}{x}$ and $\left(x-\frac{1}{x}\right)^2$**

(i)

Solution: Given that $x=2+\sqrt{3}$

$$\frac{1}{x}=\frac{1}{2+\sqrt{3}}$$

$$=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$=\frac{2-\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}$$

$$=\frac{2-\sqrt{3}}{4-3}$$

$$=\frac{2-\sqrt{3}}{1}$$

$$=2-\sqrt{3}$$

To find the value of $x-\frac{1}{x}$

$$x-\frac{1}{x}=\left(2+\sqrt{3}\right)-\left(2-\sqrt{3}\right)$$

$$=\cancel{2}+\sqrt{3}-\cancel{2}+\sqrt{3}$$

$$=\sqrt{3}+\sqrt{3}$$

$$=2\sqrt{3}$$

To find the value of $\left(x-\frac{1}{x}\right)^2$

We know that

$$x-\frac{1}{x}=2\sqrt{3}$$

Taking square on both sides

$$\left(x-\frac{1}{x}\right)^2=\left(2\sqrt{3}\right)^2$$

$$=4\left(\sqrt{3}\right)^2$$

$$=4(3)$$

$=12$ **Ans**

(ii) **If** $x=\dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$**, find the value of** $x+\dfrac{1}{x},\ x^2+\dfrac{1}{x^2}$ **and** $x^3+\dfrac{1}{x^3}$

Solution: Given that $x=\dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

$$\frac{1}{x}=\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$x+\frac{1}{x}=\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}+\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$=\frac{\left(\sqrt{5}-\sqrt{2}\right)^2+\left(\sqrt{5}+\sqrt{2}\right)^2}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)}$$

$$=\frac{\left(\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2-2\sqrt{5}\times\sqrt{2}+\left(\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2+2\sqrt{5}\times\sqrt{2}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{5+2-\cancel{2\sqrt{10}}+5+2+\cancel{2\sqrt{10}}}{5-2}$$

$$x+\frac{1}{x}=\frac{14}{3}$$

Taking square on both sides

$$\left(x+\frac{1}{x}\right)^2=\left(\frac{14}{3}\right)^2$$

$$x^2+\frac{1}{x^2}+2(\cancel{x})\left(\frac{1}{\cancel{x}}\right)=\frac{196}{9}$$

$$x^2+\frac{1}{x^2}=\frac{196}{9}-2$$

$$x^2+\frac{1}{x^2}=\frac{196-18}{9}$$

$$x^2+\frac{1}{x^2}=\frac{178}{9}$$

$$x^2+\frac{1}{x^2}=\frac{178}{9}$$

To find $x^3+\dfrac{1}{x^3}$

$$x+\frac{1}{x}=\frac{14}{3}$$

Taking cube on both sides

$$\left(x+\frac{1}{x}\right)^3=\left(\frac{14}{3}\right)^3$$

$$x^3+\frac{1}{x^3}+3(\cancel{x})\left(\frac{1}{\cancel{x}}\right)\left(x+\frac{1}{x}\right)=\frac{2744}{27}$$

$$x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)=\frac{2744}{27}$$

$$x^3+\frac{1}{x^3}+\cancel{3}\left(\frac{14}{\cancel{3}}\right)=\frac{2744}{27}$$

$$x^3+\frac{1}{x^3}+14=\frac{2744}{24}$$

$$x^3+\frac{1}{x^3}=\frac{2744}{27}-14$$

$$x^3+\frac{1}{x^3}=\frac{2744-378}{27}$$

$$x^3+\frac{1}{x^3}=\frac{2366}{27}\ \textbf{Ans}$$

Q.6 **Determine the rational numbers** a **and** b **if** $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}+\dfrac{\sqrt{3}+1}{\sqrt{3}-1}=a+b\sqrt{3}$

Solution: Given that

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}=a+b\sqrt{3}$$

$$a+b\sqrt{3}=\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$=\frac{\left(\sqrt{3}-1\right)^2+\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}$$

$$=\frac{\left(\sqrt{3}\right)^2+(1)^2-\cancel{2\sqrt{3}}+\left(\sqrt{3}\right)^2+(1)^2+\cancel{2\sqrt{3}}}{\left(\sqrt{3}\right)^2-(1)^2}$$

$$=\frac{2\left(\sqrt{3}\right)^2+2}{\left(\sqrt{3}\right)^2-1}$$

$$=\frac{2\left[\left(\sqrt{3}\right)^2+(1)^2\right]}{\left(\sqrt{3}\right)^2-(1)^2}$$

$$=\frac{2(3+1)}{3-1}$$

$$=\frac{2(4)}{2}$$

$a+b\sqrt{3}=4$

$a+b\sqrt{3}=4+0\sqrt{3}$

Comparing both sides

$a=4 \qquad b\sqrt{3}=0\sqrt{3}$

$$b=\frac{0\sqrt{3}}{\sqrt{3}}$$

$b=0$ **Ans**

Review Exercise 4

Q.1 Multiple type questions?

(i) **is an algebraic …**

(a) Expression **(b)** Sentence
(c) Equation **(d)** In-equation

(ii) **The degree of polynomial** $4x^4 + 3x^2y$ **is**

(a) 1 **(b)** 2
(c) 3 **(d)** 4

(iii) $a^3 + b^3$ **is equal to**

(a) $(a-b)(a^2+ab+b^2)$ **(b)** $(a+b)(a^2-ab+b^2)$
(c) $(a-b)(a^2-ab+b^2)$ **(d)** $(a-b)(a^2+ab+b^2)$

(iv) $(3+\sqrt{2})(3-\sqrt{2})$ **is equal to**

(a) 7 **(b)** -7
(c) -1 **(d)** 1

(v) **Conjugate of surd** $a+\sqrt{b}$ **is;**

(a) $-a+\sqrt{b}$ **(b)** $a-\sqrt{b}$
(c) $\sqrt{a}+\sqrt{b}$ **(d)** $\sqrt{a}-\sqrt{b}$

(vi) $\frac{1}{a-b} - \frac{1}{a+b}$ **is equal to**

(a) $\frac{2a}{a^2-b^2}$ **(b)** $\frac{2b}{a^2-b^2}$
(c) $\frac{-2a}{a^2-b^2}$ **(d)** $\frac{-2b}{a^2-b^2}$

(vii) $\frac{a^2-b^2}{a+b}$ **is equal to**

(a) $(a-b)^2$ **(b)** $(a+b)^2$
(c) $a+b$ **(d)** $a-b$

(viii) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})$ **is equal to**

(a) a^2+b^2 **(b)** a^2-b^2
(c) $a-b$ **(d)** $a+b$

ANSWER KEY

i	ii	iii	iv	v	vi	vii	viii
a	d	b	a	b	b	d	c

Q.2 Fill in the blanks

(i) **The degree of polynomial** $x^2y^2+3xy+y^3$ **is** __________

(ii) x^2-4 __________

(iii) $x^3+\frac{1}{x^3}=\left[x+\frac{1}{x}\right]($__________$)$

(iv) $2(a^2+b^2)=(a+b)^2+($__________$)^2$

(v) $\left[x-\frac{1}{x}\right]^2=$ __________

(vi) **Order of surd** $\sqrt[3]{x}$ **is** __________

(vii) $\frac{1}{2-\sqrt{3}}=$ __________

ANSWER KEY

(i) 4

(ii) $(x-2)(x+2)$

(iii) $x^2-1+\frac{1}{x^2}$

(iv) $a-b$

(v) $x^2+\frac{1}{x^2}-2$

(vi) 3

(vii) $2+\sqrt{3}$

Q.3 If $x+\frac{1}{x}=3$, **find**

(i) $x^2+\frac{1}{x^2}$

Solution: Given that $x+\frac{1}{x}=3$

$\therefore (a+b)^2=a^2+b^2+2ab$

Putting the values

$$\left[x+\frac{1}{x}\right]^2=(x)^2+\left(\frac{1}{x}\right)^2+2(x)\left(\frac{1}{x}\right)$$

$$(3)^2=x^2+\frac{1}{x^2}+2$$

$$9=x^2+\frac{1}{x^2}+2$$

$$9-2=x^2+\frac{1}{x^2}$$

$x^2+\frac{1}{x^2}=7$ **Ans**

(ii) $x^4+\frac{1}{x^4}$

Solution: Given that $x^2+\frac{1}{x^2}=7$

$$\left(x^2+\frac{1}{x^2}\right)^2=\left(x^2\right)^2+\left(\frac{1}{x^2}\right)^2+2\left(x^2\right)\left(\frac{1}{x^2}\right)$$

$$(7)^2=x^4+\frac{1}{x^4}+2$$

$$49=x^4+\frac{1}{x^4}+2$$

$$x^4+\frac{1}{x^4}=49-2$$

$$x^4+\frac{1}{x^4}=47 \textbf{ Ans}$$

Q.4 **If** $x-\frac{1}{x}=2$ **find**

(i) $x^2+\frac{1}{x^2}$ $\qquad$ $(ii)\, x^4+\frac{1}{x^4}$

Solution (i)

Given that $x-\frac{1}{x}=2$

$\therefore (a+b)^2=a^2+b^2+2ab$

Putting the values

$$\left(x-\frac{1}{x}\right)^2=(x)^2+\left(\frac{1}{x}\right)^2-2(x)\left(\frac{1}{x}\right)$$

$$(2)^2=x^2+\frac{1}{x^2}-2$$

$$4+2=x^2+\frac{1}{x^2}$$

$$x^2+\frac{1}{x^2}=6 \textbf{ Ans}$$

Solution (ii)

Given that $x^2+\frac{1}{x^2}=6$

$$\left(x^2+\frac{1}{x}\right)=x^4+\frac{1}{x^4}+2\left(x^2\right)\left(\frac{1}{x^2}\right)$$

$$(6)^2=x^4+\frac{1}{x^4}+2$$

$$x^4+\frac{1}{x^4}=36-2$$

$$x^4+\frac{1}{x^4}=34 \textbf{ Ans}$$

Q.5 **Find the value of** x^3+y^3 **and** xy **if** $x+y=5$ **and** $x-y=3$**.**

Solution: Given that $x+y=5$

$x-y=3$

As we know that

$\therefore (x+y)^2-(x-y)^2=4xy$

Putting the values

$$4xy=(5)^2-(3)^3$$

$$4xy=25-9$$

$$4xy=16$$

$$xy=\frac{16^4}{4}$$

$$xy=4 \textbf{ Ans}$$

As we know that

$(x+y)^3=x^3+y^3+3xy(x+y)$

Putting the values

$$(5)^3=x^3+y^3.3\times4\times5$$

$$125=x^3+y^3+60$$

$$x^3+y^3=125-60$$

$$x^3+y^3=65$$

$$x^3+y^3=65 \textbf{ Ans}$$

Q.6 **If** $P=2+\sqrt{3}$ **, find**

(i) $P+\frac{1}{P}$

Solution: Given that $P=2+\sqrt{3}$

$$\frac{1}{P}=\frac{1}{2+\sqrt{3}}$$

$$\frac{1}{P}=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$\frac{1}{P}=\frac{2-\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}$$

$$\frac{1}{P}=\frac{2-\sqrt{3}}{4-3}$$

$$\frac{1}{P}=\frac{2-\sqrt{3}}{1}$$

$$\frac{1}{P}=2-\sqrt{3}$$

$$P+\frac{1}{P}=2+\cancel{\sqrt{3}}+2-\cancel{\sqrt{3}}$$

$$P+\frac{1}{P}=4 \textbf{ Ans}$$

(ii) $P-\frac{1}{P}$

As we know that

$\frac{1}{P}=2-\sqrt{3}$ and

$$P=2+\sqrt{3}$$

$$P-\frac{1}{P}=2+\sqrt{3}-\left(2-\sqrt{3}\right)$$

$$=\cancel{2}+\sqrt{3}-\cancel{2}+\sqrt{3}$$

$$=2\sqrt{3} \textbf{ Ans}$$

(iii) $P^2+\frac{1}{P^2}$

Solution: Given that $P+\frac{1}{P}=4$

$$\therefore (a+b)^2=a^2+b^2+2ab$$

$$\left(P+\frac{1}{P}\right)^2=(P)^2+\left(\frac{1}{P}\right)^2+2\left(\cancel{P}\right)\left(\frac{1}{\cancel{P}}\right)$$

$$(4)^2=P^2+\frac{1}{P^2}+2$$

$$16-2=P^2+\frac{1}{P^2}$$

$$P^2+\frac{1}{P^2}=14 \textbf{ Ans}$$

(iv) $P^2-\frac{1}{P^2}$

Solution:

$$P^2-\frac{1}{P^2}=\left(P+\frac{1}{P}\right)\left(P-\frac{1}{P}\right)$$

$$P^2-\frac{1}{P^2}=(4)\left(2\sqrt{3}\right)$$

$$=8\sqrt{3} \textbf{ Ans}$$

Q.7 **If** $q=\sqrt{5}+2$ **find.**

(i) $q+\frac{1}{q}$

Solution: Given that $q=\sqrt{5}+2$

$$\frac{1}{q}=\frac{1}{\sqrt{5}+2}\times\frac{\sqrt{5}-2}{\sqrt{5}-2}$$

$$=\frac{\sqrt{5}-2}{\left(\sqrt{5}\right)^2-(2)^2}$$

$$=\frac{\sqrt{5}-2}{5-4}$$

$$=\sqrt{5}-2$$

$$q+\frac{1}{q}=\sqrt{5}+\cancel{2}+\sqrt{5}-\cancel{2}$$

$$q+\frac{1}{q}=2\sqrt{5} \textbf{ Ans}$$

(ii) $q-\frac{1}{q}$

Solution: Given that $q=\sqrt{5}+2$

$$\frac{1}{q}=\sqrt{5}-2$$

$$q-\frac{1}{q}=\sqrt{5}+2-\left(\sqrt{5}-2\right)$$

$$=\sqrt{5}+2-\sqrt{5}+2$$

$$q-\frac{1}{q}=4 \text{ Ans}$$

(iii) $q^2+\frac{1}{q^2}$

Solution: Given that $q-\frac{1}{q}=4$

Squaring both sides

$$\left(q-\frac{1}{q}\right)^2=(4)^2$$

$$q^2+\frac{1}{q^2}-2=16$$

$$q^2+\frac{1}{q^2}=16+2$$

$$q^2+\frac{1}{q^2}=18 \text{ Ans}$$

(iv) $q^2-\frac{1}{q^2}$

Solution: Given that $q+\frac{1}{q}=2\sqrt{5}$

$$q-\frac{1}{q}=4$$

By using formula

$$q^2-\frac{1}{q^2}=\left(q+\frac{1}{q}\right)\left(q-\frac{1}{q}\right)$$

$$=\left(2\sqrt{5}\right)(4)$$

$$=8\sqrt{5} \text{ Ans}$$

Q.8 Simplify

(i) $\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}$

Solution:

$$=\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}\times\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}+\sqrt{a^2-2}}$$

$$=\frac{\left(\sqrt{a^2+2}+\sqrt{a^2-2}\right)^2}{\left(\sqrt{a^2+2}\right)^2-\left(\sqrt{a^2-2}\right)^2}$$

$$=\frac{\left(\sqrt{a^2+2}\right)^2+\left(\sqrt{a^2-2}\right)^2+2\left(\sqrt{a^2+2}\right)\left(\sqrt{a^2-2}\right)}{a^2+2-a^2+2}$$

$$=\frac{a^2+2+a^2-2+2\left(\sqrt{a^4-2a^2+2a^2-4}\right)}{4}$$

$$=\frac{2a^2+2\sqrt{a^4-4}}{4}$$

$$=\frac{2\left(a^2+\sqrt{a^4-4}\right)}{4}$$

$$=\frac{a^2+\sqrt{a^4-4}}{2} \text{ Ans}$$

(ii) $\frac{1}{a-\sqrt{a^2-x^2}}-\frac{1}{a+\sqrt{a^2-x^2}}$

$$=\left(\frac{1}{a-\sqrt{a^2-x^2}}\times\frac{a+\sqrt{a^2-x^2}}{a+\sqrt{a^2-x^2}}\right)$$

$$-\left(\frac{(1)}{\left(a+\sqrt{a^2-x^2}\right)}\frac{\left(a-\sqrt{a^2-x^2}\right)}{\left(a-\sqrt{a^2-x^2}\right)}\right)$$

$$=\left(\frac{a+\sqrt{a^2-x^2}}{(a)^2-\left(\sqrt{a^2-x^2}\right)^2}\right)-\left(\frac{a-\sqrt{a^2-x^2}}{(a)^2-\left(\sqrt{a^2-x^2}\right)^2}\right)$$

$$=\left(\frac{a+\sqrt{a^2-x^2}}{a-\left(a^2-x^2\right)}\right)-\left(\frac{a-\sqrt{a^2-x^2}}{a-\left(a^2-x^2\right)}\right)$$

$$=\left(\frac{a+\sqrt{a^2-x^2}}{a^2-a^2+x^2}\right)-\left(\frac{a-\sqrt{a^2-x^2}}{a^2-a^2+x^2}\right)$$

$$= \left(\frac{a+\sqrt{a^2-x^2}}{x^2}\right) - \left(\frac{a-\sqrt{a^2-x^2}}{x^2}\right)$$

$$= \frac{\cancel{a} + \sqrt{a^2-x^2} - \cancel{a} + \sqrt{a^2-x^2}}{x^2}$$

$$= \frac{2\sqrt{a^2-x^2}}{x^2}$$ **Ans**

Unit 4: Algebraic Expressions and Algebraic

Formulas Overview

Algebraic expression:

An algebraic expression is that in which constants or variables or both are combined by basic operations.

Polynomial:

Polynomial means an expression with many terms.

Degree of Polynomial:

Degree of Polynomial means highest power of variable.

Rational expression:

Expression in the form $\frac{p(x)}{q(x)},(q(x)\neq 0)$ is called rational expression.

Surd:

An irrational radical with rational radicand is called a surd.

e.g., $\sqrt{3},\sqrt{\frac{2}{5}},$

Monomial Surd:

A surd which contains a single term is called monomial surd.

Binomial Surd:

A surd which contains sum or difference of two surds is called binomial surd.

Exercise 5.1

Q.1 Factorize

(i) $2abc-4abx+2abd$

Solution: $2abc-4abx+2abd$

$=2ab(c-2x+d)$

(ii) $9xy-12x^2y+18y^2$

Solution: $9xy-12x^2y+18y^2$

$=3y(3x-4x^2+6y)$

(iii) $-3x^2y-3x+9xy^2$

Solution: $-3x^2y-3x+9xy^2$

$=-3x(xy+1-3y^2)$

(iv) $5ab^2c^3-10a^2b^3c-20a^3bc^2$

Solution: $5ab^2c^3-10a^2b^3c-20a^3bc^2$

$=5abc(bc^2-2ab^2-4a^2c)$

(v) $3x^3y(x-3y)-7x^2y^2(x-3y)$

Solution: $3x^3y(x-3y)-7x^2y^2(x-3y)$

$=(x-3y)(3x^3y-7x^2y^2)$

$=(x-3y)\ x^2y(3x-7y)$

$=x^2y(x-3y)(3x-7y)$

(vi) $2xy^3(x^2+5)+8xy^2(x^2+5)$

Solution: $2xy^3(x^2+5)+8xy^2(x^2+5)$

$=(x^2+5)(2xy^3+8xy^2)$

$=(x^2+5)2xy^2(y+4)$

$=2xy^2(x^2+5)(y+4)$

Q.2 Factorize

(i) $5ax-3ay-5bx+3by$

Solution: $5ax-3ay-5bx+3by$

$=5ax-5bx-3ay+3by$

$=5x(a-b)-3y(a-b)$

$=(a-b)(5x-3y)$

(ii) $3xy+2y-12x-8$

Solution: $3xy+2y-12x-8$

$=3xy-12x+2y-8$

$=3x(y-4)+2(y-4)$

$=(y-4)(3x+2)$

(iii) $x^3+3xy^2-2x^2y-6y^3$

Solution: $x^3+3xy^2-2x^2y-6y^3$

By cyclic order

$=x^3-2x^2y+3xy^2-6y^3$

$=x^2(x-2y)+3y^2(x-2y)$

$=(x-2y)(x^2+3y^2)$

(iv) $(x^2-y^2)z+(y^2-z^2)x$

Solution: $(x^2-y^2)z+(y^2-z^2)x$

$=\ x^2z-y^2z+xy^2-xz^2$

Arrange in cyclic order

$x^2z+\ xy^2-xz^2-y^2z$

$=x^2z+xy^2-y^2z-xz^2$

$=x(xz+y^2)-z(xz+y^2)$

$=(xz+y^2)(x-z)$

Q.3 Factorize

(i) $144a^2+24a+1$

Solution: $144a^2+24a+1$

By using formula

$(a+b)^2=a^2+2ab+b^2$

$=(12a)^2+2(12a)(1)+(1)^2$

$=(12a+1)^2$

(ii) $\frac{a^2}{b^2}-2+\frac{b^2}{a^2}$

Solution: $\frac{a^2}{b^2}-2+\frac{b^2}{a^2}$

Formula $a^2-2ab+b^2=(a-b)^2$

$=\left(\frac{a}{b}\right)^2-2\left(\frac{a}{b}\right)\left(\frac{b}{a}\right)+\left(\frac{b}{a}\right)^2$

$=\left(\frac{a}{b}-\frac{b}{a}\right)^2$

(iii) $(x+y)^2-14z(x+y)+49z^2$

Solution: $(x+y)^2-14z(x+y)+49z^2$

Formula $a^2-2ab+b^2=(a-b)^2$

$=(x+y)^2-2(x+y)(7z)+(7z)^2$

$=(x+y-7z)^2$

(iv) $12x^2-36x+27$

Solution: $12x^2-36x+27$

$=3(4x^2-12x+9)$

Formula $a^2-2ab+b^2=(a-b)^2$

$=3\left[(2x)^2-2(2x)(3)+(3)^2\right]$

$=3(2x-3)^2$

Q.4 Factorize

(i) $3x^2-75y^2$

Solution: $3x^2-75y^2$

$=3(x^2-25y^2)$

Formula $a^2-b^2=(a+b)(a-b)$

$=3\left[(x)^2-(5y)^2\right]$

$=3(x+5y)(x-5y)$

(ii) $x(x-1)-y(y-1)$

Solution: $x(x-1)-y(y-1)$

$=x^2-x-y^2+y$

Arranging in cyclic order

$=x^2-y^2-x+y$

Taking common

$=(x^2-y^2)-(x-y)$

$=\left[(x+y)(x-y)\right]-(x-y)$

$=(x-y)(x+y-1)$

(iii) $128am^2-242an^2$

Solution: $128am^2-242an^2$

$=2a(64m^2-121n^2)$

$=2a\left[(8m)^2-(11n)^2\right]$

$=2a(8m+11n)(8m-11n)$

(iv) $3x-243x^3$

Solution: $3x-243x^3$

$=3x(1-81x^2)$

$=3x\left[(1)^2-(9x)^2\right]$

$=3x(1+9x)(1-9x)$

Q.5 Factorize

(i) x^2-y^2-6y-9

Solution: x^2-y^2-6y-9

$=x^2-\left[y^2+6y+9\right]$

$=x^2-[(y)^2+2(y)(3)+(3)^2]$

$=x^2-(y+3)^2$

$=(x)^2-(y+3)^2$

$=(x+y+3)[x-(y+3)]$

$=(x+y+3)(x-y-3)$

(ii) x^2-a^2+2a-1

Solution: x^2-a^2+2a-1

$=x^2-\left[a^2-2a+1\right]$

$=x^2-(a-1)^2$

$=[x+(a-1)][x-(a-1)]$

$=(x+a-1)(x-a+1)$

(iii) $4x^2-y^2-2y-1$

Solution: $4x^2-y^2-2y-1$

$=4x^2-(y^2+2y+1)$

$=4x^2-\left[(y)^2+2(y)(1)+(1)^2\right]$

$=4x^2-(y+1)^2$

$=(2x)^2-(y+1)^2$

$=\left[2x+(y+1)\right]\left[2x-(y+1)\right]$

$=(2x+y+1)(2x-y-1)$

(iv) $x^2-y^2-4x-2y+3$

Solution: $x^2-y^2-4x-2y+3$

$=x^2-4x+4-y^2-2y-1$

$=(x^2-4x+4)-(y^2+2y+1)$

$=\left[(x)^2-2(x)(2)+(2)^2\right]$

$-\left[(y)^2+2(y)(1)+(1)^2\right]$

$=(x-2)^2-(y+1)^2$

$=(x-2+y+1)\left[x-2-(y+1)\right]$

$=(x-2+y+1)(x-2-y-1)$

$=(x+y-2+1)(x-y-2-1)$

$=(x+y-1)(x-y-3)$

(v) $25x^2-10x+1-36z^2$

Solution: $25x^2-10x+1-36z^2$

$=(5x)^2-2(5x)(1)+(1)^2]-36z^2$

$=(5x-1)^2-(6z)^2$

$=\left[(5x-1)+6Z\right]\left[(5x-1)-6Z\right]$

$=(5x-1+6z)(5x-1-6z)$

(vi) $x^2-y^2-4xz+4z^2$

Solution: $x^2-y^2-4xz+4z^2$

$=x^2-4xz+4z^2-y^2$

$=\left[(x)^2-2(x)(2z)+(2z)^2\right]-y^2$

$=(x-2z)^2-(y)^2$

$=(x-2z+y)(x-2z-y)$

$=(x+y-2z)(x-y-2z)$

Exercise 5.2

Q.1 Factorize

(i) $x^4+\dfrac{1}{x^4}-3$

Solution: $x^4+\dfrac{1}{x^4}-3$

$=\left(x^2\right)^2+\left(\dfrac{1}{x^2}\right)^2-3$

By adding and subtracting by 2

$=(x^2)^2+\left(\dfrac{1}{x^2}\right)^2+2-2-3$

$=\left[(x^2)^2+\left(\dfrac{1}{x^2}\right)^2-2\right]+2-3$

$=\left[(x^2)^2+\left(\dfrac{1}{x^2}\right)^2-2\right]-1$

$=\left(x^2-\dfrac{1}{x^2}\right)^2-(1)^2$

$=\left(x^2-\dfrac{1}{x^2}+1\right)\left(x^2-\dfrac{1}{x^2}-1\right)$

(ii) $3x^4+12y^4$

Solution: $3x^4+12y^4$

$=3\left(x^4+4y^4\right)$

By adding and subtracting by $2(x^2)(2y^2)$

$=3\left[\left(x^2\right)^2+\left(2y^2\right)^2+2\left(x^2\right)\left(2y^2\right)-2\left(x^2\right)\left(2y^2\right)\right]$

$=3\left[\left(x^2\right)^2+\left(2y^2\right)^2+2\left(x^2\right)\left(2y^2\right)-2\left(x^2\right)\left(2y^2\right)\right]$

$=3\left[\left(x^2+2y^2\right)^2-4x^2y^2\right]$

$=3\left[\left(x^2+2y^2\right)^2-\left(2xy\right)^2\right]$

$=3\left[\left(x^2+2y^2+2xy\right)\left(x^2+2y^2-2xy\right)\right]$

$=3\left[\left(x^2+2xy+2y^2\right)\left(x^2-2xy+2y^2\right)\right]$

(iii) $a^4+3a^2b^2+4b^4$

Solution: $a^4+3a^2b^2+4b^4$

$=\left(a^4+4b^4\right)+3a^2b^2$

$=(a^2)^2+(2b^2)^2+3a^2b^2$

By adding and subtracting by $2\left(a^2\right)\left(2b^2\right)$

$=\left(a^2\right)^2+\left(2b^2\right)^2+2\left(a^2\right)\left(2b^2\right)-2\left(a^2\right)\left(2b^2\right)+3a^2b^2$

$=\left[\left(a^2\right)^2+\left(2b^2\right)^2+2\left(a^2\right)\left(2b^2\right)\right]-2\left(a^2\right)\left(2b^2\right)+3a^2b^2$

$=(a^2+2b^2)^2-a^2b^2$

$=(a^2+2b^2)^2-(ab)^2$

$=(a^2+2b^2+ab)(a^2+2b^2-ab)$

(iv) $4x^4+81$

Solution: $4x^4+81$

$=\left(2x^2\right)^2+\left(9\right)^2$

By adding and subtracting by $2\left(2x^2\right)\left(9\right)$

$=\left[\left(2x^2\right)^2+\left(9\right)^2+2\left(2x^2\right)\left(9\right)-2\left(2x^2\right)\left(9\right)\right]$

$=\left[\left(2x^2\right)^2+\left(9\right)^2+2\left(2x^2\right)\left(9\right)\right]-2\left(2x^2\right)\left(9\right)$

$=\left(2x^2+9\right)^2-36x^2$

$=\left(2x^2+9\right)^2-\left(6x\right)^2$

$=\left(2x^2+9+6x\right)\left(2x^2+9-6x\right)$

$=\left(2x^2+6x+9\right)\left(2x^2-6x+9\right)$

(v) x^4+x^2+25

Solution: x^4+x^2+25

$=\left(x^4+25\right)+x^2$

$=\left[\left(x^2\right)^2+\left(5\right)^2\right]+x^2$

By adding and subtracting by $2\left(x^2\right)\left(5\right)$

$=\left[\left(x^2\right)^2+\left(5\right)^2+2\left(x^2\right)\left(5\right)-2\left(x^2\right)\left(5\right)\right]+x^2$

$=\left[\left(x^2\right)^2+\left(5\right)^2+2\left(x^2\right)\left(5\right)\right]-2\left(x^2\right)\left(5\right)+x^2$

$=\left(x^2+5\right)^2-10x^2+x^2$

$=(x^2+5)^2-9x^2$

$= (x^2+5)^2 - (3x)^2$
$= \left(x^2+5+3x\right)\left(x^2+5-3x\right)$
$= (x^2+3x+5)(x^2-3x+5)$

(vi) x^4+4x^2+16
Solution: x^4+4x^2+16
$= (x^2)^2+16+4x^2$
$= (x^2)^2+(4)^2+4x^2$
By adding and subtracting by $2\left(x^2\right)(4)$
$= \left(x^2\right)^2+(4)^2+2\left(x^2\right)(4)-2\left(x^2\right)(4)+4x^2$
$= (x^2)^2+(4)^2+2(x^2)(4)-2(x^2)(4)+4x^2$
$= \left(x^2+4\right)^2-8x^2+4x^2$
$= (x^2+4)^2-4x^2$
$= (x^2+4)^2-(2x)^2$
$= \left(x^2+4+2x\right)\left(x^2+4-2x\right)$
$= \left(x^2+2x+4\right)\left(x^2-2x+4\right)$

Q.2 Factorize

(i) $x^2+14x+48$
Solution: $x^2+14x+48$
$= x^2+8x+6x+48$
$= x(x+8)+6(x+8)$
$= (x+8)(x+6)$

(ii) $x^2-21x+108$
Solution: $x^2-21x+108$
$= x^2-12x-9x+108$
$= x(x-12)-9(x-12)$
$= (x-9)(x-12)$

(iii) $x^2-11x-42$
Solution: $x^2-11x-42$
$= x^2-14x+3x-42$
$= x(x-14)+3(x-14)$
$= (x+3)(x-14)$

(iv) $x^2+x-132$
Solution: $x^2+x-132$
$= x^2+12x-11x-132$
$= x(x+12)-11(x+12)$
$= (x-11)(x+12)$

Q.3 Factorize

(i) $4x^2+12x+5$
Solution: $4x^2+12x+5$
$= 4x^2+2x+10x+5$
$= 2x(2x+1)+5(2x+1)$
$= (2x+5)(2x+1)$

(ii) $30x^2+7x-15$
Solution: $30x^2+7x-15$
$= 30x^2+25x-18x-15$
$= 5x(6x+5)-3(6x+5)$
$= (5x-3)(6x+5)$

(iii) $24x^2-65x+21$
Solution: $24x^2-65x+21$
$= 24x^2-56x-9x+21$
$= 8x(3x-7)-3(3x-7)$
$= (8x-3)(3x-7)$

(iv) $5x^2-16x-21$
Solution: $5x^2-16x-21$
$= 5x^2+5x-21x-21$
$= 5x\ (x+1)-21(x+1)$
$= (5x-21)(x+1)$

(v) $4x^2-17xy+4y^2$
Solution: $4x^2-17xy+4y^2$
$= 4x^2-16xy-xy+4y^2$
$= 4x(x-4y)-y(x-4y)$
$= (4x-y)(x-4y)$

(vi) $3x^2-38xy-13y^2$
Solution: $3x^2-38xy-13y^2$
$= 3x^2-39xy+xy-13y^2$
$= 3x(x-13y)+y(x-13y)$
$= (3x+y)(x-13y)$

(vii) $5x^2+33xy-14y^2$
Solution: $5x^2+33xy-14y^2$

$= 5x^2 + 35xy - 2xy - 14y^2$
$= 5x(x + 7y) - 2y(x + 7y)$
$= (5x - 2y)(x + 7y)$

(viii) $\left(5x - \frac{1}{x}\right)^2 + 4\left(5x - \frac{1}{x}\right) + 4, x \neq 0$

Solution: $\left(5x - \frac{1}{x}\right)^2 + 4\left(5x - \frac{1}{x}\right) + 4, x \neq 0$

$= \left(5x - \frac{1}{x}\right)^2 + 2\left(5x - \frac{1}{x}\right)(2) + (2)^2$

$= \left(5x - \frac{1}{x} + 2\right)^2$

$= \left(5x - \frac{1}{x} + 2\right)\left(5x - \frac{1}{x} + 2\right)$

Q.4

(i) $(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$

Solution: $(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$

Suppose that

$x^2 + 5x = y$

So,

$(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$
$= (y + 4)(y + 6) - 3$
$= [y(y + 6) + 4(y + 6) - 3$
$= (y^2 + 6y + 4y + 24) - 3$
$= (y^2 + 10y + 24) - 3$
$= y^2 + 10y + 24 - 3$
$= y^2 + 10y + 21$
$= y^2 + 7y + 3y + 21$
$= y(y + 7) + 3(y + 7)$
$= (y + 3)(y + 7)$

We know that $y = x^2 + 5x$

$= (x^2 + 5x + 3)(x^2 + 5x + 7)$

(ii) $(x^2 - 4x)(x^2 - 4x - 1) - 20$

Solution: $(x^2 - 4x)(x^2 - 4x - 1) - 20$

Suppose that

$x^2 - 4x = y$

So,

$= (y)(y - 1) - 20$
$= (y^2 - y) - 20$
$= y^2 - y - 20$
$= y^2 - 5y + 4y - 20$
$= y(y - 5) + 4(y - 5)$
$= (y + 4)(y - 5)$

We know that $a = x^2 - 4x$

$= (x^2 - 4x + 4)(x^2 - 4x - 5)$
$= \left[(x)^2 - 2(x)(2) + (2)^2\right]\left[x^2 - 5x + x - 5\right]$
$= (x - 2)^2\left[x(x - 5) + 1(x - 5)\right]$
$= (x - 2)^2(x - 5)(x + 1)$
$= (x - 5)(x + 1)(x - 2)^2$

(iii) $(x + 2)(x + 3)(x + 4)(x + 5) - 15$

Solution: $(x + 2)(x + 3)(x + 4)(x + 5) - 15$

$= [(x + 2)(x + 5)][(x + 3)(x + 4)] - 15$
$= [x(x + 5) + 2(x + 5)][x(x + 4) + 3(x + 4)] - 15$
$= [x^2 + 5x + 2x + 10][x^2 + 4x + 3x + 12] - 15$
$= (x^2 + 7x + 10)(x^2 + 7x + 12) - 15$

Suppose that

$x^2 + 7x = y$

So,

$(x^2 + 7x + 10)(x^2 + 7x + 12) - 15$
$= (y + 10)(y + 12) - 15$
$= [y(y + 12) + 10(y + 12)] - 15$
$= (y^2 + 12y + 10y + 120) - 15$
$= (y^2 + 22y + 120) - 15$
$= y^2 + 22y + 120 - 15$
$= y^2 + 22y + 105$
$= y^2 + 15y + 7y + 105$
$= y(y + 15) + 7(y + 15)$
$= y(y + 15) + 7(y + 15)$
$= (y + 7)(y + 15)$

We know that $y = x^2 + 7x$

$= (x^2 + 7x + 7)(x^2 + 7x + 15)$

(iv) $(x + 4)(x - 5)(x + 6)(x - 7) - 504$

Solution: $(x+4)(x-5)(x+6)(x-7)-504$

$=[(x+4)(x-5)][(x+6)(x-7)]-504$

$=[x(x-5)+4(x-5)][x(x-7)+6(x-7)]-504$

$=(x^2-5x+4x-20)(x^2-7x+6x-42)-504$

$=(x^2-x-20)(x^2-x-42)-504$

Suppose that

$x^2-x=y$

So,

$=(y-20)(y-42)-504$

$=[y(y-42)-20(y-42)]-504$

$=(y^2-42y-20y+840)-504$

$=y^2-62y+840-504$

$=y^2-62y+336$

$=y^2-56y-6y+336$

$=y(y-56)-6(y-56)$

$=(y-6)(y-56)$

We know that $a=x^2-x$

$=(x^2-x-6)(x^2-x-56)$

$=(x^2-3x+2x-6)(x^2-8x+7x-56)$

$=[x(x-3)+2(x-3)][x(x-8)+7(x-8)]$

$=(x+2)(x-3)(x+7)(x-8)$

(v) $(x+1)(x+2)(x+3)(x+6)-3x^2$

Solution: $(x+1)(x+2)(x+3)(x+6)-3x^2$

$=[(x+1)(x+6)][(x+2)(x+3)]-3x^2$

$=[x(x+6)+1(x+6)][x(x+3)+2(x+3)]-3x^2$

$=(x^2+6x+x+6)(x^2+3x+2x+6)-3x^2$

$=(x^2+6+7x)(x^2+6+5x)-3x^2$

Suppose that

$x^2+6=y$

So,

$=(y+7x)(y+5x)-3x^2$

$=[y(y+5x)+7x(y+5x)]-3x^2$

$=(y^2+5xy+7xy+35x^2-3x^2)$

$=y^2+12xy+32x^2$

$=y^2+8xy+4xy+32x^2$

$=y(y+8x)+4x(y+8x)$

$=(y+4x)(y+8)$

We know that $y=x^2+6$

$=(x^2+6+4x)(x^2+6+8x)$

$=(x^2+4x+6)(x^2+8x+6)$

Q.5

(i) $x^3+48x-12x^2-64$

Solution: $x^3+48x-12x^2-64$

$=x^3-12x^2+48x-64$

$a^3-3a^2b+3ab^2-b^3=(a-b)^3$

$=(x)^3-3(x)^2(4)+3(x)(4)^2-(4)^3$

$=(x-4)^3$

(ii) $8x^3+60x^2+150x+125$

Solution: $8x^3+60x^2+150x+125$

$a^3+3a^2b+3ab^2+b^3=(a+b)^3$

$=(2x)^3+3(2x)^2(5)+3(2x)(5)^2+(5)^3$

$=(2x+5)^3$

(iii) $x^3-18x^2+108x-216$

Solution: $x^3-18x^2+108x-216$

$a^3-3a^2b+3ab^2-b^3=(a-b)^3$

$=(x)^3-3(x)^2(6)+3(x)(6)^2-(6)^3$

$=(x-6)^3$

(iv) $8x^3-125y^3-60x^2y+150xy^2$

Solution: $8x^3-125y^3-60x^2y+150xy^2$

$=8x^3-60x^2y+150xy^2-125y^3$

$a^3-3a^2b+3ab^2-b^3=(a-b)^3$

$=(2x)^3-3(2x)^2(5y)+3(2x)(5y)^2-(5y)^3$

$=(2x-5y)^3$

Q.6

(i) $27+8x^3$

Solution: $27+8x^3$

$=(3)^3+(2x)^3$

$=(3+2x)[(3)^2-(3)(2x)+(2x)^2]$

$=(3+2x)(9-6x+4x^2)$

(ii) $125x^3-216y^3$

Solution: $125x^3-216y^3$

$=(5x)^3-(6y)^3$

$(a-b)(a^2+ab+b^2)=a^3-b^3$

$=(5x-6y)[(5x)^2+(5x)(6y)+(6y)^2]$

$=(5x-6y)(25x^2+30xy+36y^2)$

(iii) $64x^3+27y^3$

Solution: $64x^3+27y^3$

$=(4x)^3+(3y)^3$

$(a+b)(a^2+ab+b^2)=a^3+b^3$

$=(4x+3y)\left[(4x)^2-(4x)(3y)+(3y)^2\right]$

$=(4x+3y)(16x^2-12xy+9y^2)$

(iv) $(2x)^3+(5y)^3$

Solution: $(2x)^3+(5y)^3$

$(a-b)(a^2+ab+b^2)=a^3-b^3$

$=(2x+5y)\left[(2x)^2-(2x)(5y)+(5y)^2\right]$

$=(2x+5y)(4x^2-10xy+25y^2)$

Exercise 5.3

Q.1 Use the remainder theorem to find the remainder when

(i) $3x^3-10x^2+13x-6$ **is divided by** $(x-2)$.

Solution:

$P(x)=3x^3-10x^2+13x-6$

Since $P(x)$ is divided by $(x-2)$.

$\therefore P(2)=R$

$R=3(2)^3-10(2)^2+13(2)-6$

$=3(2)^3-10(2)^2+13(2)-6$

$=24-40+26-6$

R=4

Hence 4 is the remainder

(ii) $4x^3-4x+3$ **is divided by** $(2x-1)$

Solution:

$P(x)=4x^3-4x+3$

Since $P(x)$ is divided by (2x-1)

$\therefore R=P\left(\frac{1}{2}\right)$

$=4\left[\frac{1}{2}\right]^3-\not{4}^2\times\frac{1}{\not{2}}+3$

$=\not{4}\times\frac{1}{\not{8}^2}-2+3$

$=\frac{1}{2}-2+3$

$=\frac{1-4+6}{2}=\frac{3}{2}$

$R=\frac{3}{2}$

Hence $\frac{3}{2}$ is the remainder

(iii) $6x^4+2x^3-x+2$ **is divided by** $(x+2)$ **from** $x+2=0$

Solution: Given that

$P(x)=6x^4+2x^3-x+2$

Since $P(x)$ is divided by (x+2)

$\therefore R=P(-2)$

$=6(-2)^4+2(-2)^3-(-2)+2$

=96-16+2+2

R=84

Hence 84 is the remainder

(iv) $(2x-1)^3+6(3+4x)^2-10$ **is divided by** $2x+1$ **from** $2x+1=0$

$x=-\frac{1}{2}$

Solution: Given that

$P(x)=(2x-1)^3+6(3+4x)^2-10$

Since $P(x)$ is divided by 2x+1

$\therefore R=P\left(-\frac{1}{2}\right)$

$=\left[\not{2}\left(-\frac{1}{\not{2}}\right)-1\right]^3+6\left[3+\not{4}^2\left(\frac{-1}{\not{2}}\right)\right]^2-10$

$=[-1-1]^3+6[3-2]^2-10$

$=[-2]^3+6-10=-8+6-10$

$R=-12$

Hence -12 is the remainder

(v) $x^3-3x^2+4x-14$ **is divided by** $(x+2)$ **from** $x+2=0, x=-2$

Solution: Given that

$P(x)=x^3-3x^2+4x-14$

Since $P(x)$ is divided by (x+2)

$\therefore R=P(-2)$

$=(-2)^3-3(-2)^2+4(-2)-14$

$=-8-12-8-14$

$R=-42$

Hence -42 is the remainder

Q.2

(i) If (x+2) is a factor of $3x^2-4kx-4k^2$ then find the values of k $x+2=0$ $x=-2$

Solution: Given that

$P(x)=3x^2-4kx-4k^2$

P(-2)=$3(-2)^2-4k(-2)-4k^2$

P(-2)=12+8k-4k^2

If (x+2) is the factor then remainder is equal to zero

P(-2)=0

$12+8k-4k^2=0$

$4(3+2k-k^2)=0$

$-k^2+2k+3=\frac{0}{4}$

$-k^2+3k-k+3=0$

$-k(k-3)-1(k-3)=0$

$(k-3)(-k-1)=0$

$k-3=0$ $\quad -k-1=0$

$k=3$ $\quad -1=k$

$k=-1$

(ii) If (x-1) is a factor of $x^3-kx^2+11x-6$ the find the value of k from $x-1=0$ $x=1$

Solution: Given that

$P(x)=x^3-kx^2+11x-6$

P(1)=$(1)^3-k(1)^2+11(1)-6$

P(1)=$1-k+11-6$

P(1)=$6-k$

If (x-1) is the factor then remainder is equal to zero

$P(1)=0$

6-k=0

k=6

Q.3 Without long division determine whether

(i) $(x-2)$ **and** $(x-3)$ **are factor of** $P(x)=x^3-12x^2+44x-48$ **from** $x-2=0$ $x=2$

Solution: Given that

$P(x)=x^3-12x^2+44x-48$

If (x-2) is the factor then remainder is equal to zero

$P(2)=(2)^3-12(2)^2+44(2)-48=8-48+88-48=0$

Hence $x-2$ is a factor of $P(x)$

For $x-3$

$R=P(3)$

=$(3)^3$-12$(3)^2$+44(3)-48

=$(3)^3$-12$(3)^2$+44(3)-48

=27-108+132-48

=159-156

R=3

3 is remainder hence $x-3$ is not factor of $P(x)$

P(3) is not equal to zero then x-3 is not factor of $P(x)=x3-12x^2+44x-48$

(ii) $(x-2),(x+3)$ **and** $(x-4)$ **are factor of** $q(x)=x^3+2x^2-5x-6$ **from** $x-2=0$**,** $x=2$

Solution: Given that

$q(x)=x^3+2x^2-5x-6$

For (x-2), putt x-2=0

$x=2$

$R=q(2)$

$=(2)^3+2(2)^2-5(2)-6$

$R=8+8-10-6$

$R=16-16$

$R=0$

Hence $x-2$ is factor of $q(x)=x^3+2x^2-5x-6$

For (x+3), putt x+3=0

x=-3

$R=q(-3)$

$=(-3)^3+2(-3)^2-5(-3)-6$

$=-27+18+15-6$

$R=0$

Hence x-2 is factor of $q(x)=x^3+2x^2-5x-6$

For x-4, x-4=0

x=4

$R=q(4)$

$=(4)^3+2(4)^2-5(4)-6$

$=64+32-20-6$

R=70

Hence x-4 is not a factor of

$q(x)=x^3+2x^2-5x-6$

Q.4 For what value of m is the polynomial P(x) =$4x^3-7x^2+6x-3m$ exactly divisible by x+2?

Solution:

$P(x)=4x^3-7x^2+6x-3m$

From x+2=0, x=-2

$P(-2)=4(-2)^3-7(-2)^2+6(-2)-3m$

P(-2)=-32-28-12-3m=-72-3m

If (x+2) is the factor then remainder is equal to zero

$P(-2)=0$

$-72-3m=0$

$-72=3m$

$m=-\frac{72}{3}$

$m=-24$

Q.5 Determine the value of k if $P(x)=kx^3+4x^2+3x-4$ and $q(x)=x^3-4x+k$ leaves the same remainder when divided by $(x-3)$.

Solution:

$q(x)=x^3-4x+k$

from x-3=0 x=3

$R_1=q(3)$

$=(3)^3-4(3)+k$

$=27-12+k$

$=15+k$

$R_1 \quad =15+k$ ....(i)

$R_2=P(3)$

$=k(3)^3+4(3)^2+3(3)-4$

$=27k+36+9-4$

$R_2 \quad =27k+41$ ....(ii)

Since it leaves the same remainder.

Hence $R_1=R_2$

$15+k=27k+41$

$15-41=27k-k$

$-26=26k$

$k=\frac{-26}{26}$

$k=-1$

Q.6 The remainder after dividing the polynomial $P(x)=x^3+ax^2+7$ by $(x+1)$ is $2b$ calculate the value of a and b if this expression leaves a remainder of $(b+5)$ on being dividing by $(x-2)$

Solution:

Let

$P(x)=x^3+ax^2+7$

Since $P(x)$ is divided by (x+1)

Put $x+1=0$ $\quad x=-1$

R=P(−1)

$=(-1)^3+a(-1)^2+7$

$=-1+a+7$

$R=a+6$

According to first condition remainder is 2b

$2b=a+6$ ...(i)

Since $P(x)$ is divided by (x-2)

Put $x-2=0$

$x=2$

$P(2)=(2)^3+a(2)^2+7$

=8+4a+7

R=15+4a

According to second condition remainder is (b+5)

15+4a=b+5

4a-b=5-15

4a-b=-10 ...(ii)

Solving equations (i) and (ii)
From equation (ii) b=10+4a putting the value of be in equation (i)
a+6=2(10+4a)
a=20+8a-6
-8a+a=14
-7a=14

$a=\frac{14}{-7}$

a=-2
Putting the value of a in equation (ii)

$4a-b=-10$

$4(-2)-b=-10$

$-8-b=-10$

$-8+10=b$

$2=b$

$b=2$

Q.7 The polynomial $x^3+lx^2+mx+24$ has a factor $(x+4)$ and it leaves a remainder of 36 when divided by $(x-2)$
Find the values of l and m.

Solution:
Let

$P(x)=x^3+lx^2+mx+24$

From $x+4=0$ $x=-4$

$P(-4)=(-4)^3+l(-4)^2+m(-4)+24$

$P(-4)=-64+16l-4m+24$

$P(-4)=16l-4m-40$

According to condition (x+4) is the factor then

$16l-4m-40=0$

$4[4l-m-10]=0$

$4l-m-10=0$ (i)

from $x-2=0$ $x=2$

Now $P(2)=(2)^3+l(2)^2+m(2)+24$

$P(2)=8+4l+2m+24$

$P(2)=4l+2m+32$

According the condition

$4l+2m+32=36$

$4l+2m=36-32$

$4l+2m=4$

$4l+2m-4=0$ (ii)

Subtracting (i) from (ii)

$\cancel{4l}+2m-4=0$

$\pm\cancel{4l}\mp m\mp 10=0$

$3m+6=0$

$3m+6=0$

$3m=-6$

$m=\frac{-\cancel{6}2}{\cancel{3}}$

$m=-2$

Putting the value of m is equation (i)

$4l-(-2)-10=0$

$4l+2-10=0$

$4l-8=0$

$4l=8$

$l=\frac{2\cancel{8}}{\cancel{4}}$

$l=2$

Q.8 The expression lx^3+mx^2-7 leaves remainder of -3 and 12 when divided by $(x-1)$ and $(x+2)$ respectively. Calculate the value of l and m.

Solution:

$P(x)=lx^3+mx^2-7$

from x-1=0 x=1

$P(1)=l(1)^3+m(1)^2-4$

$P(1)=l+m-4$

According to conditions l+m-4=-3

$l+m=4-3$

$l=1-m$ (i)

From $x+2=0$ $x=-2$

$P(-2)=l(-2)^3+m(-2)^2-4$

$P(-2)=-8l+4m-4$

According to condition

$-8l+4m-4=12$

Putting the value of l in the equation

$-8[1-m]+4m=16$

$-8+8m+4m=16$

$12m=16+8$

$12m=24$

$m=\frac{24^{2}}{12}$

$m=2$

Putting the value of m is equation (i)

$l=1-2$

$l=-1$

$m=2$

$l=-1$

Q.9 The expression $ax^3-9x^2+bx+3a$ is exactly divisible by x^2-5x+6. Find the value of a and b.

Solution: Given that

$P(x)=ax^3-9x^2+bx+3a$

$x^2-5x+6 \quad =x^2-2x-3x+6$

$=x[x-2]-3[x-2]$

$=[x-2][x-3]$

$(x-2)(x-3)$ is divides the expression ax^3-$9x^2+bx+3a$ from $x-2=0$, $x=2$

$P(2)=a(2)^3-9(2)^2+b(2)+3a$

$P(2)=8a-36+2b+3a$

$P(2)=11a+2b-36$

According to condition (x-2) is the factor so

11a+2b-36=0 (i)

From x-3=0,x=3

$P(3)=a(3)^3-9(3)^2+b(3)+3a$

$P(3)=27a-81+3b+3a$

$P(3)=30a+3b-81$

According to condition (x-3) is the factor so

$30a+3b-81=0$ (ii)

$3(10a+b-27)=0$

$10a+b-27=\frac{0}{3}$

$b=27-10a$ (iii)

Putting the value of b in equation (i)

$11a+2[27-10a)-36=0$

$11a+54-20a-36=0$

$-9a+18=0$

$+18=9a$

$a=\frac{+18\,2}{9}$

$a=+2$

Putting the value of a in equation (iii)

$b=27-10(+2)$

$b=27-20$

$b=7$

$a=2$

Exercise 5.4

Q.1 x^3-2x^2-x+2

Solution: Given that

$P(x)=x^3-2x^2-x+2$

P=2 and possible factor of 2 are $\pm1,\pm2$.

Here q=1 and possible factor of 1 are ±1.

So possible factor of P(x) will be form $\frac{P}{q}=\pm1,\pm2$

$P(x)=x^3-2x^2-x+2$

Put x=1

$P(1)=(1)^3-2(1)^2-1+2 \quad =1-2-1+2=0$ Remainder is equal to zero so (x-1) is factor

Put x=-1

$P(-1)=(-1)^3-2(-1)^2-(-1)+2=-1-2+1+2=0$ Remainder is equal to zero so (x+1) is factor

Put x=2

$P(2)=(2)^3-2(2)^2-(2)+2=8-8-2+2=0$ Remainder is equal to zero so (x-2) is factor

$x^3-2x^2-x+2=(x-1)(x+1)(x-2)$

Q.2 $x^3-x^2-22x+40$

Solution: Given that

$P(x)=x^3-x^2-22x+40$

P=40

possible factor of 40= $\pm1,\pm2,\pm4,\pm5,\pm8,\pm10,\pm20,\pm40$

Here q=1 and possible factor of 1are ±1

So possible factor of P(x) will be from

$\frac{P}{q}=\pm1,\pm2,\pm4,\pm5,\pm8,\pm10,\pm20,\pm40$

$P(x)=x^3-x^2-22x+40$

Put $x=2$

$P(2)=(2)^3-(2)^2-22(2)+40$

$=8-4-44+40=0$

Remainder is equal to zero so (x-2) is a factor

Put x=4

$P(4)=(4)^3-(4)^2-22(4)+40$

$=64-16-88+40=0$

Remainder is not equal to zero so (x-4) is a factor

Put x=-5

$$P(-5)=(-5)^3-(-5)^2-22(-5)+40$$
$$=-125-25+110+40$$
$$=-150+150$$
$$=0$$

Remainder is equal to zero so (x+5) is a factor

Hence $x^3-x^2-22x+40=(x-2)(x-4)(x+5)$

Q.3 x^3-6x^2+3x+10

Solution: Given that

$$P(x)=x^3-6x^2+3x+10$$

P=10

So possible factor of 10 are $\pm1,\pm2,\pm5,\pm10$

Here q=1 So, possible factor of 1are ±1.

So possible of factor of P(x) can be found from $\frac{P}{q}=\pm1,\pm2,\pm5,\pm10$

P(x)=x^3-6x^2+3x+10

Put x=-1

P(-1)=$(-1)^3$-6$(-1)^2$+3(-1)+10=-1-6-3+10=0

Remainder is equal to zero so (x+1) is a factor

Put x=2

P(2)=$(2)^3$-6$(2)^2$+3(2)+10=8-24+6+10=0

Remainder is equal to zero so (x-2) is a factor

Put x=5

$P(5)=(5)^3-6(5)^2+3(5)+10=125-150+15+10=0$Remainder is equal to zero so (x-5) is a factor

Hence $x^3-6x^2+3x+10=(x+1)(x-2)(x-5)$

Q.4 $x^3+x^2-10x+8$

Solution: Given that

P(x)=x^3+x^2-10x+8

P=8 So possible factor of 8

are $\pm1,\pm2,\pm3,\pm4,\pm5,\pm6,\pm7,\pm8$.

Here q=1 So possible factor can be found from $\frac{P}{q}=\pm1,\pm2,\pm4,\pm8$

P(x)=x^3+x^2-10x+8

Put x=1

$$P(1)=(1)^3+(1)^2-10(1)+8=1+1-10+8=0$$

Remainder is equal to zero so (x-1) is a factor

Put x=2

$$P(2)=2^3+2^2-10(2)+8$$
$$=8+4-20+8$$
$$=20-20$$
$$=0$$

Remainder is equal to zero so (x-2) is a factor

Put x=-4

P(-4) $=(-4)^3+(-4)^2-10(-4)+8$

$=-64+16+40+8$

$=-64+64$

$=0$

Remainder is equal to zero so (x+4) is a factor

Hence $x^3+x^2-10x+8=(x-1)(x-2)(x+4)$

Q.5 x^3-2x^2-5x+6

Solution: Given that

$P(x)=x^3-2x^2-5x+6$

$P=6$ So factors of 3 are $\pm1,\pm2,\pm3,\pm6$

Here q=1 So factors of 1 are $\pm$ 1.

So possible factors of P(x) can be found from $\frac{P}{q}=\pm1,\pm2,\pm3,\pm6$

P(x)=x^3-2x^2-5x+6

Put x=1

P(1) $=(1)^3-2(1)^2-5(1)+6$

$=1-2-5+6$

$=-7+7$

$=0$

Remainder is equal to zero so (x-1) is a factor

Put x=-2

$P(-2)=(-2)^3-2(-2)^2-5(-2)+6$

$=-8-8+10+6$

$=-16+16$

$=0$

Remainder is equal to zero so (x+2) is a factor

Put x=3

$P(3)=(3)^3-2(3)^2-5(3)+6$

$=27-6-15+6$

27-27

$=0$

Remainder is equal to zero so (x-3) is a factor

Hence $x^3-2x^2-5x+6=(x-1)(x+2)(x-3)$

Q.6 $x^3+5x^2-2x-24$

Solution: Given that

$P(x)=x^3+5x^2-2x-24$

P= -24 So possible factors of 24 are $\pm1,\pm2,\pm3,\pm4,\pm6,\pm8,\pm12,\pm24$

Here q=1. So possible factors of 1 are $\pm$ 1.

So possible factors of P(x) will be found from

$\frac{P}{q}=\pm1,\pm2,\pm3,\pm4,\pm6,\pm8,\pm12,\pm24$

$P(x)=x^3+5x^2-2x-24$
Put $x=2$
$P(2)=(2)^3+5(2)^2-2(2)-24$
$=8+20-4-24$
$=28-28$
$=0$
Remainder is equal to zero so $(x-2)$ is a factor
Put $x=-3$
$P(-3) \quad =(-3)^3+5(-3)^2-2(-3)-24$
$=-27+45+6-24$
$=-51+51$
$=0$
Remainder is equal to zero so $(x+3)$ is a factor
Put $x=-4$
$P(-4) \quad =(-4)^3+5(-4)^2-2(-4)-24$
$=-64+80+8-24$
$=-88+88$
$=0$
Remainder is equal to zero so $(x+4)$ is a factor
Hence $x^3+5x^2-2x-24=(x-2)(x+3)(x+4)$

Q.7 $3x^3-x^2-12x+4$
Solution: Given that
$P(x)=3x^3-x^2-12x+4$

P=4 So possible factors of 4 are $\pm1,\pm2,\pm4$.
Here q=3 So possible factors of 3 are $\pm1,\pm3$.
So possible factors of P(x) can be found from
$\frac{P}{q}=\pm1,\pm2,\pm4,\frac{1}{3},\pm\frac{2}{3},\pm\frac{4}{3}$
Put $x=2$
$P(2)=3(2)^3-(2)^2-12(2)+4$
$=-24-4+24+4$
$=28-28$
$=0$
Remainder is equal to zero so $(x-2)$ is a factor
Put $x=-2$
$P(-2)=3(-2)^3-(-2)^2-12(-2)+4$
$=-24-4+24+4$
$=28-28$
$=0$
Remainder is equal to zero so $(x+2)$ is a factor
Put $x=\frac{1}{3}$
$P\left(\frac{1}{3}\right)=3\left(\frac{1}{3}\right)^3-\left(\frac{1}{3}\right)^2-\cancel{12}^4\left(\frac{1}{3}\right)+4$

$$= \not{3}\left(\frac{1}{\not{27}9}\right) - \frac{1}{9} - \not{4} + \not{4}$$

$$P\left(\frac{1}{3}\right) = \frac{\not{1}}{\not{9}} - \frac{\not{1}}{\not{9}}$$

$$= 0$$

$$x = \frac{1}{3} \Rightarrow \begin{matrix} 3x = 1 \\ 3x - 1 \end{matrix}$$

Remainder is equal to zero so (3x-1) is a factor

Hence $3x^3 - x^2 - 10x + 4 = (x-2)(x+2)(3x-1)$

Q.8 $2x^3 + x^2 - 2x - 1$

Solution: Given that

$P(x)=2x^3+x^2-2x-1$

P= -1So possible factors of -1 are $\pm 1, \pm 2$.

Here q=1. So possible factors of P(x) will be found from $\frac{P}{q}$

$\frac{P}{q} = \pm 1, \pm 2, \pm \frac{1}{2}$

$P(x)=2x^3+x^2-2x-1$

Put x=1

P(1) $=2(1)^3+(1)^2-2(1)-1$

=2+1-2-1

=3-3

=0

Remainder is equal to zero (x-1) is a factor

Put x=-1

P(-1) $=2(-1)^3+(-1)^2-2(-1)-1$

=2+1-2-1

=3-3

=0

Remainder is equal to zero (x+1) is a factor

Put $x = \frac{-1}{2}$

$$P\left(\frac{-1}{2}\right) = 2\left[\frac{-1}{2}\right]^3 + \left[\frac{-1}{2}\right]^2 - \cancel{2}\left[\frac{-1}{\cancel{2}}\right] - 1$$

$$P\left(\frac{-1}{2}\right) = 2\left[\frac{-1}{\cancel{8}4}\right] + \frac{1}{4} + \cancel{1} - \cancel{1}$$

$$P\left(\frac{-1}{2}\right) = -\cancel{\frac{1}{4}} + \cancel{\frac{1}{4}}$$

$$= 0$$

$$x = -\frac{1}{2}$$

$$2x = -1$$

$$2x + 1 = 0$$

Remainder is equal to zero so $(2x+1)$ is a factor

Hence $2x^3 + x^2 - 2x - 1 = (x-1)(x+1)(2x+1)$

Review Exercise 5

Q.1 Filling the blanks:

1. The factor of x^2-5x+6 are ________.

(a) $x+1, x-6$ **(b)** $x-2, x-3$

(c) $x+6, x-1$ **(d)** $x+2, x+3$

2. Factors of $8x^3+27y^3$ are ___________.

(a) $(2x-3y),(4x^2+9y^2)$ **(b)** $(2x-3y),(4x^2-9y^2)$

(c) $(2x+3y),(4x^2-6xy+9y^2)$ **(d)** $(2x-3y),(4x^2+6xy+9y^2)$

3. Factors of $3x^2-x-2$ are___________.

(a) $(x+1),(3x-2)$ **(b)** $(x+1),(3x+2)$

(c) $(x-1),(3x-2)$ **(d)** $(x-1),(3x+2)$

4. Factors of a^4-4b^4 are__________.

(a) $(a-b),(a+b),(a^2+4b^2)$ **(b)** $(a^2-2b^2),(a^2+2b^2)$

(c) $(a-b),(a+b)(a^2+4b^2)$ **(d)** $(a-2b),(a^2+2b^2)$

5. What will be added to complete the square of $9a^2-12ab$?.....

(a) $-16b^2$ **(b)** $16b^2$ **(c)** $4b^2$ **(d)** $-4b^2$

6. Find m so that x^2+4x+m is a complete square

(a) 8 **(b)** –8

(c) 4 **(d)** 16

7. Factors of $5x^2-17xy-12y^2$ are__________.

(a) $(x+4y),(5x+3y)$ **(b)** $(x-4y),(5x-3y)$

(c) $(x-4y),(5x+3y)$ **(d)** $(5x-4y),(x+3y)$

8. Factors of $27x^3-\frac{1}{x^3}$ are

(a) $\left(3x-\frac{1}{x}\right),\left(9x^2+3+\frac{1}{x^2}\right)$ **(b)** $\left(3x+\frac{1}{x}\right),\left(9x^2+3+\frac{1}{x^2}\right)$

(c) $\left(3x-\frac{1}{x}\right),\left(9x^2-3+\frac{1}{x^2}\right)$ **(d)** $\left(\frac{3x+1}{x}\right),\left(9x^2-3+\frac{1}{x^2}\right)$

ANSWERS KEYS

1	2	3	4	5	6	7	8
b	c	d	b	c	c	c	a

Q.2 Completion items

(i) $x^2+5x+6=$ ____________

(ii) $4a^2-16=$ ____________

(iii) $4a^2+4ab+($_____$)$ **is a complete square.**

(iv) $\frac{x^2}{y^2}-2+\frac{y^2}{x^2}=$ _________

(v) $(x+y)(x^2-xy+y^2)=$ ____________

(vi) **Factored form of** x^4-16 **is** ____________

(vii) **If x–2 is factor of** $P(x)=x^2+2kx+8$ **then** $=$ ____________

ANSWER KEYS

(i) $(x+3)(x+2)$

(ii) $(2a+4)(2a-4)=4(a+2)(a-2)$

(iii) $(b)^2$

(iv) $\left(\frac{x}{y}-\frac{y}{x}\right)^2$

(v) x^3+y^3

(vi) $(x+2)(x-2)(x^2+4)$

(vii) -3

Q.3 Factorize the following

(i) $x^2+8x+16-4y^2$

Solution: $x^2+8x+16-4y^2$

$=\left[x^2+8x+16\right]-4y^2$

$=\left[(x)^2+2(x)(4)+(4)^2\right]-(2y)^2$

$=(x+4)^2-(2y)^2$

Now arrange them

$=(x+4+2y)(x+4-2y)$

$=(x+2y+4)(x-2y+4)$

(ii) $4x^2-16y^2$

Solution: $4x^2-16y^2$

$=4\left[x^2-4y^2\right]$

$=4\left[(x)^2-(2y)^2\right]$

$=4(x-2y)(x+2y)$

(iii) $9x^2+24x+3x+8$

Solution: $=9x^2+24x+3x+8$

$=3x(3x+8)+1(3x+8)$

$=(3x+8)(3x+1)$

(iv) $1-64z^3$

Solution: $1-64z^3$

$=(1)^3-(4z)^3$

$=(1-4z)\left[(1)^2+(1)(4z)+(4z)^2\right]$

$=(1-4z)(1+4z+16z^2)$

(v) $8x^3-\left(\frac{1}{3y}\right)^3$

$=(2x)^3-\left(\frac{1}{3y}\right)^3$

$=\left(2x-\frac{1}{3y}\right)\left(4x^2+\frac{2x}{3y}+\frac{1}{9y^2}\right)$

(vi) $2y^2 + 5y - 3$

Solution: $= 2y^2 + 6y - y - 3$

$= 2y(y+3) - 1(y+3)$

$= (2y-1)(y+3)$

(vii) $x^3 + x^2 - 4x - 4$

Solution: $x^3 + x^2 - 4x - 4$

$= x^2(x+1) - 4(x+1)$

$= (x+1)(x^2 - 4)$

$= (x+1)(x-2)(x+2)$

(viii) $25m^2n^2 + 10mn + 1$

Solution: $25m^2n^2 + 10mn + 1$

$= (5mn)^2 + 2(5mn)(1) + (1)^2$

$= (5mn+1)^2$

(ix) $1 - 12pq + 36p^2q^2$

Solution: $1 - 12pq + 36p^2q^2$

$\therefore (a)^2 - 2ab + (b)^2$

$= (1)^2 - 2(1)(6pq) + (6pq)^2$

$= (1 - 6pq)^2$

Unit 5: Factorization

Overview

Factorization:

The process of expressing an algebraic expression in terms of its factors is called factorization.

Remainder Theorem:

If a polynomial $p(x)$ is divided by a linear divisor $(x-a)$, then the remainder is $p(a)$.

Zero of a Polynomial:

If a specific number $x=a$ is substituted for the variable x in a polynomial $p(x)$ so that the value $p(a)$ is zero, then $x=a$ is called a zero of the polynomial $p(x)$.

Factor Theorem:

The polynomial $(x-a)$ is a factor of the polynomial $p(x)$ if and only if $p(a)=0$.

Rational Root Theorem:

Let $a_0x^n + a_1x^{n-1} + ... + a_{n-1}x + a_n = 0,\ a_0 \neq 0$ be a polynomial equation of degree n with integral coefficients. If $\frac{p}{q}$ is a rational root (expressed in lowest terms) of the equation, then p is a factor of the constant term a_n and q is a factor of the leading coefficient a_0.

Exercise 6.1

Q.1 Find the H.C.F of the following expressions.

(i) $39x^7y^3z$ **and** $91x^5y^6z^7$

Solution:

$39x^7y^3z = 3\times13\times x.x.x.x.x.x.x.y.y.y.z$

$91x^5y^6z^7 = 7\times13\times x.x.x.x.x.y.y.y.y.y.y.z.z.z.z.z.z.z$

H.C.F $=13\times x.x.x.x.x.y.y.y.z$

H.C.F = $13x^5y^2z$

(ii) $102xy^2z, 85x^2yz$ **and** $187xyz^2$

Solution:

$102xy^2z = 2\times3\times17\times x.y.y.z$

$85x^2yz = 5\times17\times x.x.y.z$

$187xyz^2 = 11\times17\times x.y.z.z$

H.C.F $=17xyz$

Q.2 Find the H.C.F of the following expression by factorization.

(i) $x^2+5x+6, x^2-4x-12$

Solution: $x^2+5x+6, x^2-4x-12$

Factorizing x^2+5x+6

$=x^2+3x+2x+6$

$=x(x+3)+2(x+3)$

$=(x+3)(x+2)$

Factorizing $x^2-4x-12$

$=x^2-6x+2x-12$

$=x(x-6)+2(x-6)$

$=(x-6)(x+2)$

So,

H.C.F = $(x+2)$

(ii) $x^2-27, x^2+6x-27, 2x^2-18$

Solution: $x^2-27, x^2+6x-27, 2x^2-18$

Factorizing x^3-27

$=(x)^3-(3)^3$

$=(x-3)\left[(x)^2+(x)(3)+(3)^2\right]$

$=(x-3)(x^2+3x+9)$

Factorizing $x^2+6x-27$

$=x^2+9x-3x-27$

$=x(x+9)-3(x+9)$

$=(x+9)(x-3)$

Factorizing $2x^2-18$

$=2(x^2-9)$

$=2\left[(x)^2-(3)^2\right]$

$=2(x-3)(x+3)$

So,

H.C.F $=(x-3)$

(iii) $x^3-2x^2+x, x^2+2x-3, x^2+3x-4$

Factorizing x^3-2x^2+x

$=x(x^2-2x+1)$

$x(x^2-x-x+1)$

$=x\left[x(x-1)-1(x-1)\right]$

$=x(x-1)(x-1)$

Factorizing x^2+2x-3

$=x^2+3x-x-3$

$=x(x+3)-1(x+3)$

$=(x+3)(x-1)$

Factorizing x^2+3x-4

$=x^2+4x-x-4$

$=x(x+4)-1(x+4)$

$=(x+4)(x-1)$

So,

H.C.F $=(x-1)$

(iv) $18(x^3-9x^2+8x), 24(x^2+3x+2)$

Solution: $18(x^3-9x^2+8x), 24(x^2+3x+2)$

Factorizing $18(x^3-9x^2+8x)$

$=6\times3\times x(x^2-9x+8)$

$=6\times3\times x\left(x^2-8x-x+8\right)$

$=6\times3\times x\left[x(x-8)-1(x-8)\right]$

$=6\times3\times x(x-8)(x-1)$

Factorizing $24\left(x^2+3x+2\right)$

$=6\times4\left(x^2-3x+2\right)$

$=6\times4\left(x^2-2x-x+2\right)$

$=6\times4\left[x(x-2)-1(x-2)\right]$

$=6\times4(x-2)(x-1)$

So,

H.C.F = $6(x-1)$

(v) $36\left(3x^4+5x^2-2x^2\right), 54\left(27x^4-x\right)$

Factorizing $36\left(3x^4+6x^3-2x^2\right)$

$=3\times3\times2\times2\times x^2\left(3x^2+5x-2\right)$

$=3\times3\times2\times2\times x^2\left(3x^2+6x-x-2\right)$

$=3\times3\times2\times2\times x^2\left[3x(x+2)-1(x+2)\right]$

$=3\times3\times2\times2\times x^2(x+2)(3x-1)$

Factorizing $54\left(27x^4-x\right)$

$=3\times3\times3\times2\times x\left(27x^3-1\right)$

$=3\times3\times3\times2\times x\left[(3x)^3-(1)\right]$

$=3\times3\times3\times2\times x(3x-1)\left[(3x)^2+(3x)(1)+(1)^2\right]$

$=3\times3\times3\times2\times x(3x-1)\left(9x^2+3x+1\right)$

So,

H.C.F = $3\times3\times2\times x(3x-1)$

$=18x(3x-1)$

Q.3 Find the H.C.F of the following by division method.

(i) $x^3+3x^2-16x+12,\ x^3+x^2-10x+8$

Solution: $x^3+3x^2-16x+12,\ x^3+x^2-10x+8$

$$x^3+x^2-10x+8\overline{)x^3+3x^2-16x+12}\quad \text{quotient } 1$$

$$\underline{\pm x^3 \pm x^2 \mp 10x \pm 8}$$

$$2x^2-6x+4$$

$$2\left(x^2-3x+2\right)$$

$$x^2-3x+2\overline{)x^3+x^2-10x+8}\quad \text{quotient } x+4$$

$$\underline{\pm x^3 \mp 3x^2 \pm 2x}$$

$$4x^2-12x+8$$

$$\underline{\pm 4x^2 \mp 12x \pm 8}$$

$$\times$$

H.C.F = $\left(x^2-3x+2\right)$

(ii) $x^4+x^3-2x^2+x-3,\ 5x^3+3x^2-17x+6$

Solution: $x^4+x^3-2x^2+x-3,\ 5x^3+3x^2-17x+6$

$$5x^3+3x^2-17x+6\overline{)x^4+x^3-2x^2+x-3}\quad \text{quotient } x+2$$

$$\underline{\times 5}$$

$$5x^4+5x^3-10x^2+5x-15$$

$$\underline{\pm 5x^4 \pm 3x^3 \mp 17x^2 \pm 6x}$$

$$2x^3+7x^2-x-15$$

$$\underline{\times 5}$$

$$10x^3+35x^2-5x-75$$

$$\underline{\pm 10x^3 \pm 6x^2 \mp 34x \pm 12}$$

$$29x^2+29x-87$$

$$29(x^2+x-3)$$

$$x^2+x-3\overline{)5x^3+3x^2-17x+6}\quad \text{quotient } 5x-2$$

$$\pm 5x^3 \pm 5x^2 \mp 15x$$

$$-2x^2-2x+6$$

$$\underline{\mp 2x^2 \mp 2x \pm 6}$$

$$\times$$

H.C.F $=\left(x^2+x-3\right)$

(iii) $2x^5-4x^4-6x, x^5+x^4-3x^3-3x^2$

$$2x^5-4x^4-6x\overline{)\,x^5+x^4-3x^3-3x^2}\quad 1$$

$\times 2$

$2x^5+2x^4-6x^3-6x^2$

$\underline{-2x^5 \mp 4x^4 \qquad \mp 6x}$

$6x^4-6x^3-6x^2+6x$

$6\left(x^4-x^3-x^2+x\right)$

$$x^4-x^3-x^2+x\overline{)\,2x^5-4x^2-6x}\quad 2x-2$$

$\underline{\pm 2x^5 \pm 2x^2 \qquad \mp 2x^4 \mp 2x^3}$

$-2x^4+2x^3-2x^2-6x$

$\underline{\mp 2x^4 \pm 2x^3 \pm 2x^2 \mp 2x}$

$-4x^2-4x$

$-4\left(x^2+x\right)$

$$x^2+x\overline{)\,x^4-x^3-x^2+x}\quad x^2-2x+1$$

$\underline{-x^4 \pm x^3}$

$-2x^3-x^2+x$

$\underline{\mp 2x^3 \mp 2x^2}$

x^2+x

$\underline{\pm x^2 \pm x}$

$\times$

H.C.F $=x^2+x$

Q.4 Find the L.C.M of the following expressions.

(i) $39x^7y^3z$ **and** $91x^5y^6z^7$

Solution:

$39x^7y^3z=3\times 13\times x.x.x.x.x.x.x.y.y.y.z$

$91x^5y^6z^7=7\times 13\times x.x.x.x.x.y.y.y.y.y.y.z.z.z.z.z.z.z$

Common$=13x^5y^8z$

Uncommon$=3\times 7\times x^2y^3z^6$

$=21x^2y^3z^6$

L.C.M= common factors$\times$uncommon factors

$=13x^5y^3z\times 21x^2y^3z^6$

$273x^7y^6z^7$

(ii) $102xy^2z, 85x^2yz$ **and** $187xyz^2$

Solution:

$102xy^2z = 2\times3\times17.x.y.y.z$

$85x^2yz = 5\times17\times x.x.y.z$

$187xyz^2 = 11\times17.x.y.z.z$

Common=$17xyz$

Uncommon=$2\times3\times5\times11.xyz$

=$330\ xyz$

L.C.M=common x uncommon

$=17xyz\times330xyz$

$=5610x^2y^2z^2$

Q.5 Find the L.C.M of the following by factorizing.

(i) $x^2-25x+100$ **and** x^2-x-20

Solution: $x^2-25x+100$ **and** x^2-x-20

Factorizing $x^2-25x+100$

$=x^2-20x-5x+100$

$=x(x-20)-5(x-20)$

$=(x-20)(x-5)$

Factorizing x^2-x-20

$=x^2-5x+4x-20$

$=x(x-5)+4(x-5)$

$=(x-5)(x+4)$

So,

L.C.M = $(x-5)(x+4)(x-20)$

(ii) $x^2+4x+4, x^2-4, 2x^2+x-6$

Solution: $x^2+4x+4, x^2-4, 2x^2+x-6$

Factorizing x^2+4x+4

$=x^2+2x+2x+4$

$=x(x+2)+2(x+2)$

$=(x+2)(x+2)$

Factorizing x^2-4

$=(x)^2-(2)^2$

$=(x-2)(x+2)$

Factorizing $2x^2+x-6$

$=2x^2+4x-3-6$

$=2x(x+2)-3(x+2)$

$=(x+2)(2x-3)$

So,

L.C.M $= (x+2)(x+2)(x-2)(2x-3)$

$= (x+2)^2(x-2)(2x-3)$

(iii) $2(x^4-y^4), 3(x^3+2x^2y-xy^2-2y^3)$

Factorizing $2(x^4-y^4)$

$= 2\left[(x^2)^2-(y^2)^2\right]$

$= 2(x^2+y^2)(x^2-y^2)$

$= 2(x^2+y^2)(x+y)(x-y)$

Factorizing $3(x^3+2x^2y-xy^2-2y^3)$

$= 3\left[x^2(x+2y)-y^2(x+2y)\right]$

$= 3(x+2y)(x^2-y^2)$

$= 3(x+2y)(x+y)(x-y)$

So,

L.C.M $= (x+y)(x-y)(x^2+y^2)(x+2y)\times 2\times 3$

$= 6(x+y)(x-y)(x^2+y^2)(x+2y)$

$= 6(x+2y)(x^4-y^4)$

(iv) $4(x^4-1), 6(x^3-x^2-x+1)$

Solution: $4(x^4-1), 6(x^3-x^2-x+1)$

Factorizing $4(x^4-1)$

$= = 2\times 2\left[(x^2)^2-(1)^2\right]$

$= 2\times 2(x^2+1)(x^2-1)$

$= 2\times 2(x^2+1)(x+1)(x-1)$

$=6(x^3-x^2-x+1)$

$= 2\times 3\left[x^2(x-1)-1(x-1)\right]$

$=2\times 3\left[(x-1)(x^2-1)\right]$

$=2\times 3(x-1)(x-1)(x+1)$

L.C.M$=2\times 2\times 3(x-1)(x+1)(x-1)(x^2+1)$

$=12(x-1)^2(x+1)(x^2+1)$

$=12(x-1)(x^4-1)$

Q.6 **For what value of k is $(x+4)$ the H.C.F of $x^2+x-(2k+2)$ and $2x^2+kx-12$?**

Solution:

$P(x)=x^2+x-(2k+2)$

Since $x+4$ is H.C.F of $P(x)$ and $q(x)$

$\therefore x+4$ is a factor of $P(x)$

Hence $P(-4)=0$

$x^2+x-(2k+2)=0$

By putting the value of x

$(-4)^2+(-4)-(2k+2)=0$

$16-4-2k-2=0$

$-2k+10=0$

$2k=10$

$k=\frac{10^5}{2}$

$k=5$

Q.7 **If $(x+3)(x-2)$ is the H.C.F of $P(x)=(x+3)(2x^2-3x+k)$ and $q(x)=(x-2)(3x^2+7x-l)$ the find k and l**

Solution: (x-2) (x+3) will divide $P(x)$ =(x+3) (2x^2-3x+K)

(x – 2) (x+3) will divide P(x) =(x+3) (2x^2-3x+K)

x–2=0

x=2

$P(2)=(2+3)(2(2)^2-3(2)+K)$

$P(2)=(5)(2\times 4-6+K)$

$P(2)=5(8-6+K)$

$P(2)=5(2+K)$

Remainder is equal to zero

$5(2+K)=0$

$2+K=\frac{0}{5}$

$2+K=0$

$K=-2$

$q(x)=(x-2)(3x^2+7x-1)$

$(x-2)(x+3)$ will be divide $q\ (x)=(x-2)(3x^2+7x-l)$

$x+3=0$

$x=-3$

$q(-3)=(-3-2)\left[3(-3)^2+7(-3)-l\right]$

$q(-3)=(-5)\left[3(9)-21-l\right]$

$q(-3)=(-5)[27-21-l]$

$q(-3)=(-5)(6-l)$

Remainder is equal to zero

$-5(6-l)=0$

$6-l=0$

$l=6$

Q.8 **The L.C.M and H.C.F of two polynomials** $P(x)$ **and** $q(x)$ **are** $2(x^4-1)$ **and** $(x+1)(x^2+1)$ **respectively. If** $P(x)=x^3+x^2+x+1$, **find** $q(x)$

Solution: $\therefore P(x)\times q(x)=\text{L.C.M}\times\text{H.C.F}$

$\therefore P(x)\times q(x)=\text{L.C.M}\times\text{H.C.F}$

$$q(x)=\frac{L.C.M\times H.C.F}{P(x)}$$

By putting the values

$$q(x)=\frac{2(x^4-1)(x+1)(x^2+1)}{x^3+x^2+x+1}$$

$$q(x)=\frac{2(x^4-1)(x+1)(x^2+1)}{x^2(x+1)+1(x+1)}$$

$$q(x)=\frac{2(x^4-1)\cancel{(x+1)}\cancel{(x^2+1)}}{\cancel{(x+1)}\cancel{(x^2+1)}}$$

$$q(x)=2(x^4-1)$$

Q.9 **Let** $p(x)=10(x^2-9)(x^2-3x+2)$ **and** $q(x)=10x(x+3)(x-1)^2$. **if the H.C.F of** $p(x),q(x)$ **is** $10(x+3)(x-1)$, **Find their L.C.M**

Solutions: $p(x)\times q(x)=:L.C.M\times H.C.F$

$p(x)\times q(x)=:L.C.M\times H.C.F$

$$L.C.M=\frac{p(x)\times q(x)}{H.C.F}$$

By putting the values

$$L.C.M=\frac{\cancel{10}(x^2-9)(x^2-3x+2)\times 10x\cancel{(x+3)}(x-1)^{\cancel{2}}}{\cancel{10}\cancel{(x+3)}\cancel{(x-1)}}$$

$$L.C.M=10x(x^2-9)(x^2-3x+2)(x-1)$$

Q.10 Let the product of L.C.M and H.C.F of two polynomial be $(x+3)^2(x-2)(x+5)$. If one polynomial is $(x+3)(x-2)$ and the second polynomial is $x^2+kx+15$, find the value of k.

Solution: $p(x)\times q(x)=L.C.M\times H.C.F$

$p(x)\times q(x)=L.C.M\times H.C.F$

By putting the values

$(x+3)(x-2)(x^2+kx+15)=(x+3)^2(x-2)(x+5)$

$x^2+kx+15=\dfrac{(x+3)^{\not 2}(\cancel{x-2})(x+5)}{(\cancel{x+3})(\cancel{x-2})}$

$x^2+kx+15=(x+3)(x+5)$

$x^2+kx+15=x^2+8x+15$

$kx=\cancel{x^2}+8x+\cancel{15}-\cancel{x^2}-\cancel{15}$

$kx=8x$

$k=\dfrac{8\cancel{x}}{\cancel{x}}$

$k=8$

Q.11 Waqas wishes to distribute 128 bananas and also 176 apples equally among a certain number of children. Find the highest number of children who can get fruit in this way.

Solution

$$
\begin{array}{r|l}
128 & 176 \; (1 \\
 & \underline{128} \\
48 & 128 \; (2 \\
 & \underline{-96} \\
32 & 48 \\
 & \underline{-32} \\
16 & 32 \; (2 \\
 & \underline{-32} \\
 & 0
\end{array}
$$

Highest no. of children =16

Exercise 6.2

Q.1 Simplify each of the following as a rational expression.

(i) $\dfrac{x^2-x-6}{x^2-9}+\dfrac{x^2+2x-24}{x^2-x-12}$

Solution: $\dfrac{x^2-x-6}{x^2-9}+\dfrac{x^2+2x-24}{x^2-x-12}$

$=\dfrac{x^2-x-6}{x^2-9}+\dfrac{x^2+2x-24}{x^2-x-12}$

$=\dfrac{x^2-3x-2x-6}{(x)^2-(3)^2}+\dfrac{x^2+6x-4x-24}{x^2-x-12}$

$=\dfrac{x(x-3)+2(x-3)}{(x-3)(x+3)}+\dfrac{x(x+6)-4(x+6)}{x(x-4)+3(x-4)}$

$=\dfrac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x+3)}+\dfrac{(x+6)\cancel{(x-4)}}{\cancel{(x-4)}(x+3)}$

$=\dfrac{(x+2)}{(x+3)}+\dfrac{(x+6)}{(x+3)}$

$=\dfrac{x+2+x+6}{x+3}$

$=\dfrac{8+2x}{x+3}$

$=\dfrac{2(x+4)}{x+3}$

Q.2 $\left[\dfrac{x+1}{x-1}-\dfrac{x-1}{x+1}-\dfrac{4x}{x^2+1}\right]+\dfrac{4x}{x^4-1}$

Solution: $\left[\dfrac{x+1}{x-1}-\dfrac{x-1}{x+1}-\dfrac{4x}{x^2+1}\right]+\dfrac{4x}{x^4-1}$

$=\left[\dfrac{x+1}{x-1}-\dfrac{x-1}{x+1}-\dfrac{4x}{x^2+1}\right]+\dfrac{4x}{x^4-1}$

$=\left[\dfrac{(x+1)^2-(x-1)^2}{(x-1)(x+1)}-\dfrac{4x}{x^2+1}\right]+\dfrac{4x}{x^4-1}$

$=\left[\dfrac{x^2+2x+1-(x^2+1-2x)}{(x-1)(x+1)}-\dfrac{4x}{x^2+1}\right]+\dfrac{4x}{x^4-1}$

$$=\left[\frac{x^2+2x+1-x^2-1+2x}{x^2-1}-\frac{4x}{x^2+1}\right]+\left[\frac{4x}{x^4-1}\right]$$

$$=\left[\frac{4x}{x^2-1}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{4x(x^2+1)-4x(x-1)}{(x^2-1)(x^2+1)}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{4x^3+4x-4x^3+4x}{x^4-1}\right]+\frac{4x}{x^4-1}$$

$$=\frac{8x}{x^4-1}+\frac{4x}{x^4-1}$$

$$=\frac{8x+4x}{x^4-1}$$

$$=\frac{12x}{x^4-1}$$

Q.3 $\frac{1}{x^2-8x+15}+\frac{1}{x^2-4x+3}-\frac{2}{x^2-6x+5}$

Solution: $\frac{1}{x^2-8x+15}+\frac{1}{x^2-4x+3}-\frac{2}{x^2-6x+5}$

$$=\frac{1}{x^2-8x+15}+\frac{1}{x^2-4x+3}-\frac{2}{x^2-6x+5}$$

$$=\frac{1}{x^2-3x-5x+15}+\frac{1}{x^2-3x-1x+3}-\frac{2}{x^2-5x-x+5}$$

$$=\frac{1}{x(x-3)-5(x-3)}+\frac{1}{x(x-3)-1(x-3)}-\frac{2}{x(x-5)-1(x-5)}$$

$$=\frac{1}{(x-3)(x-5)}+\frac{1}{(x-3)(x-1)}-\frac{2}{(x-5)(x-1)}$$

$$=\frac{(x-1)+(x-5)-2(x-3)}{(x-3)(x-5)(x-1)}$$

$$=\frac{x-1+x-5-2x+6}{(x-3)(x-)(x-1)}$$

$$=\frac{0}{(x-3)(x-5)(x-1)}$$

$= 0$

Q.4 $\dfrac{(x+2)(x+3)}{x^2-9}+\dfrac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$

Solution: $\dfrac{(x+2)(x+3)}{x^2-9}+\dfrac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$

$$=\frac{(x+2)(x+3)}{x^2-9}+\frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$$

$$=\frac{(x+2)(x+3)}{(x)^2-(3)^2}+\frac{(x+2)\left[2(x^2-16)\right]}{(x-4)(x^2-3x+2x-6)}$$

$$=\frac{(x+2)\cancel{(x+3)}}{(x-3)\cancel{(x+3)}}+\frac{(x+2)\left[2(x)^2-(4)^2\right]}{(x-4)\left[x(x-3)+2(x-3)\right]}$$

$$=\frac{(x+2)}{(x-3)}+\frac{\cancel{(x+2)}\left[2(x+4)\cancel{(x-4)}\right]}{\cancel{(x-4)}(x-3)\cancel{(x+2)}}$$

$$=\frac{(x+2)}{(x-3)}+\frac{2(x+4)}{(x-3)}$$

$$=\frac{x+2}{x-3}+\frac{2x+8}{x-3}$$

$$=\frac{x+2+2x+8}{x-3}$$

$$=\frac{3x+10}{x-3}$$

Q.5 $=\dfrac{x+3}{2x^2+9x+9}+\dfrac{1}{2(2x-3)}-\dfrac{4x}{4x^2-9}$

Solution: $=\dfrac{x+3}{2x^2+9x+9}+\dfrac{1}{2(2x-3)}-\dfrac{4x}{4x^2-9}$

$$=\frac{x+3}{2x^2+9x+9}+\frac{1}{2(2x-3)}-\frac{4x}{4x^2-9}$$

$$=\frac{x+3}{2x^2+6x+3x+9}+\frac{1}{2(2x-3)}-\frac{4x}{(2x)^2-(3)^2}$$

$$=\frac{x+3}{2x(x+3)+3(x+3)}+\frac{1}{2(2x-3)}-\frac{4x}{(2x-3)(2x+3)}$$

$$=\frac{\cancel{(x+3)}}{\cancel{(x+3)}(2x+3)}+\frac{1}{2(2x-3)}-\frac{4x}{(2x-3)(2x+3)}$$

$$=\frac{1}{2x+3}+\frac{1}{2(2x-3)}-\frac{4x}{(2x-3)(2x+3)}$$

$$=\frac{2(2x-3)+(2x+3)-4x\times 2}{2(2x-3)(2x+3)}$$

$$=\frac{4x-6+2x+3-8x}{2(2x-3)(2x+3)}$$

$$=\frac{-2x-3}{2(2x-3)(2x+3)}$$

$$=\frac{-1(2x+3)}{2(2x-3)(2x+3)}$$

$$=\frac{-1}{2(2x-3)}$$

Q.6 $A-\frac{1}{A}$, **Where** $A=\frac{a+1}{a-1}$

Solution: $A-\frac{1}{A}$, Where $A=\frac{a+1}{a-1}$

$$=A-\frac{1}{A}=?$$

Put the value of A

$$=\frac{a+1}{a-1}-\frac{a-1}{a+1}$$

$$=\frac{(a+1)^2-(a-1)^2}{(a-1)(a+1)}$$

$$=\frac{a^2+2a+1-(a^2-2a+1)}{a^2-1}$$

$$=\frac{a^2+2a+1-a^2+2a-1}{a^2-1}$$

$$=\frac{4a}{a^2-1}$$

Q.7 $\left[\frac{x-1}{x-2}+\frac{2}{2-x}\right]-\left[\frac{x+1}{x+2}+\frac{4}{4-x^2}\right]$

Solution: $\left[\frac{x-1}{x-2}+\frac{2}{2-x}\right]-\left[\frac{x+1}{x+2}+\frac{4}{4-x^2}\right]$

$$=\left[\frac{x-1}{x-2}+\frac{2}{2-x}\right]-\left[\frac{x+1}{x+2}+\frac{4}{4-x^2}\right]$$

$$=\left[\frac{x-1}{x-2}+\frac{2}{-x+2}\right]-\left[\frac{x+1}{x+2}+\frac{4}{-x^2+4}\right]$$

$$=\left[\frac{x-1}{x-2}-\frac{2}{x-2}\right]-\left[\frac{x+1}{x+2}+\frac{4}{-\left(x^2-4\right)}\right]$$

$$=\left[\frac{x-1}{x-2}-\frac{2}{x-2}\right]-\left[\frac{x+1}{x+2}-\frac{4}{x^2-4}\right]$$

$$=\left[\frac{x-1-2}{x-2}\right]-\left[\frac{x+1}{x+2}-\frac{4}{(x+2)(x-2)}\right]$$

$$=\frac{(x-3)}{(x-2)}-\frac{(x+1)(x-2)-4}{(x+2)(x-2)}$$

$$=\frac{x-3}{x-2}-\frac{x^2-x-2-4}{(x+2)(x-2)}$$

$$=\frac{x-3}{x-2}-\frac{x^2-x-6}{(x-2)(x+2)}$$

$$=\frac{x-3}{x-2}-\frac{x^2-3x+2x-6}{(x-2)(x+2)}$$

$$=\frac{x-3}{x-2}-\frac{x(x-3)2(x-3)}{(x-2)(x+2)}$$

$$=\frac{x-3}{x-2}-\frac{(x-3)(x+2)}{(x-2)(x+2)}$$

$$=\frac{x-3}{x-2}-\frac{x-3}{x-2}$$

0 **Ans**

Q.8 What rational number should be subtracted from

$$=\frac{2x^2+2x-7}{x2+x-6}\text{ to get }\frac{x-1}{x-2}$$

Solution: let required rational number be *P*(*x*)

According to condition

$$\frac{2x^2+2x-7}{x^2+x-6}-P(x)=\frac{x-1}{x-2}$$

$$P(x)=\frac{2x^2+2x-7}{x^2+x-6}-\frac{x-1}{x-2}$$

$$=\frac{2x^2+2x-7}{x^2+3x-2x-6}-\frac{x-1}{x-2}$$

$$=\frac{2x^2+2x-7}{x(x+3)-2(x+3)}-\frac{x-1}{x-2}$$

$$=\frac{2x^2+2x-7}{(x+3)(x-2)}-\frac{x-1}{x-2}$$

$$=\frac{2x^2+2x-7-(x-1)(x+3)}{(x+3)(x-2)}$$

$$=\frac{2x^2+2x-7-(x^2+2x-3)}{(x+3)(x-2)}$$

$$=\frac{2x^2+2x-7-x^2-2x+3}{(x+3)(x-2)}$$

$$=\frac{x^2-4}{(x+3)(x-2)}$$

$$=\frac{x^2-2^2}{(x+3)(x-2)}$$

$$=\frac{(x+2)\,\cancel{(x-2)}}{(x+3)\,\cancel{(x-2)}}$$

$$=\frac{x+2}{x+3}$$

Q.9 $=\frac{x^2+x-6}{x^2-x-6}\times\frac{x^2-4}{x^2-9}$

Solution: $=\frac{x^2+x-6}{x^2-x-6}\times\frac{x^2-4}{x^2-9}$

$$=\frac{x^2+x-6}{x^2-x-6}\times\frac{x^2-4}{x^2-9}$$

$$=\frac{x^2+3x-2x-6}{x^2-3x+2x-6}\times\frac{x^2-2^2}{x^2-3^2}$$

$$=\frac{x(x+3)-2(x+3)}{2(x-3)+2(x-3)}\times\frac{(x-2)(x+3)}{(x-3)(x+3)}$$

$$=\frac{\cancel{(x+3)}(x-2)}{(x-3)\cancel{(x+2)}}\times\frac{(x-2)\cancel{(x+2)}}{(x-3)\cancel{(x+3)}}$$

$$=\frac{(x-2)^2}{(x-3)^2}$$

Q.10 $\frac{x^3-8}{x^2-4}\times\frac{x^2+6x+8}{x^2-2x+1}$

Solution: $\frac{x^3-8}{x^2-4}\times\frac{x^2+6x+8}{x^2-2x+1}$

$$=\frac{x^3-8}{x^2-4}\times\frac{x^2+6x+8}{x^2-2x+1}$$

$$=\frac{(x)^3-(2)^3}{(x^2)-(2)^2}\times\frac{x^2+4x+2x+8}{x^2-x-x+1}$$

$$=\frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}\times\frac{x(x+4)+2(x+4)}{x(x-1)-1(x-1)}$$

$$=\frac{x^2+2x+4}{(\cancel{x+2})}\times\frac{(x+4)(\cancel{x+2})}{(x-1)(x-1)}$$

$$=\frac{(x^2+2x+4)(x+4)}{(x-1)^2}$$

Q.11 $\frac{x^4-8x}{2x^2+5x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x^2-2x}$

Solution: $\frac{x^4-8x}{2x^2+5x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x^2-2x}$

$$=\frac{x^4-8x}{2x^2+5x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x^2-2x}$$

$$=\frac{x(x^3-8)}{2x^2+6x-x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x(x-2)}$$

$$=\frac{x\left[(x)^3-(2)^3\right]}{2x(x+3)-1(x+3)}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x(x-2)}$$

$$=\frac{\cancel{x}(\cancel{x-2})(\cancel{x^2+2x+4})}{(\cancel{2x-1})(\cancel{x+3})}\times\frac{\cancel{2x-1}}{\cancel{x^2+2x+4}}\times\frac{\cancel{x+3}}{\cancel{x}(\cancel{x-2})}$$

$=1$ **Ans**

Q.12 $\frac{2y^2+7y-4}{3y^2-13y+4}\div\frac{4y^2-1}{6y^2+y-1}$

Solution: $\frac{2y^2+7y-4}{3y^2-13y+4}\div\frac{4y^2-1}{6y^2+y-1}$

$$= \frac{2y^2 + 7y - 4}{3y^2 - 13y + 4} \div \frac{4y^2 - 1}{6y^2 + y - 1}$$

$$= \frac{2y^2 + 8y - 1y - 4}{3y^2 - 12y - y + 4} \div \frac{(2y)^2 - (1)^2}{6y^2 + 3y - 2y - 1}$$

$$= \frac{2y(y+4) - 1(y+4)}{3y(y-4) - 1(y-4)} \div \frac{(2y-1)(2y+1)}{3y(2y+1) - 1(2y+1)}$$

$$= \frac{(y+4)(2y-1)}{(3y-1)(y-4)} \div \frac{(2y-1)(2y+1)}{(3y-1)(2y+1)}$$

$$= \frac{(y+4)(2y-1)}{(3y-1)(y-4)} \times \frac{(3y-1)}{(2y-1)}$$

$$= \frac{y+4}{y-4}$$

Q.13 $\left[\frac{x^2 + y^2}{x^2 - y^2} - \frac{x^2 - y^2}{x^2 + y^2}\right] \div \left[\frac{x+y}{x-y} - \frac{x-y}{x+y}\right]$

Solution: $\left[\frac{x^2 + y^2}{x^2 - y^2} - \frac{x^2 - y^2}{x^2 + y^2}\right] \div \left[\frac{x+y}{x-y} - \frac{x-y}{x+y}\right]$

$$= \left[\frac{x^2 + y^2}{x^2 - y^2} - \frac{x^2 - y^2}{x^2 + y^2}\right] \div \left[\frac{x+y}{x-y} - \frac{x-y}{x+y}\right]$$

$$= \left[\frac{(x^2 + y^2)^2 - (x^2 - y^2)^2}{(x^2 - y^2)(x^2 + y^2)}\right] \div \left[\frac{(x+y)^2 - (x-y)^2}{(x-y)(x+y)}\right]$$

$$= \left[\frac{(x^4 + 2x^2y^2 + y^4) - (x^4 - 2x^2y^2 + y^4)}{(x^2 - y^2)(x^2 + y^2)}\right] \div \left[\frac{(x^2 + 2xy + y^2) - (x^2 - 2xy + y^2)}{x^2 - y^2}\right]$$

$$= \left[\frac{x^4 + 2x^2y^2 + y^4 - x^4 + 2x^2y^2 - y^4}{(x^2 - y^2)(x^2 + y^2)}\right] \div \left[\frac{x^2 + 2xy + y^2 - x^2 + 2xy - y^2}{x^2 - y^2}\right]$$

$$= \left[\frac{4x^2y^2}{(x^2 - y^2)(x^2 + y^2)}\right] \div \left[\frac{4xy}{x^2 - y^2}\right]$$

$$= \frac{4x^2y^2}{(x^2 - y^2)(x^2 + y^2)} \times \frac{x^2 - y^2}{4xy}$$

$$= \frac{4xy.xy}{(x^2 - y^2)(x^2 + y^2)} \times \frac{x^2 - y^2}{4xy}$$

$$= \frac{xy}{x^2 + y^2}$$ **Ans**

Exercise 6.3

Q.1 Use factorization to find the square root of the following expression.

(i) $4x^2-12xy+9y^2$

Solution: $4x^2-12xy+9y^2$

$4x^2-12xy+9y^2=4x^2-6xy-6xy+9y^2$

$=2x(2x-3y)-3y(3x-3y)$

$=(2x-3y)(2x-3y)$

$4x^2-12y+9y^2=(2x-3y)^2$

Taking square root on both side

$\sqrt{4x^2-12xy+9y^2}=\sqrt{[2x-3y]^2}$

$=\pm(2x-3y)$

(ii) $x^2-1+\frac{1}{4x^2}$

Solution: $x^2-1+\frac{1}{4x^2}$

$=(x)^2-2(x)\left[\frac{1}{2x}\right]+\left[\frac{1}{2x}\right]^2$

$=\left[x-\frac{1}{2x}\right]^2$

Taking square root

$\sqrt{x^2-1+\frac{1}{4x^2}}=\sqrt{\left[x-\frac{1}{2x}\right]^2}$

$\sqrt{x^2-1+\frac{1}{4x^2}}=\pm\left(x-\frac{1}{2x}\right)$

(iii) $\frac{1}{16}x^2-\frac{1}{12}xy+\frac{1}{36}y^2$

Solution: $\frac{1}{16}x^2-\frac{1}{12}xy+\frac{1}{36}y^2$

$=\left(\frac{1}{4}x\right)^2-2\left(\frac{1}{4}x\right)\left(\frac{1}{6}y\right)+\left(\frac{1}{6}y\right)^2$

$=\left(\frac{x}{4}-\frac{y}{6}\right)^2$

Taking the square root

$$\sqrt{\frac{1}{16}x^2 - \frac{1}{12}xy + \frac{1}{36}y^2} = \sqrt{\left(\frac{1}{4}x - \frac{1}{6}y\right)^2}$$
$$= \pm\left(\frac{1}{4}x - \frac{1}{6}y\right)$$
$$= \pm\left(\frac{x}{4} - \frac{y}{6}\right)$$

(iv) $4(a+b)^2 - 12(a2+b^2) + 9(a-b)^2$

Solution: $4(a+b)^2 - 12(a2+b^2) + 9(a-b)^2$

$= \left[2(a+b)^2\right] - 2[2(a+b)][3(a-b)] + [3(a-b)]^2$

$= \left[2(a+b) - 3(a-b)\right]^2$

Taking square root

$\sqrt{4(a+b)^2 - 12(a2+b^2) + 9(a-b)^2} = \sqrt{[2(a+b) - 3(a-b)]^2}$

$= \pm[2a + 2b - 3a + 3b]$
$= \pm(5b - a)$

(v) $\dfrac{4x^6 - 12x^3y^3 + 9y^6}{9x^4 + 24x^2y^2 + 16y^4}$

Solution: $\dfrac{4x^6 - 12x^3y^3 + 9y^6}{9x^4 + 24x^2y^2 + 16y^4}$

$= \dfrac{(2x^3)^2 - 2(2x^3)(3y^3) + (3y^3)^2}{(3x^3)^2 + 2(3x^2)(4y^2) + (4y^2)^2}$

=

$= \dfrac{\left[2x^3 - 3y^3\right]^2}{\left[3x^3 + 4y^2\right]^2}$

Taking square root

$= \sqrt{\dfrac{4x^6 - 12x^3y^3 + 9y^6}{9x^4 + 24x^2y^2 + 16y^4}}$

$= \pm\left(\dfrac{2x^3 - 3y^3}{3x^2 + 4y^2}\right)$

(vi) $\left(x + \frac{1}{x}\right)^2 - 4\left(x - \frac{1}{x}\right), (x \neq 0)$

Solution: $\left(x + \frac{1}{x}\right)^2 - 4\left(x - \frac{1}{x}\right), (x \neq 0)$

By adding and substituting 4

$$= x^2 + \frac{1}{x^2} + 2 - 4\left(x - \frac{1}{x}\right)$$

$$= x^2 + \frac{1}{x^2} + 2 - 4\left(x - \frac{1}{x}\right) - 4 + 4$$

$$= x^2 + \frac{1}{x^2} - 2 - 4\left(x - \frac{1}{x}\right) + 4$$

$$= \left(x - \frac{1}{x}\right)^2 - 2\left(x - \frac{1}{x}\right)(2) + (2)^2$$

$$\left[\left(x - \frac{1}{x}\right) - 2\right]^2$$

Taking square root

$$\sqrt{\left(x + \frac{1}{x}\right)^2 - 4\left(x - \frac{1}{x}\right)} = \sqrt{\left[x - \frac{1}{x} - 2\right]^2}$$

$$= \pm\left(x - \frac{1}{x} - 2\right)$$

(vii) $\left(x^2 + \frac{1}{x^2}\right)^2 - 4\left(x + \frac{1}{x}\right)^2 + 12$

Solution: $\left(x^2 + \frac{1}{x^2}\right)^2 - 4\left(x + \frac{1}{x}\right)^2 + 12$

$$= \left[x^2 + \frac{1}{x^2}\right]^2 - 4\left[x^2 + \frac{1}{x^2} + 2\right] + 12$$

$$= \left[x^2 + \frac{1}{x^2}\right]^2 - 4x^2 - \frac{4}{x^2} - 8 + 12$$

$$= \left(x^2 + \frac{1}{x^2}\right)^2 - 4\left(x^2 + \frac{1}{x^2}\right) + 4$$

$$= \left[x^2 + \frac{1}{x^2}\right]^2 -2\left[x^2 + \frac{1}{x^2}\right](2) + (2)^2$$

$$= \left[x^2 + \frac{1}{x^2} -2\right]^2$$

Taking square root

$$= \sqrt{\left[x^2 + \frac{1}{x^2}\right] -4\left[x + \frac{1}{x}\right]2 + 12}$$

$$= \sqrt{\left[x^2 \frac{1}{x^2} -2\right]^2}$$

$$= \pm\left(x^2 + \frac{1}{x^2} -2\right)$$

(viii) $(x^2+3x+2)(x^2+4x+3)(x^2+5x+6)$

Solution: $(x^2+3x+2)(x^2+4x+3)(x^2+5x+6)$

$=[x^2+2x+x+2][x^2+3x+x+3][x^2+3x+2x+6]$

$=[x(x+2)+1(x+2)][x(x+3)+1(x+3)][x(x+3)+2(x+3)]$

$=(x+2)(x+1)(x+3)(x+1)(x+3)(x+2)$

$=(x+2)^2(x+1)^2(x+3)^2$

Taking square root

$=\sqrt{(x^2+3x+2)(x^2+4x+3)(x^2+5x+6)}$

$=\sqrt{(x+2)^2(x+1)^2(x+3)^2}$

$=\pm(x+1)(x+2)(x+3)$ **Ans**

(ix) $(x^2+8x+7)(2x^2-x-3)(2x^2+11x-21)$

Solution: $(x^2+8x+7)(2x^2-x-3)(2x^2+11x-21)$

$=(x^2+7x+1x+7)(2x^2-3x+2x-3)(2x^2+14x-3x-21)$

$=[(x(x+7)+1(x+7)][x(2x-x)+1(2x-3)][(2x(x+7)-3(x+7)]$

$=(x+7)(x+1)(2x-3)(x+1)(x+7)(2x-3)$

$=(x+7)^2(x+1)^2(2x-3)^2$

Taking square root

$=\sqrt{(x^2+8x+7)(2x^2-x-3)(2x^2+11x-21)}$

$=\sqrt{(x+7)^2(x+1)^2(2x-3)^2}$

$=\pm(x+1)(x+7)(2x-3)$ **Ans**

Q.2 Use division method to find the square root of the following expression.

(i) $4x^2+12xy+9y^2+16x+24y+16$

Solution: $4x^2+12xy+9y^2+16x+24y+16$

$2x+3y+4$

$2x \overline{)\, 4x^2+12xy+9y^2+16x+24y+16}$

$\pm 4x^2$

$4x+3y \overline{)\, 12xy+9y^2+16x+24y+16}$

$\pm 12xy \pm 9y^2$

$4x+6y+4 \overline{)\, 16x\ 24y+16}$

$16x \pm 24y \pm 16$

0

Square root $=\pm(2x+3y+4)$

(ii) $x^4 - 10x^3 + 37x^2 - 60x + 36$

Solution: $x^4 - 10x^3 + 37x^2 - 60x + 36$

$$x^2 - 5x + 6$$

$$x^2 \overline{\big)\, x^4 - 10x^3 + 37x^2 - 60x + 36}$$

$$\pm x^4$$

$$2x^2 - 5x \overline{\big)\, -10x^3 + 37x^2 - 60x + 36}$$

$$\pm 10x^3 \pm 25x^2$$

$$2x^2 - 10x + 6 \overline{\big)\, 12x^2 + 60x + 36}$$

$$\pm 12x^2 \pm 60x \pm 36$$

$$\times$$

Square root $= \pm\left(x^2 - 5x + 6\right)$

(iii) $9x^4 - 6x^3 + 7x^2 - 2x + 1$

Solution: $9x^4 - 6x^3 + 7x^2 - 2x + 1$

$$3x^2 - x + 1$$

$$3x^2 \overline{\big)\, 9x^4 - 6x^3 + 7x^2 - 2x + 1}$$

$$\pm 9x^4$$

$$6x^2 - x \overline{\big)\, 6x^3 + 7x^2 - 2x + 1}$$

$$\pm 6x^3 \pm 7x^2$$

$$6x^2 - 2x + 1 \overline{\big)\, 6x^2 - 2x + 1}$$

$$\pm 6x^2 \pm 2x \pm 1$$

$$\times$$

Square root $\pm\left(= 3x^2 - x + 1\right)$

(iv) $4 + 25x^2 + 7x^2 - 2x + 1$

Solution: $4+25x^2-12x-24x^3+16x^4$

$$\begin{array}{l}
\text{Quotient: } 4x^2-3x+2 \\
4x^2 \,\big)\, 16x^4-24x^3+25x^2-12x+4 \\
\qquad \pm 16x^4 \\
\hline
8x^2-3x \,\big)\, -24x^3+25x^2-12x+4 \\
\qquad \pm 24x^3 \pm 9x^2 \\
\hline
8x^2-6x+2 \,\big)\, 16x^2-12x+4 \\
\qquad \pm 16x^2 \pm 12x \pm 4 \\
\hline
\qquad \times
\end{array}$$

Square root $=\pm\left(4x^2-3x+2\right)$

(v) $\dfrac{x^2}{y^2}-\dfrac{10x}{y}+27-\dfrac{10y}{x}+\dfrac{y^2}{x^2},(x\neq 0,y\neq 0)$

Solution: $\dfrac{x^2}{y^2}-\dfrac{10x}{y}+27-\dfrac{10y}{x}+\dfrac{y^2}{x^2},(x\neq 0,y\neq 0)$

$$\begin{array}{l}
\text{Quotient: } \dfrac{x}{y}-5+\dfrac{y}{x} \\
\dfrac{x}{y} \,\Big)\, \dfrac{x^2}{y^2}-\dfrac{10x}{y}+27-\dfrac{10y}{x}+\dfrac{y^2}{x^2}, \\
\qquad \pm\dfrac{x^2}{y^2} \\
\hline
\dfrac{2x}{y}-5 \,\Big)\, -\dfrac{10x}{y}+27-\dfrac{10y}{x}+\dfrac{y^2}{x^2} \\
\qquad \pm\dfrac{x^2}{y^2} \pm 25 \\
\hline
\dfrac{2x}{y}-10+\dfrac{y}{x} \,\Big)\, +2-\dfrac{10y}{c}+\dfrac{y^2}{x^2} \\
\qquad \pm 2 \mp \dfrac{10y}{x} \pm \dfrac{y^2}{x^2} \\
\hline
\qquad \times
\end{array}$$

Square root $=\pm\left(\dfrac{x}{y}-5+\dfrac{y}{x}\right)$

Q.3 Find the value of k for which the following expressions will become a perfect square.

(i) $4x^4 - 12x^3 + 37x^2 - 42x + k$

Solution: $4x^4 - 12x^3 + 37x^2 - 42x + k$

$$2x^2 - 3x + 7$$

$$2x^2 \overline{)\cancel{4x^4} - 12x^3 + 37x^2 - 42x + k}$$

$$\pm \cancel{4x^4}$$

$$4x^2 - 3x \overline{)-\cancel{12x^3} + 37x^2 - 42x + k}$$

$$\pm \cancel{12x^3} \pm 9x^2$$

$$4x^2 - 6x + 7 \overline{)\cancel{28x^2} - \cancel{42x} + k}$$

$$\pm \cancel{28x^2} \pm \cancel{42x} \pm 49$$

$$k - 49$$

In the case of perfect square remainder is always is equal to zero so

$k - 49 = 0$

$k = 49$

(ii) $x^4 - 4x^3 + 10x^2 - kx + 9$

Solution: $x^4 - 4x^3 + 10x^2 - kx + 9$

$$x^2 - 2x + 3$$

$$= x^2 \overline{)\cancel{x^4} - 4x^3 + 10x^2 - kx + 9}$$

$$\pm \cancel{x^4}$$

$$2x^2 - 2x \overline{)-\cancel{4x^3} + 10x^2 - kx + 9}$$

$$\pm \cancel{4x^3} \pm 4x^2$$

$$2x^2 - 4x + 3 \overline{)6x^2 - kx + 9}$$

$$-6x^2 \mp 12x \pm 9$$

$-kx + 12x = 0$

In the case of square root remainder is always equal to zero

$-x(k - 12) = 0$

$k - 12 = \dfrac{0}{-x}$

$k - 12 = 0$

$k = 12$

Q.4 Find the value of l and m for which the following expression will be perfect square

(i) $x^4+4x^3+16x^2+ln+m$

Solution: $x^4+4x^3+16x^2+ln+m$

$$x^2+2x+6$$

$$=x^2\overline{)\;\cancel{x^4}+4x^3+16x^2+lx+m}$$

$$\pm\cancel{x^4}$$

$$2x^2+2x\overline{)\;\cancel{4x^3}+16x^2+lx+m}$$

$$\pm\cancel{4x^3}\pm4x^2$$

$$2x^2+4x+6\overline{)\;\cancel{12x^2}+lx+m}$$

$$\pm\cancel{12x^2}\pm24x-36$$

In the case of square root remainder is always zero

$(lx-24x),\qquad m-36=0$

$x(l-24)=0,\qquad m=36$ **Ans**

$l-24=\dfrac{0}{x}$

$l-24=0$

$l=24$ **Ans**

(ii) $49x^4-70x^3+109x^2+lx-m$

Solution: $49x^4-70x^3+109x^2+lx-m$

$$7x^2-5x+6$$

$$7x^2\overline{)\;\cancel{49x^4}-70x^3+109x^2+lx-m}$$

$$\pm\cancel{49x^4}$$

$$14x^2-5x\overline{)\;-\cancel{70x^3}+109x^2+lx-m}$$

$$\mp\cancel{70x^3}\pm25x^2$$

$$14x^2-10x+6\overline{)\;\cancel{84x^2}+lx-m}$$

$$\pm\cancel{84x^2}\mp60x\pm36$$

$$lx+60x-m-36$$

$$(l+60)x-m-36$$

In the case of square root remainder is always equal to zero

$-m-36=0$

$-m=36$

$l+60=0\qquad m=-36$

$l=-60$ **Ans**

Q.5 **To make the expression** $9x^4 - 12x^3 + 22x^2 - 13x + 12$ **a perfect square**

Solution: $9x^4 - 12x^3 + 22x^2 - 13x + 12$

$$3x^2 - 2x + 3$$

$$= 3x^2 \,\big)\, \cancel{9x^4} - 12x^3 - + 22x^2 - 13x + 12$$

$$\pm \cancel{9x^4}$$

$$6x^2 - 2x \,\big)\, -\cancel{12x^3} + 22x^2 - 13x + 12$$

$$\pm \cancel{12x^3} \pm 4x^2$$

$$6x^2 - 4x + 3 \,\big)\, \cancel{18x^2} - 13x + 12$$

$$\pm \cancel{18x^2} \mp 12x \pm 9$$

$$-x + 3$$

(i) $+x - 3$ **is to be added**

(ii) $-x + 3$ **is to be subtract from it**

(iii) $-x + 3 = 0$

$x = 3$

Review Exercise 6

Q.1 Choose the correct answer.

(i) **H.C.F of** $p^3q - pq^3$ **and** $p^5q^2 - pq^5$ **is** ________

(a) $pq(p^2 - q^2)$ **(b)** $pq(p-q)$

(c) $p^2q^2(p-q)$ **(d)** $pq(p^3 - q^3)$

(ii) **H.C.F of** $5x^2y^2$ *and* $20x^3y^3$ is ________

(a) $5x^2y^2$ **(b)** $20x^3y^3$

(c) $100x^5y^5$ **(d)** $5xy$

(iii) **H.C.F of** $x-2$ **and** x^2+x-6 ______

(a) x^2+x-6 **(b)** $x+3$

(c) $x-2$ **(d)** $x+2$

(iv) **H.C.F of** a^3+b^3 **and** a^2-ab+b^2 ______

(a) $a+b$ **(b)** a^2-ab+b^2

(c) $(a-b)^2$ **(d)** a^2+b^2

(v) **H.C.F of** x^2-5x+6 **and** x^2-x-6 is______

(a) $x-3$ **(b)** $x+2$

(c) x^2-4 **(d)** $x-2$

(vi) **H.C.F of** a^2-b^2 **and** a^3-b^3 is______

(a) $a-b$ **(b)** $a+b$

(c) a^2+ab+b^2 **(d)** a^2-ab+b^2

(vii) **H.C.F of** x^2+3x+2, x^2+4x+3 **and** x^2+5x+4 is______

(a) $x+1$ **(b)** $(x+1)(x+2)$

(c) $x+3$ **(d)** $(x+4)(x+1)$

(viii) **L.C.M of** $15x^2, 45xy$ **and** $30xyz$ is______

(a) $90xyz$ **(b)** $90x^2yz$

(c) $15xyz$ **(d)** $15x^2yz$

(ix) **L.C.M of** a^2+b^2 **and** a^4-b^4 is______

(a) a^2+b^2 **(b)** a^2-b^2

(c) a^4-b^4 **(d)** $a-b$

(x) **The product of two algebraic expression is equal to the ________ of their H.C.F and L.C.M**

(a) Sum **(b)** Difference

(c) Product **(d)** Quotitent

(xi) **Simplify** $\frac{a}{9a^2-b^2}+\frac{1}{3a-b}$ is_______

(a) $\frac{4a}{9a^2-b^2}$ (b) $\frac{4a-b}{9a^2-b^2}$

(c) $\frac{4a+b}{9a^2-b^2}$ (d) $\frac{b}{9a^2-b^2}$

(xii) **Simplify** $\frac{a^2+5a-14}{a^2-3a-18} \times \frac{a+3}{a-2} =$________

(a) $\frac{a+7}{a-6}$ (b) $\frac{a+7}{a-2}$

(c) $\frac{a+3}{a-6}$ (d) $\frac{a-2}{a+3}$

(xiii) **Simplify the** $\frac{a^3-b^3}{a^4-b^4} \div \frac{a^2+ab+b^2}{a^2+b^2} =$________

(a) $\frac{1}{a+b}$ (b) $\frac{1}{a-b}$

(c) $\frac{a-b}{a^2+b^2}$ (d) $\frac{a+b}{a^2+b^2}$

(xiv) **Simplify** $\left(\frac{2x+y}{x+y}-1\right) \div \left(1-\frac{x}{x+y}\right) =$__________

(a) $\frac{x}{x+y}$ (b) $\frac{y}{x+y}$

(c) $\frac{y}{x}$ (d) $\frac{x}{y}$

(xv) **The square root of** a^2-2a+1 is________

(a) $\pm(a+1)$ (b) $\pm(a-1)$

(c) $a-1$ (d) $a+1$

(xvi) **What should be added to complete the square of** x^4+64 ?______

(a) $8x^2$ (b) $-8x^2$

(c) $16x^2$ (d) $4x^2$

(xvii) **The square root to** $x^4+\frac{1}{x^4}+2$ **is**________

(a) $\pm\left(x+\frac{1}{x}\right)$ (b) $\pm\left(x^2+\frac{1}{x^2}\right)$

(c) $\pm\left(x-\frac{1}{x}\right)$ (d) $\pm\left(x^2-\frac{1}{x^2}\right)$

ANSWER KEYS

1	b	5	a	9	c	13	a	17	b
2	a	6	a	10	c	14	d		
3	c	7	a	11	c	15	b		
4	b	8	b	12	a	16	c		

Q.2 Find the H.C.F of the following by factorization.

$8x^4 - 128, 12x^3 - 96$

Solution:

$8x^4 - 128 \quad = 8(x^4 - 16) \quad = 8\left[(x)^2 - (4)^2\right]$

$= 2 \times 2 \times 2(x^2 + 4)(x^2 - 4)$

$= 2 \times 2 \times 2(x^2 + 4)(x + 2)(x - 2)$

$12x^3 - 96 \quad = 12(x^3 - 8)$

$= (12(x^3 - 2^3)$

$= 12(x - 2)(x^2 + 2x + 4)$

$2 \times 2 \times 3(x - 2)(x^2 + 2x + 4)$

H.C.F = $2 \times 2(x - 2)$

$= 4(x - 2)$

Q.3 Find the H.C.F of the following by division method $y^3 + 3y^2 - 3y - 9, 3y^2 - 8y - 2y$**.**

Solution: $y^3 + 3y^2 - 3y - 9,$

$= y^3 + 3y^2 - 3y - 9$

$$y^3 + 3y^2 - 8y - 24 \overline{\big)\, y^3 + 3y^2 - 3y - 9} \quad \text{quotient } 1$$

$$\pm y^3 \pm 3y^2 \pm 8y \pm 24$$

$$5y + 15$$

$5(y + 3)$

$$y + 3 \overline{\big)\, y3 + 3y - 8y - 24} \quad \text{quotient } y^2 - 8$$

$$\pm y3 \pm 3y$$

$$-8y - 24$$

$$\pm 8y \pm 24$$

$$\times$$

H.C.F $= (y + 3)$

Q.4 Find the L.C.M of the following by factorization.

$12x^2-75,\ 6x^2-13x-5,\ 4x^2-20x+25$

Solution:

$12x^2-75 \quad = 3(4x^2-25)$

$= 3\left[(2x)^2-(5)^2\right]$

$= 3(2x-5)(2x+5)$

$6x^2-15x+2x-5 \quad = 3x(2x-5)+1(2x-5)$

$= (2x-5)(3x+1)$

$4x^2-20x+25 = 4x^2-10x-10x+25$

$= 2x(2x-5)-5(2x-5)$

$= (2x-5)(2x-5)$

Common factor = (2*x*-5)

Non common factor $=3(3x+1)(2x-5)2x+5)$

L.C.M = common factor× non common factor

L.C.M = $(2x-5)3(3x+1)(2x+5)(2x-5)$

L.C.M $= 3(2x+5)(2x-5)^2(3x+1)$

Q.5 If H.C.F of $x^4+3x^3+5x^2+26x+56$ and $x^4+2x^3-4x^2-x+28$ is x^2+5x+7 find their L.C.M.

Solution: $p(x)=x^4+3x^3+5x^2+26x+56$ and $q(x)=x^4+2x^3-4x^2-x+28$

$HCF = x^2+5x+7$, LCM=?

$$L.C.M = \frac{P(x)\times q(n)}{H.C.F}$$

$$\text{L.C.M} = \frac{(x^4+3x^2+5x^2+26x+56)\times(x^4+2x^3-4x^2-x+28)}{(x^2+5x+7)}$$

$$\begin{array}{r|l} & x^2-3x+4 \\ x^2+5x+7 & x^4+2x^3-4x^2-x+28 \\ & \pm x^4 \pm 5x^3 \pm 7x^2 \\ \hline & -3x^3-11x^2-x+28 \\ & \mp 3x^3 \mp 15x^2 \mp 21x \\ \hline & +4x^2+20x+28 \\ & \pm 4x^2 \pm 20x \pm 28 \\ \hline & 0 \end{array}$$

$$x^2-3x+4$$

$$L.C.M=\frac{(x^4+3x^3+5x^2+26x+56)\,(x^4+2x^3-4x^2-x+28)}{(x2+5x+7)}$$

$$L.C.M=(x^4+3x^3+5x^2+26x+56)(x^2-3x+4)$$

Q.6 Simplify:

Solution:

(i) $\dfrac{3}{x^3+x^2+x+1}-\dfrac{3}{x^3-x^2+x-1}$

Solution: $\dfrac{3}{x^3+x^2+x+1}-\dfrac{3}{x^3-x^2+x-1}$

$$=\frac{3}{x^2(x+1)+1(x+1)}-\frac{3}{x^2(x-1)+1(x-1)}$$

$$=\frac{3}{(x^2+1)(x+1)}-\frac{3}{(x-1)(x^2+1)}$$

$$=\frac{3(x-1)-3(x+1)}{(x+1)(x-1)(x^2+1)}$$

$$=\frac{3x-3-3x-3}{(x-1)(x-1)(x^2+1)}$$

$$=\frac{-6}{(x+1)(x-1)(x^2+1)}=\frac{-6}{(x^2-1)(x^2+1)}$$

$$=\frac{-6}{(x^4-1)}$$

$$=\frac{6}{1-x^4}$$

(ii) $\dfrac{a+b}{a^2-b^2}\div\dfrac{a^2-ab}{a^2-2ab+b^2}$

Solution: $\dfrac{a+b}{a^2-b^2}\div\dfrac{a^2-ab}{a^2-2ab+b^2}$

$$=\frac{a+b}{a^2-b^2}\times\frac{a^2-2ab+b^2}{a^2-ab}$$

$$=\frac{a+b}{(a-b)(a+b)}\times\frac{(a-b)^2}{a(a-b)}$$

$$=\frac{(\cancel{a-b})^2}{a(\cancel{a-b})^2}$$

$$=\frac{1}{a}$$

Q.7 Find the square root by using factorization. $\left(x^2+\frac{1}{x^2}\right)+10\left(x+\frac{1}{x}\right)+27 \quad (x\neq 0)$.

Solution: $(x^2+\frac{1}{x^2}+10\left(x+\frac{1}{x}\right)+27$

$$=(x^2+\frac{1}{x^2}+10\left(x+\frac{1}{x}\right)+25+2$$

$$=x^2+\frac{1}{x^2}+2+10\left(x+\frac{1}{x}\right)+25$$

$$=\left(x+\frac{1}{x}\right)^2+2\left(x+\frac{1}{x}\right)\times 5+(5)^2$$

$$=\left[x+\frac{1}{x}+5\right]^2$$

Taking the square root

$$\sqrt{\left(x^2+\frac{1}{x^2}\right)+10\left(x+\frac{1}{x}\right)+27}=\sqrt{\left[x+\frac{1}{x}+5\right]^2}$$

$$=\pm\left(x+\frac{1}{x}+5\right)$$

Q.8 Find the square roots by using division method. $\frac{4x^2}{y^2}+\frac{20x}{y}+13-\frac{30y}{x}+\frac{9y^2}{x}$

Solution:

$$\frac{2x}{y}+5-\frac{3y}{x}$$

$$\frac{2x}{y}\Big)\ \frac{4x^2}{y^2}+\frac{20x}{4}+13-\frac{30y}{x}+\frac{9y^2}{x^2}$$

$$\pm\frac{4x^2}{y^2}$$

$$\frac{4x}{y}+5\Big)\ \frac{20x}{y}+13-\frac{30y}{x}+\frac{9y^2}{x^2}$$

$$\pm\frac{20x}{y}\pm 25$$

$$\frac{4x}{y}+10-\frac{3y}{x} \overline{\left) -12-\frac{30y}{x}+\frac{9y^2}{x^2} \right.}$$

$$\mp 12 \mp \frac{30y}{x} \pm \frac{9y^2}{x^2}$$

Square root $= \pm\left[\frac{2x}{y}+5-\frac{3y}{x}\right]$

Unit 6: Algebraic Manipulation

Overview

Highest Common Factor:

If two or more algebraic expressions are given then their common factor of highest power is called the H.C.F. of the expression.

Least Common Multiple(L.C.M):

The Least common Multiple (L.C.M) is the product of common factors together with non-common factors of the given expressions.

Exercise 7.1

Q.1 Solve the following equations

(i) $\frac{2}{3}x-\frac{1}{2}x=x+\frac{1}{6}$

Solution: $\frac{2}{3}x-\frac{1}{2}x=x+\frac{1}{6}$

$\frac{4x-3x}{6}=\frac{6x+1}{6}$

$x=6\frac{(6x+1)}{6}$

$x=6x+1$

$-6x+x=1$

$-5x=1$

$x=\frac{1}{-5}$

$x=-\frac{1}{5}$

To check

Substitution $x=-\frac{1}{5}$

$\frac{2}{3}\times\frac{-1}{5}-\frac{1}{2}\times\frac{-1}{5}=\frac{-1}{5}+\frac{1}{6}$

$\frac{-2}{15}+\frac{1}{10}=\frac{-6+5}{30}$

$\frac{-2\times2+1\times3}{30}=\frac{-1}{30}$

$\frac{-4+3}{30}=\frac{-1}{30}$

$\frac{-1}{30}=\frac{-1}{30}$

Solution Set $=\left\{-\frac{1}{5}\right\}$

(ii) $\frac{x-3}{3}-\frac{x-2}{2}=-1$

Solution $\frac{x-3}{3}-\frac{x-2}{2}=-1$

By taking L.C.M

$\frac{2(x-3)-3(x-2)}{6}=-1$

$2x-6-3x+6=-6$

$-x=-6$

Multiplying both sets by –1

$-1\times-x=-1\times-6$

$x=6$

To check

$\frac{x-3}{3}-\frac{x-2}{2}=-1$

When $x=6$

$\frac{6-3}{3}-\frac{6-2}{2}=-1$

$\frac{3}{3}-\frac{4}{2}=-1$

$\frac{6-12}{6}=-1$

$\frac{\cancel{-6}}{\cancel{6}}=-1$

$-1=-1$

Solution Set $=\{6\}$

(iii) $\frac{1}{2}\left(x-\frac{1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1}{2}-3x\right)$

Solution $\frac{1}{2}\left(x-\frac{1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1}{2}-3x\right)$

Taking L.C.M of brackets

$\frac{1}{2}\left(\frac{6x-1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1-6x}{2}\right)$

$\frac{6x-1}{12}+\frac{2}{3}=\frac{5}{6}+\frac{1-6x}{6}$

$\frac{6x-1+8}{12}=\frac{5+1-6x}{6}$

$\frac{\cancel{6}(6x+7)}{\cancel{12}^{2}}=6-6x$

$\frac{6x+7}{2}=6-6x$

$6x+7=2(6-6x)$

$6x+7=12-12x$

$6x+12x=12-7$

$18x=5$

$x=\frac{5}{18}$

To check

$$\frac{1}{2}\left(x-\frac{1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1}{2}-3x\right)$$

When $x=\frac{5}{18}$

$$\frac{1}{2}\left[\frac{15}{18}-\frac{1}{6}\right]+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left[\frac{1}{2}-3\left(\frac{5}{18}\right)\right]$$

$$\frac{1}{2}\left[\frac{5-3}{18}\right]+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left[\frac{1}{2}-\frac{5}{6}\right]$$

$$\frac{1}{2}\left[\frac{\cancel{2}}{18}\right]+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left[\frac{3-5}{6}\right]$$

$$\frac{1}{\cancel{2}}\left[\frac{\cancel{2}}{18}\right]+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left[\frac{-\cancel{2}^{1}}{\cancel{6}^{3}}\right]$$

$$\frac{1}{18}+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left[\frac{-1}{3}\right]$$

$$\frac{1+12}{18}=\frac{5}{6}-\frac{1}{9}$$

$$\frac{13}{18}=\frac{15-2}{18}$$

$$\frac{13}{18}=\frac{13}{18}$$

Solution Set $=\left\{\frac{5}{18}\right\}$

(iv) $x+\frac{1}{3}=2\left[x-\frac{2}{3}\right]-6x$

Solution $x+\frac{1}{3}=2\left[x-\frac{2}{3}\right]-6x$

$$\frac{3x+1}{3}=2\left[\frac{3x-2}{3}\right]-6x$$

$$\frac{3x+1}{3}=\frac{6x-4}{3}-6x$$

Taking L.C.M of right side

$$\frac{3x+1}{3}=\frac{6x-4-18x}{3}$$

$$\frac{3x+1}{\cancel{3}}=\frac{(-12x-4)}{\cancel{3}}$$

$$3x+1=-12x-4$$
$$3x+12x=-4-1$$
$$15x=-5$$
$$x=\frac{-5}{15}$$
$$x=\frac{-1}{3}$$

To check

$$x+\frac{1}{3}=2\left[x-\frac{2}{3}\right]-6x$$

When $x=\frac{-1}{3}$

$$\frac{-\cancel{1}}{\cancel{3}}+\frac{\cancel{1}}{\cancel{3}}=2\left[\frac{-1}{3}-\frac{2}{3}\right]-6\left(\frac{-1}{3}\right)$$

$$0=2\left[\frac{-1-2}{3}\right]+\frac{\cancel{6}^{2}}{\cancel{3}}$$

$$0=2\left[\frac{-\cancel{3}}{\cancel{3}}\right]+2$$

$$0=2(-1)+2$$
$$0=-\cancel{2}+\cancel{2}$$
$$0=0$$

Solution Set $=\left\{\frac{-1}{3}\right\}$

(v) $\frac{5(x-3)}{6}-x=1-\frac{x}{9}$

Solution $\frac{5(x-3)}{6}-x=1-\frac{x}{9}$

$$\frac{5x-15-6x}{6}=\frac{9-x}{9}$$

$$\frac{-15-x}{6}=\frac{9-x}{9}$$

$$9(-15-x)=6(9-x)$$
$$-135-9x=54-6x$$
$$-135-54=-6x+9x$$
$$-189=3x$$
$$\frac{-189}{3}=x$$
$$x=-63$$

To check

$$\frac{5(x-3)}{6}-x=1-\frac{x}{9}$$

When $x=-63$

$$\frac{5(-63-3)}{6}-(-63)=1-\frac{(-\cancel{63}^{7})}{\cancel{9}}$$

$$\frac{5(-\cancel{66}^{11})}{\cancel{6}}+63=1+7$$

$-55+63=8$
$8=8$
Solution Set $=\{-63\}$

(vi) $\frac{x}{3x-6}=2-\frac{2x}{x-2}, x\neq 2$

Solution $\frac{x}{3x-6}=2-\frac{2x}{x-2}, x\neq 2$

$\frac{x}{3(x-2)}=\frac{2(x-2)-2x}{x-2}$

$\frac{x}{3(x-2)}=\frac{2x-4-2x}{x-2}$

$\frac{x}{3(x-2)}=\frac{-4}{x-2}$

$x(x-2)=-4\times 3(x-2)$

$x(x-2)=-12(x-2)$

$x(x-2)+12(x-2)=0$

$(x-2)(x+12)=0$

$x-2=0$, or $x+12=0$

$x=2$, or $x=-12$

$x=2$ (Rejected because $x\neq 2$)

Hence $x=-12$

To check

$\frac{x}{3x-6}=2-\frac{2x}{x-2}$

When $x=-12$

$\frac{-12}{3(-12)-6}=2-\frac{2(-12)}{-12-2}$

$\frac{-12}{-36-6}=2+\frac{24}{-14}$

$\frac{-12}{-42}=2-\frac{24}{14}$

$\frac{12}{42}=2-\frac{12}{7}$

$\frac{2}{7}=\frac{14-12}{7}$

$\frac{2}{7}=\frac{2}{7}$

Solution Set $=\{-12\}$

(vii) $\frac{2x}{2x+5}=\frac{2}{3}-\frac{5}{4x+10}$

Solution $\frac{2x}{2x+5}=\frac{2}{3}-\frac{5}{4x+10}$

$\frac{2x}{2x+5}=\frac{2(4x+10)-3\times 5}{3(4x+10)}$

$\frac{2x\times 3(4x+10)}{2x+5}=8x+20-15$

$\frac{6x\times 2\cancel{(2x+5)}}{\cancel{(2x+5)}}=8x+5$

$12x=8x+5$

$12x-8x=5$

$4x=5$

$x=\frac{5}{4}$

To check

$\frac{2x}{2x+5}=\frac{2}{3}-\frac{5}{4x+10}$

When $x=\frac{5}{4}$

$\frac{2\left(\frac{5}{4}\right)}{2\left(\frac{5}{4}\right)+5}=\frac{2}{3}-\frac{5}{4\left(\frac{5}{4}\right)+10}$

$\frac{\frac{5}{2}}{\frac{5}{2}+5}=\frac{2}{3}-\frac{5}{5+10}$

$\frac{\frac{5}{2}}{\frac{5+10}{2}}=\frac{2}{3}-\frac{5}{15}$

$\frac{\frac{5}{2}}{\frac{15}{2}}=\frac{2}{3}-\frac{1}{3}$

$\frac{\cancel{5}}{\cancel{2}}\times\frac{\cancel{2}}{\cancel{15}^{3}}=\frac{2-1}{3}$

$\frac{1}{3}=\frac{1}{3}$

Solution Set $=\left\{\frac{5}{4}\right\}$

(viii) $\frac{2x}{x-1}+\frac{1}{3}=\frac{5}{6}+\frac{2}{x-1} \quad x\neq 1$

Solution $\frac{2x}{x-1}+\frac{1}{3}=\frac{5}{6}+\frac{2}{x-1} \quad x\neq 1$

$$\frac{3\times 2x+1(x-1)}{3(x-1)}=\frac{5(x-1)+2\times 6}{6(x-1)}$$

$$\frac{6x+x-1}{3(x-1)}=\frac{5x-5+12}{6(x-1)}$$

$$\frac{7x-1}{3(x-1)}=\frac{5x-5+12}{6(x-1)}$$

$$7x-1=\frac{5\cancel{(x-1)}(5x+7)}{\cancel{(x-1)}}$$

$$2(7x-1)=5x+7$$

$$14x-2=5x+7$$

$$14x-5x=4+2$$

$$9x=9$$

$$x=\frac{9}{9}$$

$$x=1$$

No solution because $x\neq 1$.

(ix) $\frac{2}{x^2-1}-\frac{1}{x+1}=\frac{1}{x+1} \quad x\neq \pm 1$

Solution $\frac{2}{x^2-1}-\frac{1}{x+1}=\frac{1}{x+1} \quad x\neq \pm 1$

$$\frac{2}{(x-1)(x+1)}-\frac{1}{x+1}=\frac{1}{x+1}$$

$$\frac{2-(x-1)}{(x-1)(x+1)}=\frac{1}{x+1}$$

$$\frac{2-(x-1)}{(x-1)(x+1)}=\frac{1}{x+1}$$

$$2-x+1=\frac{(x-1)(x+1)}{(x+1)}$$

$$3-x=x-1$$

$$1+3=x+x$$

$$4=2x$$

$$\frac{4}{2}=x$$

$$x=2$$

To check

$$\frac{2}{2^2-1}-\frac{1}{2+1}=\frac{1}{2+1}$$

$$\frac{2}{4-1}-\frac{1}{3}=\frac{1}{3}$$

$$\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$$

$$\frac{2-1}{3}=\frac{1}{3}$$

$$\frac{1}{3}=\frac{1}{3}$$

Solution Set $=\{2\}$

(x) $\frac{2}{3x+6}=\frac{1}{6}-\frac{1}{2x+4}$

Solution $\frac{2}{3x+6}=\frac{1}{6}-\frac{1}{2x+4}$

$$\frac{2}{3(x+2)}=\frac{1}{6}-\frac{1}{2(x+2)}$$

$$\frac{2}{3(x+2)}=\frac{x+2-3}{6(x+2)}$$

$$\frac{2\times 6(x+2)}{3(x+2)}=x-1$$

$4=x-1$

$4+1=x$

$x=5$

Check

$$\frac{2}{3(5)+6}=\frac{1}{6}-\frac{1}{2(5)+4}$$

$$\frac{2}{15+6}=\frac{1}{6}-\frac{1}{10+4}$$

$$\frac{2}{21}=\frac{1}{6}-\frac{1}{14}$$

$$\frac{2}{21}=\frac{7-3}{42}$$

$$\frac{2}{21}=\frac{\cancel{4}^{2}}{\cancel{42}^{21}}$$

$$\frac{2}{21}=\frac{2}{21}$$

Solution Set $=\{5\}$

Q.2 Check each equation and check for extraneous solution, if any

(i) $\sqrt{3x+4}=2$

Solution: $\sqrt{3x+4}=2$

Taking square on both side

$\left(\sqrt{3x+4}\right)^2=(2)^2$

$3x+4=4$

$3x=4-4$

$3x=0$

$x=\dfrac{0}{3}$

$x=0$

To check

$\sqrt{3x+4}=2$

When $x=0$

$\sqrt{3(0)+4}=2$

$\sqrt{4}=2$

$2=2$

L.H.S = R.H.S

Solution Set $=\{0\}$

(ii) $\sqrt[3]{2x-4}-2=0$

Solution: $\sqrt[3]{2x-4}-2=0$

$\sqrt[3]{2x-4}=2$

Taking cube on both sides

$\left(\sqrt[3]{2x-4}\right)^3=(2)^3$

$2x-4=8$

$2x=8+4$

$2x=12$

$x=\dfrac{12}{2}$

$x=6$

To check

$\sqrt[3]{2x-4}-2=0$

When $x=6$

$\sqrt[3]{2x-4}-2=0$

$\sqrt[3]{2(6)-4}-2=0$

$\sqrt{12-4}-2=0$

$\sqrt[3]{8}-2=0$

$\sqrt[3]{2^3}-2=0$

$2-2=0$

$0=0$

L.H.S = R.H.S

Solution Set $=\{6\}$

(iii) $\sqrt{x-3}-7=0$

Solution: $\sqrt{x-3}-7=0$

$\sqrt{x-3}=7$

Taking square on both side

$\left(\sqrt{x-3}\right)^2=(7)^2$

$x-3=49$

$x=49+3$

$x=52$

To check

$\sqrt{x-3}-7=0$

When $x=52$

$\sqrt{52-3}-7=0$

$\sqrt{49}-7=0$

$7-7=0$

$0=0$

L.H.S = R.H.S

Solution Set $=\{52\}$

(iv) $2\sqrt{t+4}=5$

Solution: $2\sqrt{t+4}=5$

Taking square on both side

$\left(2\sqrt{t+4}\right)^2=(5)^2$

$4(t+4)=25$

$t+4=\dfrac{25}{4}$

$t=\dfrac{25}{4}-4$

$t=\dfrac{25-16}{4}$

$t=\dfrac{9}{4}$

To check

$2\sqrt{t+4}=5$

When $t=\dfrac{9}{4}$

$2\sqrt{\dfrac{9}{4}+4}=5$

$2\sqrt{\dfrac{9+16}{4}}=5$

$2\sqrt{\dfrac{25}{4}}=5$

$2\times\dfrac{5}{2}=5$

$5=5$
L.H.S = R.H.S
Solution Set $=\left\{\frac{9}{4}\right\}$

(v) $\sqrt[3]{2x+3}=\sqrt[3]{x-2}$
Solution: $\sqrt[3]{2x+3}=\sqrt[3]{x-2}$
Taking cube on both sides
$\left(\sqrt[3]{2x+3}\right)^3=\left(\sqrt[3]{x-2}\right)^3$
$2x+3=x-2$
$2x-x=-2-3$
$x=-5$
To check
$\sqrt[3]{2x+3}=\sqrt[3]{x-2}$
When $x=-5$
$\sqrt[3]{2(-5)+3}=\sqrt[3]{-5-2}$
$\sqrt[3]{-10+3}=\sqrt[3]{-7}$
$\sqrt[3]{-7}=\sqrt[3]{-7}$
L.H.S = R.H.S
Solution Set $=\{-5\}$

(vi) $\sqrt[3]{2-t}=\sqrt[3]{2t-28}$
Solution: $\sqrt[3]{2-t}=\sqrt[3]{2t-28}$
Taking cube on both sides
$\left(\sqrt[3]{2-t}\right)^3=\left(\sqrt[3]{2t-28}\right)^3$
$2-t=2t-28$
$2+28=2t+t$
$30=3t$
$\frac{30}{3}=t$
$t=10$
To check
$\sqrt[3]{2-t}=\sqrt[3]{2t-28}$
When $t=10$
$\sqrt[3]{2-10}=\sqrt[3]{2(10)-28}$
$\sqrt[3]{-8}=\sqrt[3]{20-28}$
$\sqrt[3]{-8}=\sqrt[3]{-8}$
L.H.S = R.H.S
Solution Set $=\{10\}$

(vii) $\sqrt{2t+6}-\sqrt{2t-5}=0$
Solution: $\sqrt{2t+6}-\sqrt{2t-5}=0$
$\sqrt{2t+6}=\sqrt{2t-5}$
Taking square on both side
$\left(\sqrt{2t+6}\right)^2=\left(\sqrt{2t-5}\right)^2$
$2t+6=2t-5$
$2t-2t=-5-6$
$0=-11$
Solution is not possible
Solution Set $=\{\}$ or ϕ

(viii) $\sqrt{\frac{x+1}{2x+5}}=2 \quad x\neq\frac{-5}{2}$
Solution: $\sqrt{\frac{x+1}{2x+5}}=2 \quad x\neq\frac{-5}{2}$
Taking square on both side
$\left(\sqrt{\frac{x+1}{2x+5}}\right)^2=(2)^2$
$\frac{x+1}{2x+5}=4$
$x+1=4(2x+5)$
$x+1=8x+20$
$1-20=8x-x$
$-19=7x$
$-\frac{19}{7}=x$
Or, $x=\frac{-19}{7}$
To check
$\sqrt{\frac{x+1}{2x+5}}=2$
When $x=\frac{-19}{7}$
$\sqrt{\left(\frac{-19}{7}+1\right)\div\left[2\times\frac{-19}{7}+5\right]}=2$
$\sqrt{\frac{-19+7}{7}\div\left[\frac{-38}{7}+5\right]}=2$
$\sqrt{\frac{-12}{7}\div\left[\frac{-38+35}{7}\right]}=2$
$\sqrt{\frac{-12}{7}\div\frac{-3}{7}}=2$

$$\sqrt{\frac{-\cancel{12}^{4}}{\cancel{7}}\times\frac{\cancel{7}}{-\cancel{3}}}=2$$

$$\sqrt{4}=2$$

$$2=2$$

L.H.S = R.H.S

Solution Set $=\left\{\frac{-19}{7}\right\}$

Exercise 7.2

Q1) Identify the following statements as true or

(i)	$\lvert x\rvert = 0$ has only one solution	True
(ii)	All absolute value equations have two solutions	False
(iii)	The equation $\lvert x\rvert = 2$ is equivalent to $x = 2$ or $x = -2$	True
(iv)	The equation $\lvert x-4\rvert = -4$ has no solution	True
(v)	The equation $\lvert 2x-3\rvert = 5$ is equivalent to $2x-3=5$ or $2x+3=5$	False

Q2)

(i) $|3x-5|=4$

Solution $|3x-5|=4$

$3x-5 = \pm 4$

$3x-5=4$

$3x=4+5$

$3x=9$

$x=\frac{9}{3}$

$x=3$

$3x-5=-4$

$3x=-4+5$

$3x=1$

$x=\frac{1}{3}$

To check

$x=3$

$|3(3)-5|=4$

$|9-5|=4$

$4=4$

To check

$x=\frac{1}{3}$

$\left|3\times\frac{1}{3}-5\right|=4$

$|1-5|=4$

$|-4|=4$

$4=4$

Solution Set $=\left\{3,\frac{1}{3}\right\}$

(ii) $\frac{1}{2}|3x+2|-4=11$

Solution $\frac{1}{2}|3x+2|-4=11$

$\frac{1}{2}|3x+2|-4=11$

$\frac{1}{2}|3x+2|=11+4$

$\frac{1}{2}|3x+2|=15$

$|3x+2|=2\times 15$

$|3x+2|=30$

$3x+2=\pm 30$

$3x+2=30$

$3x=30-2$

$3x=28$

$x=\frac{28}{3}$

$3x+2=-30$

$3x=-30-2$

$3x=-32$

$x=\frac{-32}{3}$

Check

$\frac{1}{2}|3x+2|-4=11$

$\frac{1}{2}\left|3\times\frac{28}{3}+2\right|-4=11$

$\frac{1}{2}|28+2|-4=11$

$\frac{1}{2}\times 30-4=11$

$15-4=11$

$11=11$

$\frac{1}{2}\left|3\times\frac{32}{3}+2\right|-4=11$

$\frac{1}{2}|-32+2|-4=11$

$\frac{1}{2}|-30|-4=11$

$\frac{1}{2}(30)-4=11$

$15-4=11$

$11=11$

Solution Set $=\left\{\frac{28}{3},\frac{-32}{3}\right\}$

(iii) $|2x+5|=11$

Solution $|2x+5|=11$

$2x+5=\pm 11$

$2x+5=11$

$2x=11-5$

$2x=6$

$x=\frac{6}{2}$

$x=3$

$2x+5=-11$

$2x=-11-5$

$2x=-16$

$x=\frac{-16}{2}$

$x=-8$

To check

$|2x+5|=11 \qquad |2(-8)-8+5|=11$

$|2\times 3+5|=11 \qquad |-16+5|=11$

$6+5 \;=11 \qquad |-11|=11$

$11=11 \qquad 11=11$

Solution Set $=\{-8,3\}$

(iv) $|3+2x|=|6x-7|$

Solution $|3+2x|=|6x-7|$

$3+2x=\pm(6x-7)$

$3+2x=6x-7 \qquad 3+2x=-(6x\;-7)$

$3+7=6x-7 \qquad 3+2x=-6x+7$

$10=4x \qquad 2x+6x=7-3$

$\frac{10}{4}=x \qquad \frac{4}{8}=x$

$x=\frac{5}{2} \qquad x=\frac{1}{2}$

To check

$|3+2x|=|6x-7| \qquad |3+2x|=|6x-7|$

$\left|3+2\left(\frac{5}{2}\right)\right|=\left|\cancel{6}^{3}\left(\frac{5}{\cancel{2}}\right)-7\right|$

$\left|3+2\times\frac{1}{2}\right|=\left|\cancel{6}^{3}\times\frac{1}{\cancel{2}}-7\right|$

$|3+5|=|15-7| \qquad |3+1|=|3-7|$

$|8|=|8| \qquad |4|=|-4|$

$8=8 \qquad 4=4$

Solution Set $=\left\{\frac{5}{2},\frac{1}{2}\right\}$

(v) $|x+2|-3=5-|x+2|$

Solution $|x+2|-3=5-|x+2|$

$|x+2|+|x+2|=5+3$

$2|x+2|=8$

$|x+2|=\frac{8}{2}$

$|x+2|=4$

$x+2=\pm 4$

$x+2=4 \qquad x+2=-4$

$x=4-2 \qquad x=-4-2$

$x=2 \qquad x=-6$

To check

$|x+2|-3=5-|x+2| \qquad |x+2|-3=5-|x+2|$

$|2+2|-3=5-|2+2| \qquad |-6+2|-3=5-|-6+2|$

$14-3=5-|4| \qquad |-4|-3=5-|-4|$

$4-3=5-4 \qquad 4\text{-}3=5\text{-}4$

$1=1 \qquad 1=1$

Solution Set $=\{-6,2\}$

(vi) $\frac{1}{2}|x+3|+21=9$

Solution $\frac{1}{2}|x+3|+21=9$

$\frac{1}{2}|x+3|=9-21$

$\frac{1}{2}|x+3|=-12$

$|x+3|=-12\times 2$

$|x+3|=-24$

Value of absolute in never negative so solution is not possible

Solution Set $=\{\;\}$

(vii) $\left|\frac{3-5x}{4}\right|-\frac{1}{3}=\frac{2}{3}$

Solution $\left|\frac{3-5x}{4}\right|-\frac{1}{3}=\frac{2}{3}$

$\left|\frac{3-5x}{4}\right|=\frac{2}{3}+\frac{1}{3}$

$\left|\frac{3-5x}{4}\right|=\frac{2+1}{3}$

$\left|\frac{3-5x}{4}\right|=\frac{3}{3}$

$\left|\frac{3-5x}{4}\right|=1$

$\frac{3-5x}{4}=\pm 1$

$\frac{3-5x}{4}=1$ and $\frac{3-5x}{4}=-1$

$3-5x=4 \qquad 3-5x=-4$

$-5x=4-3 \qquad -5x=-4-3$

$-5x=1 \qquad -5x=-7$

$x = \frac{1}{-5}$ $\quad$ $x = \frac{-7}{-5}$

$x = -\frac{1}{5}$ $\quad$ $x = \frac{7}{5}$

$\left|\frac{3-5\times\left(-\frac{1}{5}\right)}{4}\right| - \frac{1}{3} = \frac{2}{3}$ $\quad$ $\left|\frac{3-5\times\left(+\frac{7}{5}\right)}{4}\right| - \frac{1}{3} = \frac{2}{3}$

$\left|\frac{3+1}{4}\right| - \frac{1}{3} = \frac{2}{3}$ $\quad$ $\left|\frac{3-7}{4}\right| - \frac{1}{3} = \frac{2}{3}$

$\left|\frac{4}{4}\right| - \frac{1}{3} = \frac{2}{3}$ $\quad$ $\left|\frac{-4}{4}\right| - \frac{1}{3} = \frac{2}{3}$

$\quad$ $|-1| - \frac{1}{3} = \frac{2}{3}$

$1 - \frac{1}{3} = \frac{2}{3}$ $\quad$ $1 - \frac{1}{3} = \frac{2}{3}$

$\frac{3-1}{3} = \frac{2}{3}$ $\quad$ $\frac{3-1}{3} = \frac{2}{3}$

$\frac{2}{3} = \frac{2}{3}$ $\quad$ $\frac{2}{3} = \frac{2}{3}$

Solution Set $= \left\{\frac{-1}{5}, \frac{7}{5}\right\}$

(viii) $\left|\frac{x+5}{2-x}\right| = 6$

Solution $\left|\frac{x+5}{2-x}\right| = 6$

$\frac{x+5}{2-x} = \pm 6$

$\frac{x+5}{2-x} = 6$

$x + 5 = 6(2 - x)$

$x + 5 = 12 - 6x$

$x + 6x = 12 - 5$

$7x = 7$

$x = \frac{7}{7}$

$x = 1$

$\frac{x+5}{2-x} = -6$

$x + 5 = -6(2 - x)$

$x + 5 = -12 + 6x$

$5 + 12 = 6x - x$

$17 = 5x$

$\frac{17}{5} = x$

$x = \frac{17}{5}$

To check

$\left|\frac{x+5}{2-x}\right| = 6$

$\left|\frac{1+5}{2-1}\right| = 6$

$\left|\frac{6}{1}\right| = 6$

$6 = 6$

$\left|\left(\frac{17}{5} + 5\right) \div \left(2 - \frac{17}{5}\right)\right| = 6$

$\left|\frac{17+25}{5} \div \frac{10-17}{5}\right| = 6$

$\left|\frac{42}{5} \div \frac{-7}{5}\right| = 6$

$|-6| = 6$

$6 = 6$

Solution Set $= \left\{1, \frac{17}{5}\right\}$

Exercise 7.3

Q1) Solve the following inequalities.

(i) $3x+1<5x-4$

Solution: $3x+1<5x-4$

$3x<5x-4-1$

$3x-5x<-5$

$-2x<-5$

Case-I When negative is eliminated from both sides of inequality the symbol will be change.

Case-II When negative is transferred from variable to constant side, symbol will also change.

$x>\frac{-5}{-2}$

$x>\frac{5}{2}$

Solution Set $=\left\{x \mid x>\frac{5}{2}\right\}$

(ii) $4x-10.3\le 21x-1.8$

Solution: $4x-10.3\le 21x-1.8$

$4x-21x\le -8.5+10.3$

$-17x\le 8.5$

When negative value is shifted to other side its symbol changes.

$x\ge\frac{8.5}{-17}$

$x\ge-\frac{8.5}{17}$

$x\ge-0.5$

Solution Set $=\{x \mid x\ge-0.5\}$

(iii) $4-\frac{1}{2}x\ge-7+\frac{1}{4}x$

Solution: $4-\frac{1}{2}x\ge-7+\frac{1}{4}x$

$-\frac{1}{2}x-\frac{1}{4}\ge-7-4$

$\frac{-2x-x}{4}\ge-11$

$-3x\ge-44$

When negative value is shifted the symbol changes

$x\le\frac{-44}{-3}$

$x\le\frac{44}{3}$

Solution Set $=\{x \mid x\le\frac{44}{3}\}$

(iv) $x-2(5-2x)\ge 6x-3\frac{1}{2}$

Solution: $x-2(5-2x)\ge 6x-3\frac{1}{2}$

$x-10+4x\ge 6x-\frac{7}{2}$

$5x-6x\ge-\frac{7}{2}+10$

$-1x\ge\frac{-7+20}{2}$

$-x\ge-\frac{13}{2}$

When negative is shifted other side symbol changes

$x\le\frac{13}{-1\times 2}$

$x\le-\frac{13}{2}$

$x\le-6.5$

Solution Set $=\{x \mid x\le-6.5\}$

(v) $\frac{3x+2}{9}-\frac{2x+1}{3}>-1$

Solution: $\frac{3x+2}{9}-\frac{2x+1}{3}>-1$

$\frac{3x+2-3(2x+1)}{9}>-1$

$3x+2-6x-3>-9$

$-3x>-9+1$

$3x > -8$

Negative value is shifted to other side its symbols changes

$$x < \frac{-8}{-3}$$

$$x < \frac{8}{3}$$

Solution Set $= \left\{x \mid x < \frac{8}{3}\right\}$

(vi) $3(2x+1)-2(2x+5) < 5(3x-2)$

Solution: $3(2x+1)-2(2x+5) < 5(3x-2)$

$$6x+3-4x-10 < 15x-10$$

$$2x-7-15x < -10$$

$$-13x < -10+7$$

$$-13x < -3$$

The value is negative when shifted to other side it changes its symbols

$$x > \frac{-3}{-13}$$

$$x > \frac{3}{13}$$

Solution Set $= \left\{x \mid x > \frac{3}{13}\right\}$

(vii) $3(x-1)-(x-2) > -2(x+4)$

Solution: $3(x-1)-(x-2) > -2(x+4)$

$$3x-3-x+2 > -2x-8$$

$$2x-1 > -2x-8$$

$$2x+2x > -8+1$$

$$4x > -7$$

$$x > \frac{-7}{4}$$

Solution Set $= \left\{x \mid x > \frac{-7}{4}\right\}$

(viii) $2\frac{2}{3}x + \frac{2}{3}(5x-4) > -\frac{1}{3}(8x+7)$

Solution: $2\frac{2}{3}x + \frac{2}{3}(5x-4) > -\frac{1}{3}(8x+7)$

$$\frac{8}{3}x + \frac{10x-8}{3} > -\frac{(8x+7)}{3}$$

$$\frac{8x+10x-8}{3} > -\frac{8x+7}{3}$$

Multiplying both side by 3

$$\cancel{3} \times \frac{18x-8}{\cancel{3}} > -\cancel{3} \times \frac{8x+7}{\cancel{3}}$$

$$18x-8 > -(8x+7)$$

$$18x-8 > -8x-7$$

$$18x+8x > -7+8$$

$$26x > 1$$

$$x > \frac{1}{26}$$

Solution Set $= \left\{x \mid x > \frac{1}{26}\right\}$

Q2) Solve the following inequalities

(i) $-4 < 3x+5 < 8$

Solution: $-4 < 3x+5 < 8$

$-4 < 3x+5$	and	$3x+5 < 8$
$-4-5 < 3x$		$3x < 8-5$
$-9 < 3x$		$3x < 3$
$\frac{-9}{3} < x$		$x < \frac{3}{3}$
$-3 < x$		$x < 1$

$-3<x<1$

Solution Set $= \{x \mid -3 < x < 1\}$

(ii) $-5 \le \frac{4-3x}{2} < 1$

Solution: $-5 \le \frac{4-3x}{2} < 1$

$-5 \le \frac{4-3x}{2}$	and	$\frac{4-3x}{2} < 1$
$-10 \le 4-3x$		$4-3x < 2$
$3x-10 \le 4$		$-3x < 2-4$
$3x \le 4+10$		$-3x < -2$
$3x \le 14$		$x > \frac{-2}{-3}$
$x \le \frac{14}{3}$		$x > \frac{2}{3}$

$$\frac{2}{3} < x$$

$\frac{2}{3} < x \leq \frac{14}{3}$

Solution Set $= \{x \mid \frac{2}{3} < x \leq \frac{14}{3}\}$

(iii) $-6 < \frac{x-2}{4} < 6$

Solution: $-6 < \frac{x-2}{4} < 6$

$-6 < \frac{x-2}{4}$

$-24 < x - 2$

$-24 + 2 < x$

$-22 < x$

and

$\frac{x-2}{4} < 6$

$x - 2 < 24$

$x < 24 + 2$

$x < 26$

$-22 < x < 26$

Solution Set $= \{x \mid -22 < x < 26\}$

(iv) $3 \geq \frac{7-x}{2} \geq 1$

Solution: $3 \geq \frac{7-x}{2} \geq 1$

$3 \geq \frac{7-x}{2}$

$6 \geq 7 - x$

$6 - 7 \geq -x$

$-1 \geq -x$

Negative sign change the symbols

$1 \leq x$

and

$\frac{7-x}{2} \geq 1$

$7 - x \geq 2$

$-x \geq 2 - 7$

$-x \geq -5$

$x \leq 5$

$1 \leq x \leq 5$

Solution Set $= \{x \mid 1 \leq x \leq 5\}$

(v) $3x - 10 \leq 5 < x + 3$

Solution $3x - 10 \leq 5 < x + 3$

$3x - 10 \leq 5$ and $5 < x+3$

$3x \leq 5+10$ $\quad$ $5-3 < x$

$3x \leq 15$ $\quad$ $2 < x$

$\frac{3x}{3} \leq \frac{15}{3}$

$x \leq 5$

$2 < x \leq 5$

Solution Set $= \{x \mid 2 < x \leq 5\}$

(vi) $-3 \leq \frac{x-4}{-5} < 4$

Solution $-3 \leq \frac{x-4}{-5} < 4$

$-3 \leq \frac{x-4}{-5}$ and $\frac{x-4}{-5} < 4$

$-3 \times -5 \geq x - 4$

$x - 4 > 4(-5)$

$15 \geq x - 4$ $\quad$ $x > -20 + 4$

$15 + 4 \geq x$ $\quad$ $x > -16$

$19 \geq x$ $\quad$ $-16 < x$

$x \leq 19$

$-16 < x \leq 19$

Solution Set $= \{x \mid -16 < x \leq 19\}$

(vii) $1 - 2x < 5 - x \leq 25 - 6x$

Solution: $1 - 2x < 5 - x \leq 25 - 6x$

$1 - 2x < 5 - x$ and $5 - x \leq 25 - 6x$

$-x + 6x \leq 25 - 5$ $\quad$ $6x - x \leq 20$

$1 - 2x + x < 5$ $\quad$ $5x \leq 20$

$-x < 5 - 1$ $\quad$ $x \leq \frac{20}{5}$

$-x < 4$ $\quad$ $x \leq 4$

Due negative sign

Symbol change

$-4 < x$

$-4 < x \leq 4$

Solution Set $= \{x \mid -4 < x \leq 4\}$

(viii) $3x - 2 < 2x + 1 < 4x + 17$

Solution: $3x - 2 < 2x + 1 < 4x + 17$

$3x - 2 < 2x + 1$ $\quad$ $2x + 1 < 4x + 17$

$3x - 2x - 2 < +1$ $\quad$ $2x - 4x < 17 - 1$

$x < 1 + 2$ $\quad$ $-2x < 16$

$x < 3$

$$x > \frac{16}{-2}$$
$$x > -8$$
$$-8 < x$$

$-8 < x < 3$

Solution Set $= \{x \mid -8 < x < 3\}$

Review Exercise 7

Q.1 Choose the correct answer

(i) Which of the following is the solution of the inequality $3-4x \le 11$**?**

(a) -8 **(b)** -2

(c) $-\frac{14}{4}$ **(d)** None of these

(ii) A statement involving any of the symbols $<$, $>$, $\le$or $>$, is called------------

(a) Equation **(b)** Identity

(c) Inequality **(d)** Linear equation

(iii) $x =$ **------- is a solution of the inequality** $-z < x > \frac{3}{2}$

(a) -5 **(b)** 3

(c) 0 **(d)** $\frac{3}{2}$

(iv) If x is no larger than 10, then ----------

(a) $x \le 8$ **(b)** $x \ge 10$

(c) $x < 10$ **(d)** $x > 10$

(v) If the capacity $<$ of an elevator is at most 1600 pounds then ----------

(a) $c < 1600$ **(b)** $c \ge 1600$

(c) $c \le 1600$ **(d)** $c > 1600$

(vi) x = 0 is a solution of the inequality -----------

(a) $x > 0$ **(b)** $3x+5<0$

(c) $x+\frac{z}{2}<0$ **(d)** $x-2<0$

ANSWER KEY

i	ii	iii	iv	v	vi
b	c	c	b	c	d

Q.2 Identify the following statement as true or false

(i) The equation $3x-5=7-x$ is a linear equation. (True)

(ii) The equation $x-0.3x=0.7x$ is an identity (True)

(iii) The equation $-2x+3=8$ is equivalent to $-2x=11$ (False)

(iv) To eliminate fractions we multiply each side of an equation by the L.C.M of denominators (True)

(v) $4(x+3)=x+3$ is a conditional equations (True)

(vi) The equation $2(3x+5)=6x+12$ is an in consistent equation (True)

(vii) To solve $\frac{2}{3}x=12$, we should multiply each side by $\frac{2}{3}$ (False)

(viii) Equations having exactly the same solution are called equivalent equations. (True)

(ix) A solution that does not satisfy the original equation is called extra solution (True)

Q.3 **Answer the following short question.**

(i) **Define a linear inequality in one variable**

Ans A linear inequality in one variable x is an inequality in which the variable x occurs only to the first power and has the standard form $ax+b<0, a\neq 0$

(ii) **State the trichotomy and transitive properties of in equalities**

Ans **Trichotomy Property**

For any $a,b\in R$ one and only one of the following statements in true. $a<b$ or $a=b$, or $a>b$

Transitive Property

Let $a,b,c\in R$.

(a) If $a>b$ and $b>c$, then $a>c$

(b) If $a<b$ and $b<c$, then $a<c$

(iii) **The formula relating degree Fahrenheit to degree Celsius is F= $\frac{9}{5}c+32$ for what value of c is F< O was**

Ans F=$\frac{9}{5}c+32$

$\frac{9}{5}c+32=\text{F}$

Since F <0

So $\frac{9}{5}c+32<0$

$\frac{9c+160}{5}<0$

Or 9c + 160 < 0 × 5

Or 9c + 160 < 0

Or 9c< – 160

Or c $<-\frac{160}{9}$

(iv) **Seven times the sum of an integer and 12 is at least 50 and at most 60. Write and solve the inequality that expresses this relation ship**

Solution: Let the integer $=y$

Sum of integer and $12=y+12$

Seven times sum of integer and $12=7(y+12)$

According to condition

$50\leq 7(y+12)\leq 60$

$\frac{50}{7}\leq 7\frac{(y+12)}{7}\leq\frac{60}{7}$

$\frac{50}{7}\leq y+12\leq\frac{60}{7}$

$\frac{50}{7}-12\leq y+\cancel{12}-\cancel{12}\leq\frac{60}{7}-12$

$$\frac{50-84}{7} \le y \le \frac{60-84}{7}$$

$$\frac{-34}{7} \le y \le \frac{-24}{7}$$

Solution Set $= \left\{ y \mid \frac{-34}{7} \le y - \frac{24}{7} \right\}$

Q.4 Solve each of the following and check for extraneous solution if any

(i) $\sqrt{2t+4} = \sqrt{t-1}$

Solution: $\sqrt{2t+4} = \sqrt{t-1}$

Taking square on both side

$$\left(\sqrt{2t+4}\right)^2 = \left(\sqrt{t-1}\right)^2$$

$$2t+4 = t-1$$

$$2t-t = -1-4$$

$$t = -5$$

To check

$$\sqrt{2t+4} = \sqrt{t-1}$$

When $t = -5$

$$\sqrt{2(-5)+4} = \sqrt{t-5-1}$$

$$\sqrt{-10+4} = \sqrt{-6}$$

$$\sqrt{-6} = \sqrt{-6}$$

L.H.S = R.H.S

Solution Set $= \{-5\}$

(ii) $\sqrt{3x-1} - 2\sqrt{8-2x} = 0$

Solution: $\sqrt{3x-1} - 2\sqrt{8-2x} = 0$

$$\sqrt{3x-1} = 2\sqrt{8-2x}$$

Taking square on both side

$$\left(\sqrt{3x-1}\right)^2 = \left(2\sqrt{8-2x}\right)^2$$

$$3x-1 = 4(8-2x)$$

$$3x-1 = 32-8x$$

$$3x+8x = 32+1$$

$$11x = 33$$

$$x = \frac{33}{11}$$

$$x = 3$$

To check

$$\sqrt{3x-1} - 2\sqrt{8-2x} = 0$$

When $x=3$

$\sqrt{3(3)-1}-2\sqrt{8-2(3)}=0$

$\sqrt{9-1}-2\sqrt{8-6}=0$

$\sqrt{8}-2\sqrt{2}=0$

$2\sqrt{2}-2\sqrt{2}=0$

$0=0$

L.H.S = R.H.S

Solution Set $=\{3\}$

Q.5 Solve for x

(i) $|3x+14|-2=5x$

Solution: $|3x+14|-2=5x$

$|3x+14|=5x+2$

$3x+14=\pm(5x+2)$

$3x+14=5x+2$

$14-2=5x-3x$

$12=2x$

$\frac{12}{2}=x$

$x=6$

To check

$|3x+14|-2=5x$

When $x=6$

$|3(6)+14|-2=5(6)$

$|18+14|-2=30$

$32-2=30$

$30=30$

$3x+14=-(5x+2)$

$3x+14=-5x-2$

$3x+5x=-2-14$

$8x=\frac{-16}{8}$

$x=-2$

$|3x+14|-2=5x$

when $x=-2$

$|3(-2)+14|-2=5(-2)$

$|-6+14|-2=-10$

$8-2=-10$

$6=-10$

Solution Set $=\{6\}$

(ii) $\frac{1}{3}|x-3|=\frac{1}{2}|x+2|$

Solution $\frac{1}{3}|x-3|=\frac{1}{2}|x+2|$

$\frac{2}{3}|x-3|=|x+2|$

$\frac{2}{3}=\frac{|x+2|}{|x-3|}$

$\frac{x+2}{x-3}=\pm\frac{2}{3}$

$\frac{x+2}{x-3}=\frac{2}{3}$

$3(x+2)=2(x-3)$

$3x+6=2x-6$

$3x-2x=-6-6$

$x=-12$

To check

$\frac{1}{3}|x-3|=\frac{1}{2}|x+2|$

When $x=-12$

$\frac{1}{3}|-12-3|=\frac{1}{2}|-12+2|$

$\frac{1}{3}|-15|=\frac{1}{2}|-10|$

$\frac{1}{\cancel{3}}\left(\cancel{15}^{5}\right)=\frac{1}{\cancel{2}}\left(\cancel{10}^{5}\right)$

$5=5$

and

$\frac{x+2}{x-3}=-\frac{2}{3}$

$3(x+2)=-2(x-3)$

$3x+6=-2x+6$

$3x+2x=+6-6$

$5x=0$

$x=\frac{0}{5} \Rightarrow x=0$

$\frac{1}{3}|x-3|=\frac{1}{2}|x+2|$

when $x=0$

$\frac{1}{3}|0-3|=\frac{1}{2}|0+2|$

$\frac{1}{3}|-3|=\frac{1}{2}|2|$

$\frac{1}{\cancel{3}}\left(\cancel{3}^{1}\right)=\frac{1}{\cancel{2}}\left(\cancel{2}^{1}\right)$

$\frac{1}{3}(3)=1$

$1=1$

Solution Set $=\{-12,0\}$

Q.6 Solve the following inequality

(i) $-\frac{1}{3}x+5\leq 1$

Solution $-\frac{1}{3}x+5\leq 1$

$-\frac{1}{3}x\leq 1-5$

$-\frac{1}{3}x\leq -4$

$x\geq -4\times(-3)$

$x\geq 12$

Solution Set $=\{x\,|\,x\geq 12\}$

(ii) $-3<\frac{1-2x}{5}<1$

Solution $-3<\frac{1-2x}{5}<1$

$-3<\frac{1-2x}{5}$ $\qquad$ $\frac{1-2x}{5}<1$

$-15 < 1 - 2x$ $\qquad$ $1 - 2x < 5$

$-15 - 1 < -2x$ $\qquad$ $-2x < 5 - 1$

$-16 < -2x$ $\qquad$ $-2x < 4$

$\frac{-16}{-2} > x$ $\qquad$ $x > \frac{4}{-2}$

$8 > x$ $\qquad$ $x > -2$

$x < 8$ $\qquad$ $-2 < x$

$$-2 < x < 8$$

Solution Set $= \{x \mid -2 < x < 8\}$

Unit 7: Linear Equations and Inequalities

Overview

Linear Equation:

A linear equation in one unknown variable x is an equation of the form $ax + b = 0$, where $a, b \in R$ and $a \neq 0$.

Example:

(i) $5x - 3 = 0$

(ii) $\frac{1}{2}x + 18 = 0$

Radical equations:

When the variable in an equation occurs under a radical the equation is called a radical equation.

Example:

(i) $\sqrt{2x-3} - 7 = 0$

Absolute value:

The absolute value of a real number 'a' denoted by $|a|$, is defined as

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

e.g.,
$$|6| = 6,$$
$$|0| = 0$$
$$|-6| = -(-6) = 6$$

Extraneous Roots:

If the solutions (roots) obtained from the equation does not satisfy the original equations are called extraneous roots.

Linear inequality:

A linear inequality in one variable x is an inequality in which the variable x occurs only to the first power and has the standard form. $ax + b < 0$, $a \neq 0$ $a, b \in R$ we may replace the symbol $<$by$>$, $\leq$or $\geq$also.

Inconsistent equation:

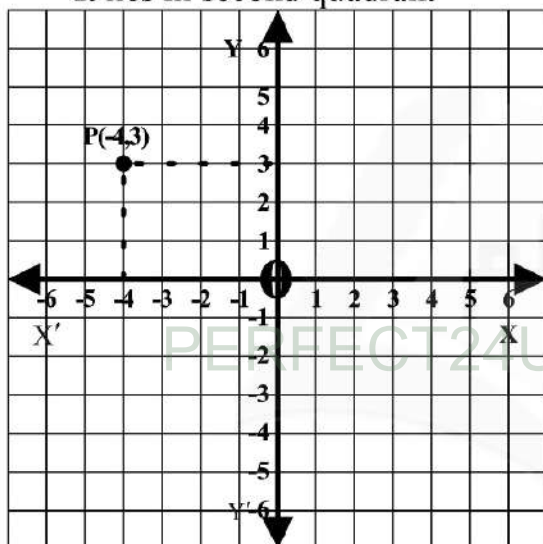
An inconsistent equation is that whose solution set is ϕ.

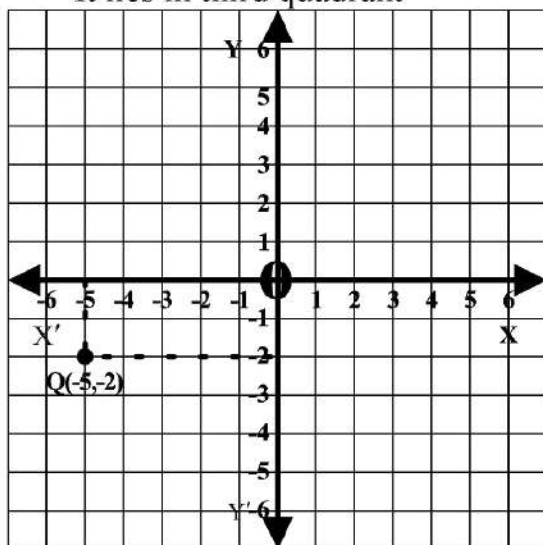
Exercise 8.1

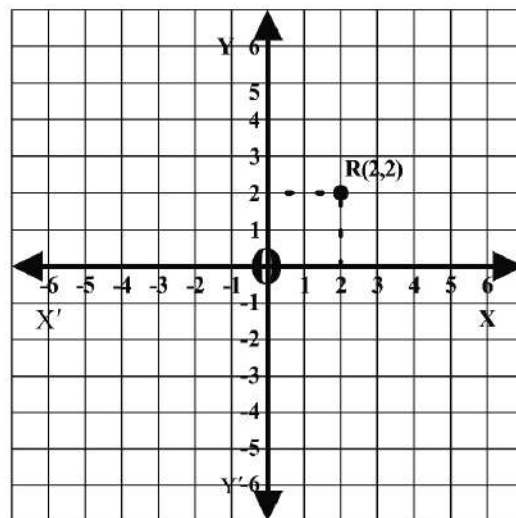
Q.1

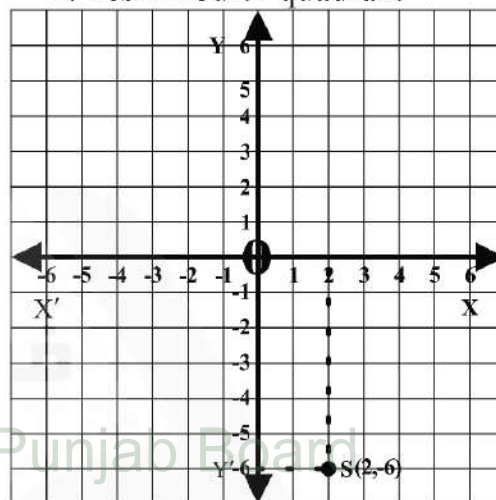
(i) **Determine the quadrant of coordinate plane in which the following points lies**

P (–4, 3)

It lies in second quadrant

Q (–5, –2)

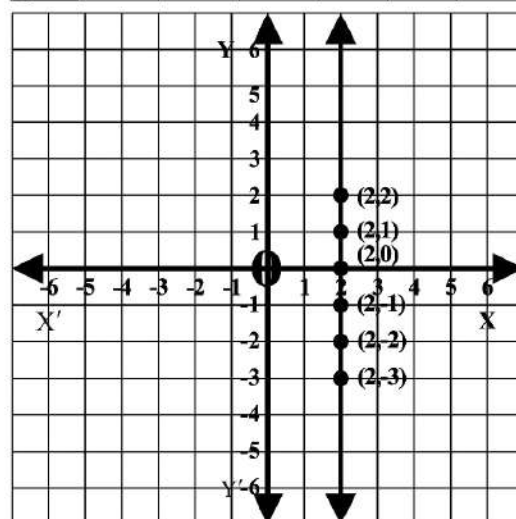
It lies in third quadrant

R (2, 2)

It lies in first quadrant

S (2, –6)

It lies in fourth quadrant

Q.2 Draw the graph of each of the following i.e.

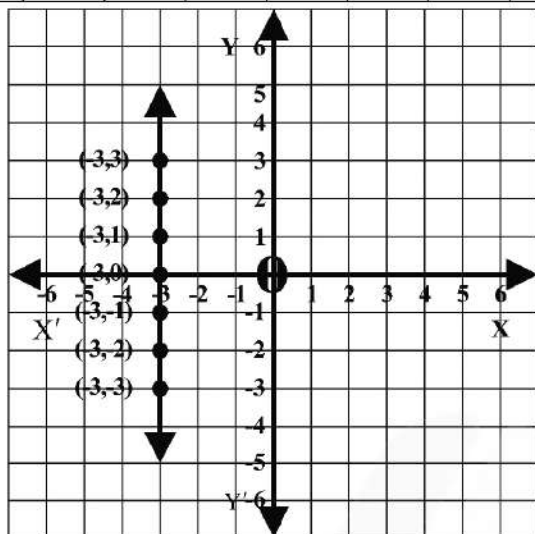
(i) $x = 2$

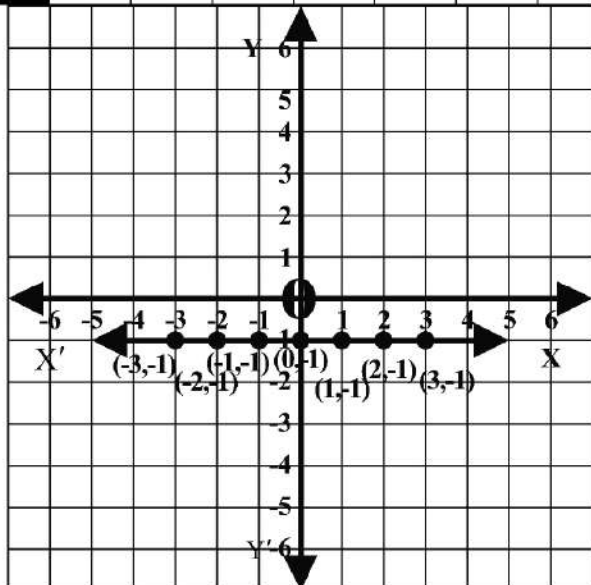
The table for the points of equation $x = 2$ is as under

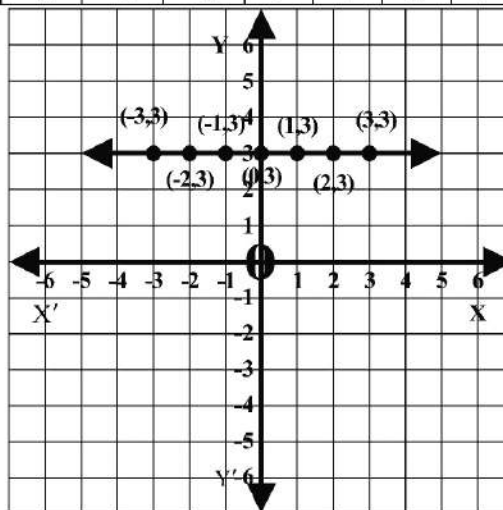
x	2	2	2	2	2	2
y	-3	-2	-1	0	1	2

(ii) $x = -3$

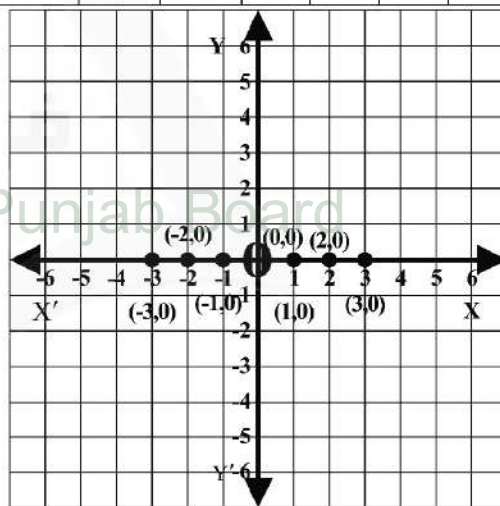
The table for the points of equation

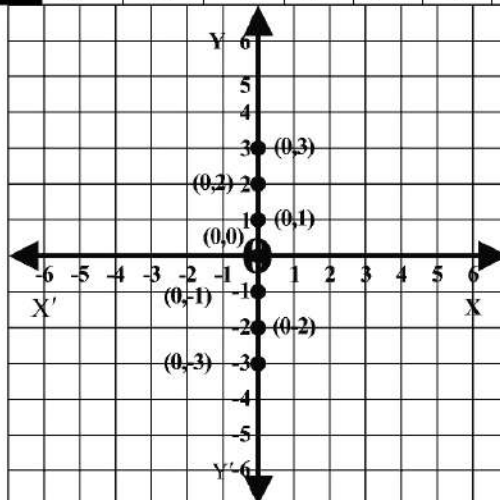
$x = -3$ is as under

x	-3	-3	-3	-3	-3	-3	-3
y	-3	-2	-1	0	1	2	3

(iii) $y = -1$

x	-1	-1	-1	-1	-1	-1	-1
y	-3	-2	-1	0	1	2	3

(iv) $y = 3$

x	3	3	3	3	3	3	3	3
y	-3	-2	-1	0	1	2	3	4

(v) $y = 0$

x	-3	-2	-1	0	1	2	3	4
y	0	0	0	0	0	0	0	0

(vi) $x = 0$

x	0	0	0	0	0	0	0
y	-3	-2	-1	0	1	2	3

(vii) $y = 3x$

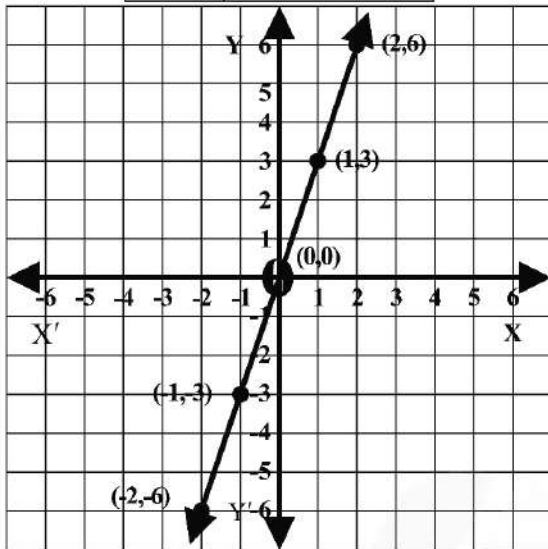
x	$y = 3x$
....	....
-2	$3(-2) = -6$
-1	$3(-1) = -3$
0	$3(0) = 0$
1	$3(1) = 3$
2	$3(2) = 6$
...	...

(viii) $-y = 2x$

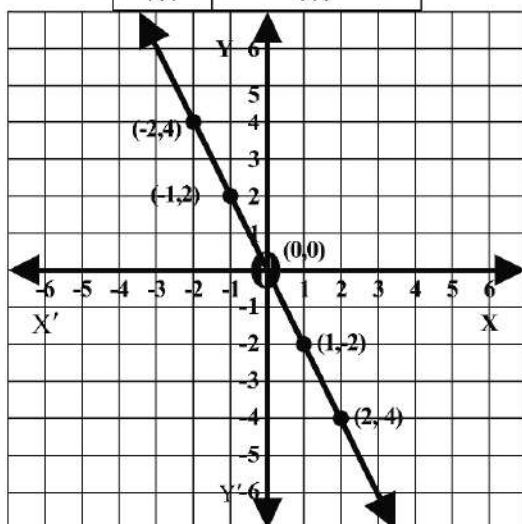
Multiply both sides by (–)

$-(-y) = -2x$

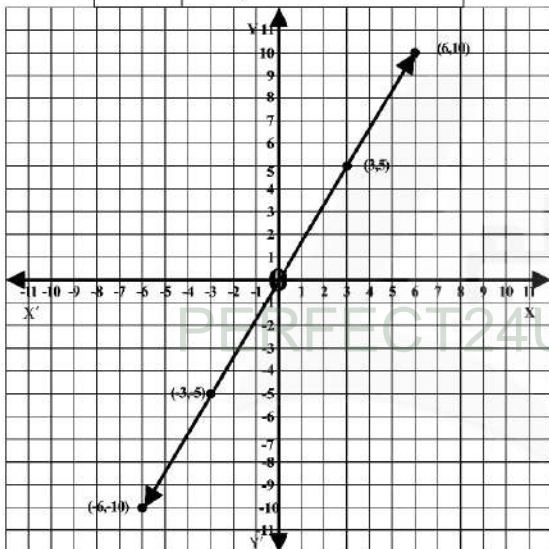
$y = -2x$

x	$y = -2x$
....	....
-2	$-2(-2) = 4$
-1	$-2(-1) = 2$
0	$-2(0) = 0$
1	$-2(1) = -2$
2	$-2(2) = -4$
...	...

(ix) $\frac{1}{2} = x$

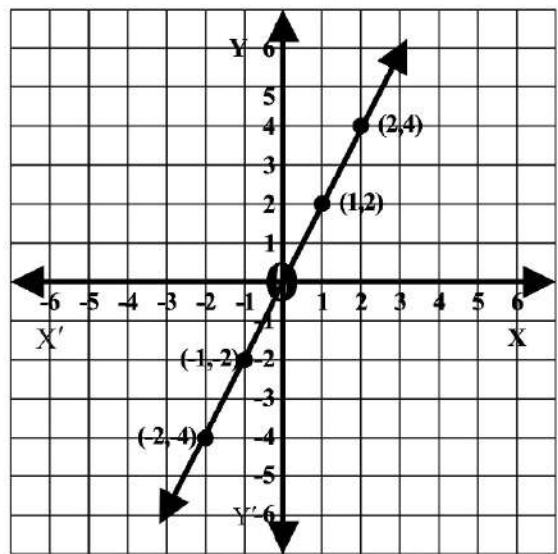
Or $x = \frac{1}{2}$

x	y
$\frac{1}{2} = 0.5$	-3
$\frac{1}{2} = 0.5$	-2
$\frac{1}{2} = 0.5$	-1
$\frac{1}{2} = 0.5$	0
$\frac{1}{2} = 0.5$	1
$\frac{1}{2} = 0.5$	2
$\frac{1}{2} = 0.5$	.....

(x) $3y = 5x$

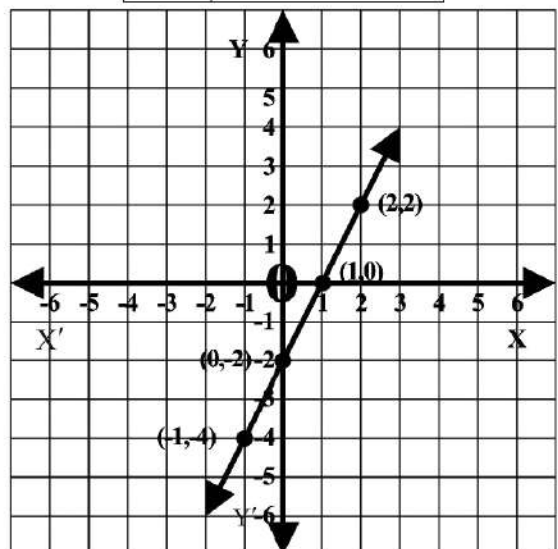
$$y = \frac{5}{3}x$$

x	$y = \frac{5}{3}x$
-6	$\frac{5}{\not{3}} \times -\not{6}^{2} = -10$
–3	$\frac{5}{\not{3}} \times \not{-3} = -5$
0	$\frac{5}{3} \times 0 = 0$
3	$\frac{5}{\not{3}} \times \not{3} = 5$
6	$\frac{5}{\not{3}} \times \not{6}^{2} = 10$

(xi) $2x - y = 0$

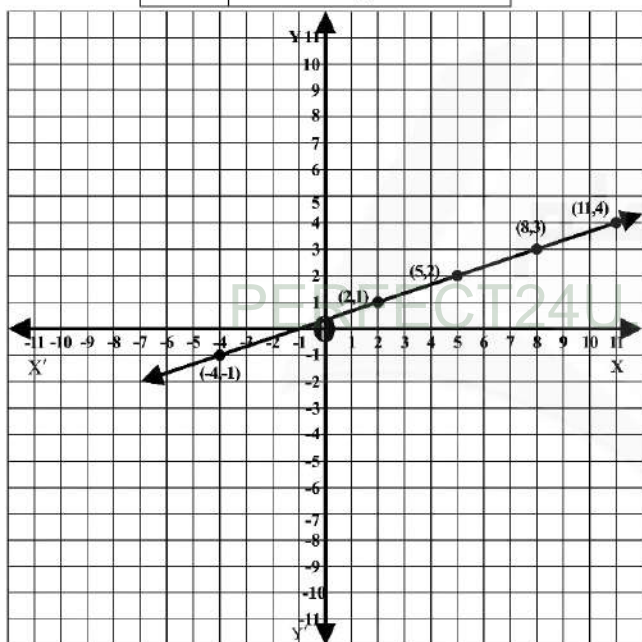
$2x = y$ or $y = 2x$

x	y = $2x$
–2	2(–2)= –4
–1	2(–1)= –2
0	2(0)= 0
1	2(1)= 2
2	2(2)= 4

(xii) $2x - y = 2$

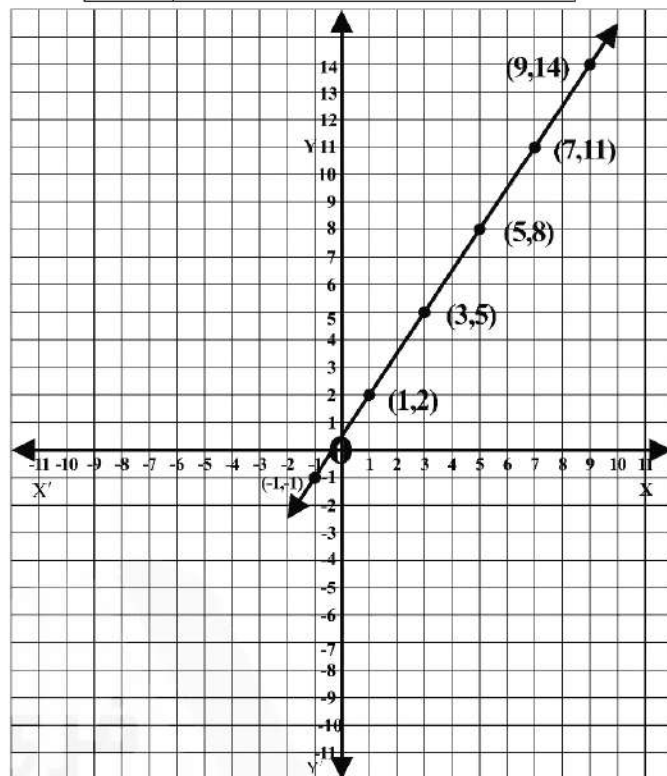
$2x - 2 = y$ or $y = 2x - 2$

x	y = $2x$ -2
–1	2(–1)–2= –4
0	2(0)–2= –2
1	2(1)–2= 0
2	2(2)–2= 2

(xiii) $x-3y+1=0 \Rightarrow x+1=+3y$

$$y=\frac{x+1}{3}$$

x	$y=\frac{x+1}{3}$
−4	$y=\frac{-4+1}{3}=-1$
2	$y=\frac{2+1}{3}=1$
5	$y=\frac{5+1}{3}=2$
8	$y=\frac{8+1}{3}=3$
11	$y=\frac{11+1}{3}=4$

(xiv) $3x-2y+1=0$

$$y=\frac{3x+1}{2}$$

x	$y=\frac{3x+1}{2}$
−1	$y=\frac{3(-1)+1}{2}=\frac{-2}{2}=-1$
1	$y=\frac{3(1)+1}{2}=\frac{4}{2}=2$
3	$y=\frac{3(3)+1}{2}=\frac{10}{2}=5$
5	$y=\frac{3(5)+1}{2}=\frac{16}{2}=8$
7	$y=\frac{3(7)+1}{2}=\frac{22}{2}=11$
9	$y=\frac{3(9)+1}{2}=\frac{28}{2}=14$

Q.3 Are the following lines (i) parallel to $x-axis$ (ii) parallel to $y-axis$

Solution:

(i) $2x-1=3$

$2x=3+1$

$2x=4$

$x=\frac{4}{2}$

$x=2$ it is a line parallel to y-axis

(ii) $x+2=-1$

$x=-1-2$

$x=-3$ it is a line parallel to $y-axis$

(iii) $2y+3=2$

$2y=2-3$

$2y=-1$

$y=\frac{-1}{2}$ it is a line parallel to $x-axis$

(iv) $x+y=0$

$x=-y$ It is neither parallel to $x-axis$ nor $y-axis$

(v) $2x-2y=0$

$2x=2y$

$x=\frac{2y}{2}$

$x=y$

$y=x$

It is neither parallel to $x-axis$ nor $y-axis$

Q.4 Find the value of m and c of the following lines by expressing them in the form $y=mx+c$

Solution:

(a) $2x+3y-1=0$

$3y=-2x+1$

$y=\frac{-2x+1}{3}$

$y=\frac{-2x}{3}+\frac{1}{3}$

$m=-\frac{2}{3}$ and $c=\frac{1}{3}$

(b) $x-2y=-2$

$x+2=2y$

$\frac{x+2}{2}=y$

Or

$y=\frac{x+2}{2}$

$y=\frac{1}{2}x+\frac{2}{2}$

$y=\frac{1}{2}x+1$

So, $m=\frac{1}{2}$ $c=1$

(c) $3x+y-1=0$

$y=1-3x$

or

$y=-3x+1$

$m=-3$ $c=1$

(d) $2x-y=7$

$2x-7=y$

Or

$y=2x-7$

$m=2$ $c=-7$

(e) $3-2x+y=0$

$y=2x-3$

$m=2$ $c=-3$

(f) $2x=y+3$

$2x-3=y$

Or

$y=2x-3$

$m=2$ $c=-3$

Q.5 Verify whether the following point lies on the line 2x–y + 1 =0 or not

Solution:

(i) (2, 3)

$2x-y+1=0$

$2(2)-3+1=0$

$4-3+1=0$

$2\neq 0$

$\therefore$ The point does not lie on the line

(ii) (0, 0)

$2x-y+1=0$

$2(0)-0+1=0$

$0-0+1=0$

$1\neq 0$

$\therefore$ The point does not lie on the line

(iii) (–1, 1)
$2x - y + 1 = 0$
$2(-1) - 1 + 1 = 0$
$-2 - 1 + 1 = 0$
$-2 \neq 0$
$\therefore$ The point does not lie on the line

(iv) (2, 5)
$2x - y + 1 = 0$
$2(2) - 5 + 1 = 0$
$4 - 5 + 1 = 0$
$0 = 0$
$\therefore$ It lies on the line

(v) (5, 3)
$2x - y + 1 = 0$
$2(5) - 3 + 1 = 0$
$10 - 3 + 1 = 0$
$8 \neq 0$
$\therefore$ It does not lie on the line

Exercise 8.2

Q.1

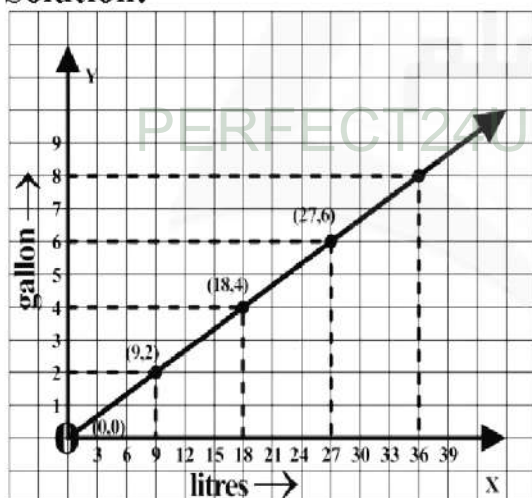
Draw the conversion graph between liters and gallons using the relation 9 liters = 2 gallons (approximately) and taking liters along horizontal axis and gallons along vertical axis from the graph read.

(i) The number of gallons in 18 liters.

(ii) The number of liters in 8 gallons.

We know 9 liters = 2 gallons

$1 \text{liters} = \frac{2}{9}$ gallons

Solution:

$y = \frac{2}{9}x$

x	0	9	18	27
y	0	2	4	6

18 litters=4 gallons

Scale

Along *X-axis*

3 litters = 1 box

Along *Y-axis*

1 gallon = 1 box

(i) The number of gallons in 18 liters.

Ans: =4 Gallons

(ii) The number of liters in 8 gallons.

Ans: =36 Liters

Q.2 **On** 15-03-2008 **the exchange rate of Pakistan currency and Saudi Riyal was as under 1SRial = 16.70 rupees**

If Pakistani currency y is an expression of S. Riyal x expressed under. The rule $y = 16.70\ x$ then draw the conversion graph between these two currencies by taking S. riyal along x axis.

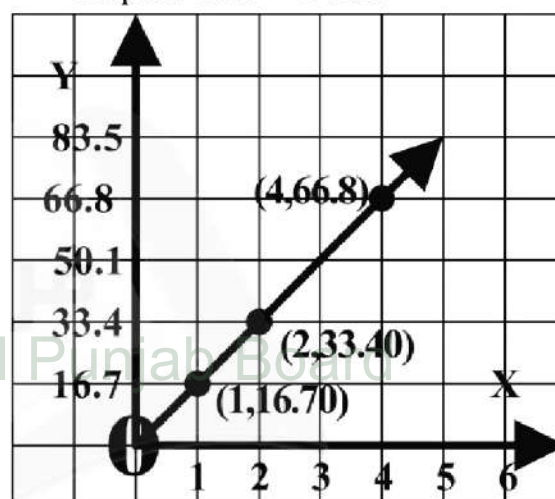
1SR = 16.70 Rupees

Scale

Along *X-axis*

1 SR =1 box

Along *Y-axis*

Rupees 16.7 = 1 box

x	1	2	3	4
y	16.70	33.4	50.1	66.8

Q.3 **Sketch the graph of each of the following lines.**

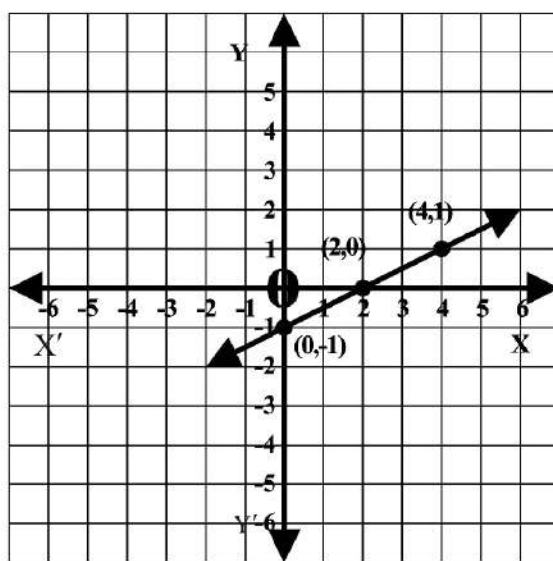
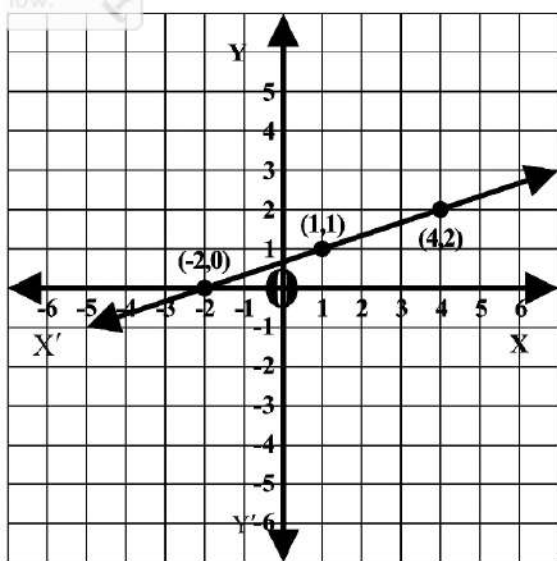
(a) $x-3y+2=0$

$x+2=3y$

$\frac{x+2}{3} = y$

Or

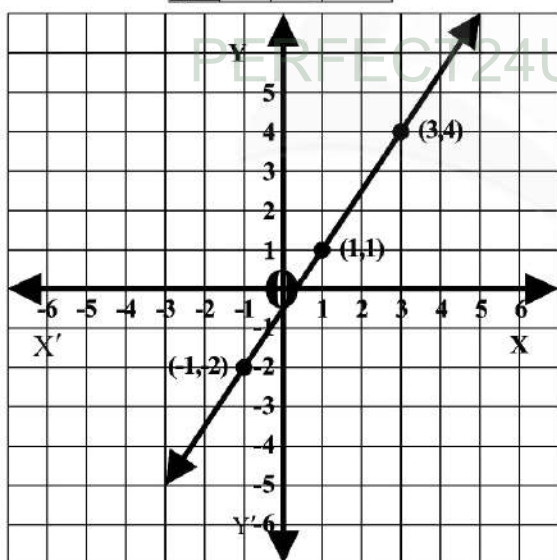
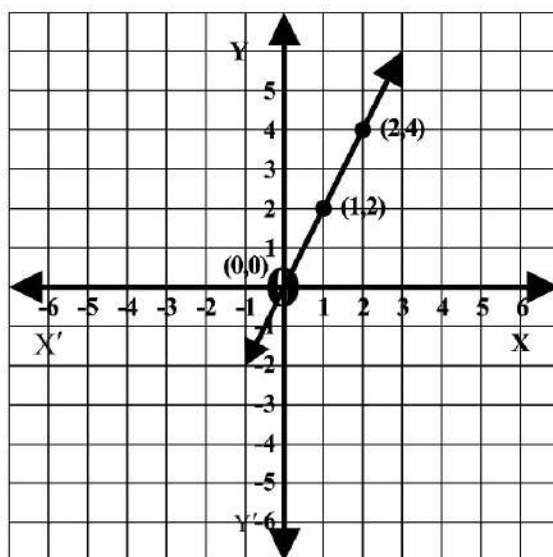
$y = \frac{x+2}{3}$

x	1	4	-2
$y = \frac{x+2}{3}$	1	2	0

(b) $3x-2y-1=0$

$3x-1=2y$

$\frac{3x-1}{2}=y$

$y=\frac{3x-1}{2}$

x	1	3	-1
y	1	4	-2

(c)

$2y-x+2=0$

$2y=x-2$

$y=\frac{x-2}{2}$

x	0	2	4
y	-1	0	1

(d) $y-2x=0$

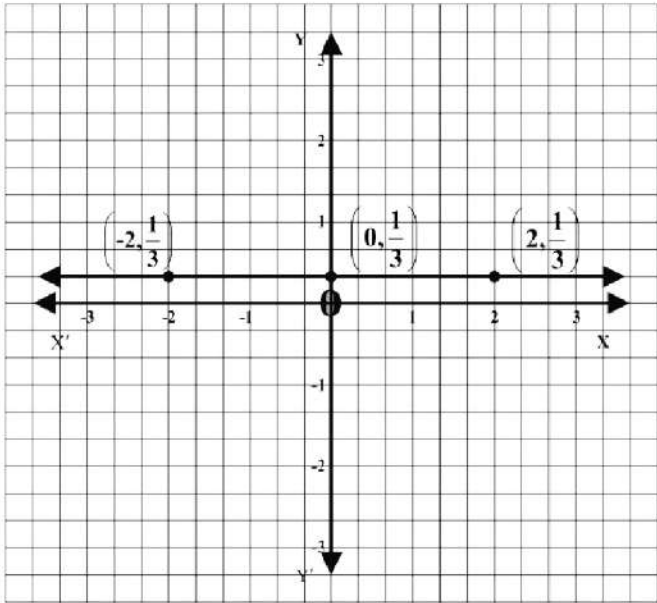
$y=2x$

x	0	1	2
y	0	2	4

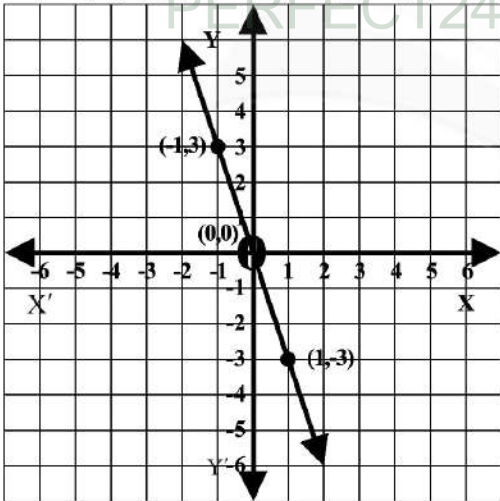
(e) $3y-1=0$

$3y=1$

$y=\frac{1}{3}$

x	-2	0	2
y	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

(f) $y + 3x = 0$

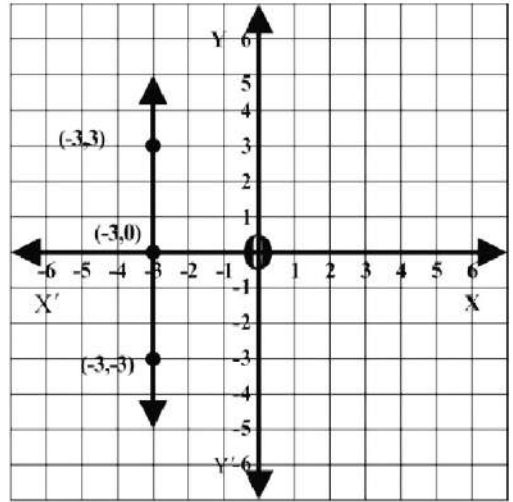
$y = -3x$

x	1	-1	0
y	-3	3	0

(g) $2x + 6 = 0$

$2x = -6$

$x = \frac{-6}{2}$

$x = -3$

x	-3	-3	-3
y	3	0	-3

Q.4 Draw the graph for following relations

(i) One mile $= 1.6km$

$y = 1.6x$

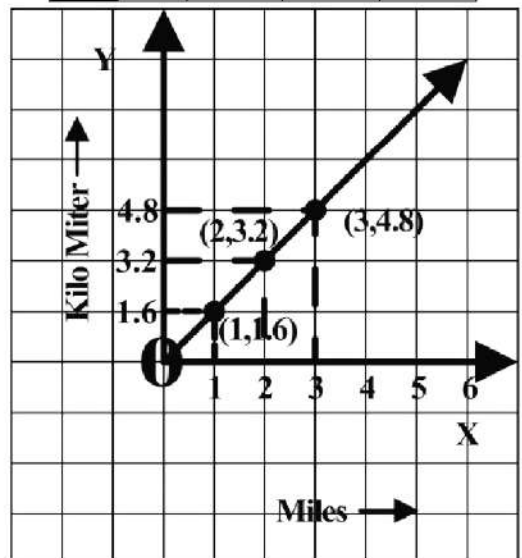
Scale

Along *x-axis*

1 Big Square = 1 Unit

Along *y-axis*

1 Big Square = 1.6 Units

x	0	1	2	3
y	0	1.6	3.2	4.8

(ii) One acre = 0.4 hectare

$y = 0.4x$

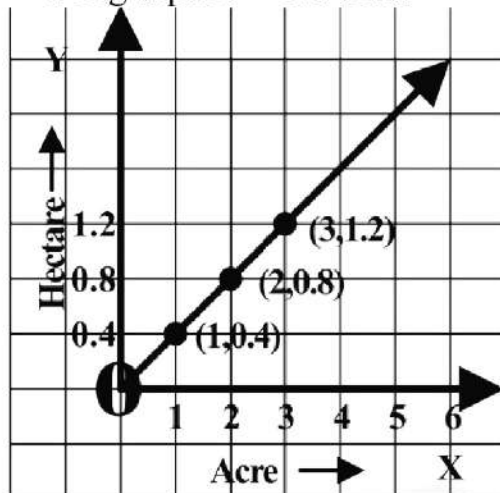
x	2	4
y	0.8	1.6

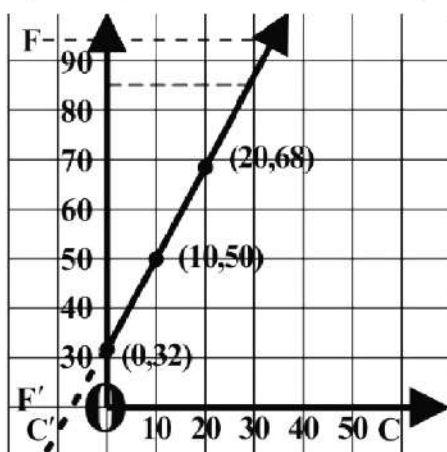
Scale

Along *x-axis*

1 Big Square= 1 Unit

Along *y-axis*

1 Big Square = 0.4 Units

(iii) $F = \frac{9}{5}c + 32$

C	$F = \frac{9}{5}C + 32$
5	$\frac{9}{5} \times 5 + 32 = 41$
10	$\frac{9}{5} \times 10 + 32 = 50$
15	$\frac{9}{5} \times 15 + 32 = 59$
20	$\frac{9}{5} \times 20 + 32 = 68$

10° = Length of square

Where value of c = x and value of f = y

x	5	10	15	20
y	41	50	59	68

(iv) $1\text{Rupee} = \frac{1}{86}\$$

Scale

Along *x-axis*

1 Big Square = 1 Unit

Along *y-axis*

1 Big Square = $1/86$ Units

$y = \frac{1}{86}x$

x	0	1	2	3
y	0	$1/86$	$2/86$	$3/86$

Exercise 8.3

Q.1 $x+y=0$ ——— (I) and

$2x-y+3=0$ – (II)

From equation I from equation II

$y=-x$

$2x-y+3=0$

$2x+3=y$

$y=2x+3$

x	$y=-x$	(x,y)
1	$-1(1)=-1$	(1,-1)
2	-(2)=-2	(2,-2)

x	$y=2x+3$	(x,y)
1	2(1)+3 = 5	(1,5)
–1	2(-1)+3=1	(-1,1)

The point of intersection is a solution set

Solution Set $=\{(-1,1)\}$

Q.2 $x-y+1=0$

$x-2y=-1$

$x+1=y$

$x+1=2y$

$y=x+1$

$\frac{x+1}{2}=y$

Or

$y=\frac{x+1}{2}$

x	$y=x+1$	(x,y)
1	1+1 = 2	(1,2)
2	2+1 = 3	(2,3)

x	$y=\frac{x+1}{2}$	(x,y)
1	$\frac{1+1}{2}=\frac{2}{2}=1$	(1,1)
3	$\frac{3+1}{2}=\frac{4}{2}=2$	(3,2)

Point of intersection is a solution set

Solution Set $=\{(-1,0)\}$

Q.3 $2x+y=0$

$x+2y=2$

$y=-2x$

$2y=2-x$

$y=\frac{2-x}{2}$

x	$y=-2x$	(x,y)
1	$-2(1)=-2$	(1,-2)
2	$-2(2)=-4$	(2,-4)

x	$y=\frac{2-x}{2}$	(x,y)
0	$\frac{2-0}{2}=\frac{2}{2}=1$	(0,1)
2	$\frac{2-2}{2}=\frac{0}{2}=0$	(2,0)

Point of intersection is a solution

Solution Set $=\left(\frac{-2}{3},\frac{4}{3}\right)$

Q.4 $x+y-1=0$

$x-y+1=0$

$y=1-x$

$x+1=y$ Or $y=x+1$

x	$y=1-x$	x,y
1	$1-1=0$	(1,0)
2	$1-2=-1$	(2,-1)

x	$y=x+1$	x,y
1	$1+1=2$	(1,2)
2	$2+1=3$	(2,3)

Point of intersection is a solution set

Solution Set $=\{(0,1)\}$

Q.5 $2x+y-1=0$

$x=-y$

$y=1-2x$

$y=-x$

x	$y=1-2x$	(x,y)
1	$1-2(1)=-1$	(1,-1)
2	$1-2(2)=-3$	(2,-3)

x	$y=-x$	(x,y)
1	$-(1)=-1$	(1,-1)
2	$-(2)=-2$	(2,-2)

Point of intersection is a solution set

Solution Set $=\{(1,-1)\}$

Review Exercise 8

Q.1 Choose the correct answer

(i) **If** $(x-1, y+1) = (0,0)$, **Then** (x, y) **is**

(a) (1,-1) **(b)** (-1,1)
(c) (1,1) **(d)** (-1,-1)

(ii) **If** $(x,0) = (0, y)$ **Then** (x, y) **is**

(a) (0,1) **(b)** (1,0)
(c) (0,0) **(d)** (1,1)

(iii) **Point** $(2,-3)$ **lies in quadrant**

(a) I **(b)** II
(c) III **(d)** IV

(iv) **Point** $(-3,-3)$ **lies in quadrant**

(a) I **(b)** II
(c) III **(d)** IV

(v) **If** $y = 2x+1, x = 2$ **Then** y **is**

(a) 2 **(b)** 3
(c) 4 **(d)** 5

(vi) **Which order pair satisfy the equation** $y = 2x$

(a) (1,2) **(b)** (2,1)
(c) (2,2) **(d)** (0,1)

ANSWER KEYS

1	2	3	4	5	6
a	c	d	c	d	a

Q.2 Identify the following statement as true or false

1. The point O(0,0) is in quadrant II — False
2. The point p (2,0) lies on x-axis — True
3. The graph of x=-2 is a vertical line — True
4. 3-y =0 is a horizontal line — True
5. The point Q (-1,2) is in quadrant II — True
6. The point R (-1,-2) is in quadrant IV — False
7. $y = x$ is a line on which origin lies — True
8. The point p (1,1) lies on the line x + y = 0 — False
9. The point S (1,-3) lies in quadrant III — False
10. The point R (0,1) lien on the x-axis — False

Q.3 Draw the following points on the graph paper

(i) $(-3,-3) =>$

(ii) $(-6,4) =>$

(iii) $(4,-5) =>$

(iv) $(5,3)$

Q.4 Draw the graph of the following

(i) $x = -6$

x	-6	-6	-6
y	1	2	3

(-6,3)
(-6,2)
(-6,1)

(ii) $y = 7$

x	1	2	3
y	7	7	7

(iii) $x = \frac{5}{2}$

$x = 2.5$

x	2.5	2.5	2.5
y	1	2	3

(iv) $y = -\frac{9}{2}$

$y = -4.5$

x	-1	0	1
y	-4.5	-4.5	-4.5

(v) $y = 4x$

x	0	1	2
$y = 4x$	$4\times0 = 0$	$4\times1=4$	$4\times2=8$

(vi) $y = -2x + 1$

x	0	1	2
y	1	-1	-3

Q.5 Draw the following graph

(i) $y = 0.62x$

x	$y = 0.62x$
1	$0.62 \times 1 = 0.62$
2	$0.62 \times 2 = 1.24$
3	$0.62 \times 3 = 1.86$

Scale
Along *x-axis*
1 Big Square= 1 Unit
Along *y-axis*
1 Big Square = 0.62 Units

(ii) $y = 2.5x$

x	$y = 2.5x$
1	$2.5(1) = 2.5$
2	$2.5(2) = 5.0$
3	$2.5(3) = 7.5$

Scale

Along *x-axis*

1 Big Square= 1 Unit

Along *y-axis*

1 Big Square = 2.5 Units

Q.6

(i) $x - y = 1$ $\qquad\qquad$ $x + y = \frac{1}{2}$

$x - 1 = y$ $\qquad\qquad$ $y = \frac{1}{2} - x$

or $y = x - 1$ $\qquad\qquad$ $y = \frac{1-2x}{2}$

x	$y = x - 1$
1	$1-1 = 0$
2	$2-1=1$
3	$3-1=2$

x	$y = \frac{1-x}{2}$
1	$\frac{1-2}{2} = -\frac{1}{2}$
3	$\frac{1-6}{2} = \frac{-5}{2}$
5	$\frac{1-10}{2} = \frac{-9}{2}$

Point of intersection is a solution set

Solution Set $= \left\{ \left(\frac{3}{4}, -\frac{1}{4} \right) \right\}$

(ii) $x = 3y$

$y = \frac{1}{3}x$

x	$y = \frac{1}{3}x$
3	$\frac{1}{\cancel{3}} \times \cancel{3} = 1$
6	$\frac{1}{\cancel{3}} \times \cancel{6}^2 = 2$

$$2x - 3y = -6$$
$$2x + 6 = 3y$$
$$\frac{2x+6}{3} = y$$
$$y = \frac{2x+6}{3}$$

Point of intersection is a solution set

Solution Set $= \{(-6, -2)\}$

x	$y = \frac{2x+6}{3}$
0	$\frac{2(0)+6}{3} = \frac{\cancel{6}^2}{\cancel{3}} = 2$
3	$\frac{2(3)+6}{3} = \frac{\cancel{12}^4}{\cancel{3}} = 4$
6	$\frac{2(6)+6}{3} = \frac{\cancel{18}^6}{\cancel{3}} = 6$

(iii) $\frac{1}{3}(x+y)=2 \qquad \frac{1}{2}(x-y)=-1$

$x+y=6 \qquad x-y=-2$

$y=6-x \qquad x+2=y$

x	y=6-x
1	6–1 = 5
2	6–2=4
3	6–3=3

x	y= x+2
1	1+2 = 3
2	2+2 = 4
3	3+2 = 5

Point of intersection is a solution set

Solution Set $=\{(2,4)\}$

Unit 8: Linear Graph & Their Application

Overview

Ordered pair:

An ordered pair of real numbers x and y is a pair (x, y) in which elements are written in specific order.

For example $(2,3)$, $(-1,-3)$

Cartesian Plane:

In plane two mutually perpendicular straight lines are drawn. The lines are called the coordinate axes. The point O, where the two lines meet is called origin. This plane is called the coordinate plane or the Cartesian plane.

Abscissa:

First value of the order pair (x, y) is called abscissa.

Ordinate:

Second value of the order pair (x, y) is called ordinate.

For Example $(5,-3)$

5 is abscissa and -3 is an ordinate

Exercise 9.1

Q.1 Find the distance between the following pairs of points

Solution:

(a) $A(9,2), B(7,2)$

Distance $= \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{|7-9|^2 + |2-2|^2}$

$|AB| = \sqrt{(-2)^2 + (0)^2}$

$|AB| = \sqrt{4}$

$|AB| = 2$

(b) $A(2,-6), B(3,-6)$

Distance $= \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{|3-2|^2 + |-6-(-6)|^2}$

$|AB| = \sqrt{(1)^2 + (-6+6)^2}$

$|AB| = \sqrt{1 + (0)^2}$

$|AB| = \sqrt{1}$

$AB = 1$

(c) $A(-8,1), B(6,1)$

Distance $= \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{|6-(-8)|^2 + |1-1|^2}$

$|AB| = \sqrt{(6+8)^2 + (0)^2}$

$|AB| = \sqrt{(14)^2}$

$|AB| = 14$

(d) $A(-4, \sqrt{2}), B(-4,-3)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{|-4-(-4)|^2 + |-3-\sqrt{2}|^2}$

$|AB| = \sqrt{(-4+4)^2 + \left(-(3+\sqrt{2})\right)^2}$

$|AB| = \sqrt{(0)^2 + (3+\sqrt{2})^2}$

$|AB| = \sqrt{(3+\sqrt{2})^2}$

$|AB| = 3 + \sqrt{2}$

(e) $A(3,-11), B(3,-4)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{|3-3|^2 + |-4-(-11)|^2}$

$|AB| = \sqrt{(0)^2 + (-4+11)^2}$

$|AB| = \sqrt{(7)^2}$

$|AB| = 7$

(f) $A(0,0), B(0,-5)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{|0-0|^2 + |-5-0|^2}$

$|AB| = \sqrt{(-5)^2}$

$|AB| = \sqrt{25}$

$|AB| = 5$

Q.2 Let P be the print on x-axis with x-coordinate a and Q be the point on y-axis with y coordinate b as given below. Find the distance between P and Q

Solution:

(i) $a = 9, b = 7$

P is (9, 0) and Q (0, 7)

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|PQ| = \sqrt{|0-9|^2 + |7-0|^2}$

$|PQ| = \sqrt{(-9)^2 + (7)^2}$

$|PQ| = \sqrt{81+49}$

$|PQ| = \sqrt{130}$

(ii) $a = 2, b = 3$

$P(2,0), Q(0,3)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|PQ| = \sqrt{|0-2|^2 + |3-0|^2}$

$|PQ| = \sqrt{(-2)^2 + (3)^2}$

$|PQ| = \sqrt{4+9}$

$|PQ| = \sqrt{13}$

(iii) $a = -8, b = 6$

$P(-8,0), Q(0,6)$

$|d| = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|PQ| = \sqrt{|0-(-8)|^2 + |6-0|^2}$

$|PQ| = \sqrt{(8)^2 + (6)^2}$

$|PQ| = \sqrt{64+36}$

$|PQ| = \sqrt{100}$

$|PQ| = 10$

(iv) $a = -2, b = -3$

$P(-2\ ,0)\ ,Q(0,-3)$

$|d| = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$d = \sqrt{|0-(-2)|^2 + |-3-0|^2}$

$d = \sqrt{(2)^2 + (-3)^2}$

$d = \sqrt{4+9}$

$d = \sqrt{13}$

(v) $a = \sqrt{2}, b = 1$

$P(\sqrt{2},0), Q(0,1)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$d = \sqrt{|0-\sqrt{2}|^2 + |1-0|^2}$

$d = \sqrt{(-\sqrt{2})^2 + (1)^2}$

$d = \sqrt{2+1}$

$d = \sqrt{3}$

(vi) $a = -9, b = -4$

$P(-9,0), Q(0,-4)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|PQ| = \sqrt{|0-(-9)|^2 + |-4-0|^2}$

$|PQ| = \sqrt{(9)^2 + (-4)^2}$

$|PQ| = \sqrt{81+16}$

$|PQ| = \sqrt{97}$

Exercise 9.2

Q.1 Show whether the points with vertices (5,-2),(5,4) and (-4,1) are the vertices of equilateral triangle or an isosceles triangle

$P(5,-2), Q(5,4), R(-4,1)$

Solution:

We know that the distance formula is

$=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$

We have $P(5,-2), Q(5,4)$

$|PQ|=\sqrt{|5-5|^2+|4-(-2)|^2}$

$|PQ|=\sqrt{(0)^2+(4+2)^2}$

$|PQ|=\sqrt{(6)^2}$

$|PQ|=6$

$Q(5,4), R(-4,1)$

$|QR|=\sqrt{|-4-5|^2+|1-4|^2}$

$|QR|=\sqrt{(-9)^2+(-3)^2}$

$|QR|=\sqrt{81+9}$

$|QR|=\sqrt{90}$

$|QR|=\sqrt{9\times10}=3\sqrt{10}$

$R(-4,1), P(5,-2)$

$|RP|=\sqrt{|5-(-4)|^2+|-2-1|^2}$

$|RP|=\sqrt{(5+4)^2+(-3)^2}$

$|RP|=\sqrt{(9)^2+9}$

$|RP|=\sqrt{81+9}$

$|RP|=\sqrt{90}$

$|RP|=\sqrt{9\times10}=3\sqrt{10}$

$|QR|=|PR|$

Two lengths of triangle are equal

So it is a isosceles triangle

Q.2 Show whether or not the points with vertices $(-1,1),(2,-2)$ and $(-4,1)$ form a Square

Solution:

$P(-1,1)Q(5,4)R(2,-2)S(-4,1)$

Distance $=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$

$|PQ|=\sqrt{\left|5-(-1)^2\right|+|4-1|^2}$

$|PQ|=\sqrt{|5+1|^2+|3|^2}$

$|PQ|=\sqrt{6^2+9}$

$|PQ|=\sqrt{36+9}$

$|PQ|=\sqrt{45}$

$|PQ|=\sqrt{9\times5}$

$|PQ|=3\sqrt{5}$

$|QR|=\sqrt{|2-5|^2+|-2-4|^2}$

$|QR| = \sqrt{(-3)^2 + (6)^2}$

$|QR| = \sqrt{9+36}$

$|QR| = \sqrt{45}$

$|QR| = \sqrt{9\times 5}$

$|QR| = 3\sqrt{5}$

$|RS| = \sqrt{|-4-2|^2 + |1-(-2)|^2}$

$|RS| = \sqrt{(-6)^2 + (1+2)^2} = \sqrt{36+(3)^2}$

$|RS| = \sqrt{36+9}$

$|RS| = \sqrt{45}$

$|RS| = \sqrt{9\times 5}$

$|RS| = 3\sqrt{5}$

$|SP| = \sqrt{|-4-(-1)|^2 + |1-1|^2}$

$|SP| = \sqrt{(-4+1)^2 + (0)^2}$

$|SP| = \sqrt{(-3)^2}$

$|SP| = \sqrt{9}$

$|SP| = 3$

If all the length are same then it will be a Square all the length are not equal so it is not square.

$|PQ| = |QR| = |RS| \neq |SP|$

Q.3 Show whether or not the points with coordinates $(1,3), (4,2)$ and $(-2,6)$ are vertices of a right triangle?

Solution:

$A(1,3), B(4,2), C(-2,6)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{|4-1|^2 + |2-3|^2}$

$|AB| = \sqrt{(3)^2 + (-1)^2}$

$|AB| = \sqrt{9+1}$

$|AB| = \sqrt{10}$

$|BC| = \sqrt{|-2-4|^2 + |6-2|^2}$

$|BC| = \sqrt{(-6)^2 + (4)^2}$

$|BC| = \sqrt{36+16}$

$|BC| = \sqrt{52}$

$|CA| = \sqrt{|-2-1|^2 + |6-3|^2} = \sqrt{(-3)^2 + (3)^2}$

$|CA| = \sqrt{9+9}$

$|CA| = \sqrt{18}$

By Pythagoras theorem

$(\text{Hyp})^2 = (\text{Base})^2 + (\text{Perp})^2$

$(\sqrt{52})^2 = (\sqrt{18})^2 + (\sqrt{10})^2$

$52 = 18 + 10$
52=28
Since $52 \neq 28$
So it not right angle triangle.

Q.4 Use distance formula to prove whether or not the points $(1,1), (-2,-8)$ and $(4,10)$ lie on a straight line?

Solution:

$A(1,1), B(-2,-8), C(4,10)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{|-2-1|^2 + |-8-1|^2}$

$|AB| = \sqrt{(-3)^2 + (-9)^2}$

$|AB| = \sqrt{9+81}$

$|AB| = \sqrt{90}$

$|AB| = \sqrt{9 \times 10}$

$|AB| = 3\sqrt{10}$

$|BC| = \sqrt{|4-(-2)|^2 + |10-(-8)|^2}$

$|BC| = \sqrt{(4+2)^2 + (10+8)^2}$

$|BC| = \sqrt{(6)^2 + (18)^2}$

$|BC| = \sqrt{36+324}$

$|BC| = \sqrt{360}$

$|BC| = \sqrt{36 \times 10}$

$|BC| = 6\sqrt{10}$

$|AC| = \sqrt{|4-1|^2 + |10-1|^2}$

$|AC| = \sqrt{(3)^2 + (9)^2}$

$|AC| = \sqrt{9+81}$

$|AC| = \sqrt{90}$

$|AC| = \sqrt{9 \times 10}$

$|AC| = 3\sqrt{10}$

$|AC| + |AB| = |BC|$

$3\sqrt{10} + 3\sqrt{10} = 6\sqrt{10}$

$6\sqrt{10} = 6\sqrt{10}$

It means that they lie on same line so they are collinear.

Q.5 Find K given that point (2, K) is equidistance from (3, 7) and (9,1)

Solution: $M(2,K), A(3,7)$ and $B(9,1)$

(3,7) A (2,K) M (9,1) B

$|\overline{AM}| = |\overline{BM}|$

$\sqrt{|2-3|^2 + |K-7|^2} = \sqrt{|9-2|^2 + |1-K|^2}$

$\sqrt{(-1)^2 + (K-7)^2} = \sqrt{(7)^2 + (1-K)^2}$

Taking square on both Side

$\left(\sqrt{1+K^2+49-14K}\right)^2 = \left(\sqrt{49+1+K^2-2K}\right)^2$

$K^2 - 14K + 50 = 50 + K^2 - 2K$

$\cancel{K^2} - 14K \cancel{+50} \cancel{-50} \cancel{-K^2} + 2K = 0$

$-12K = 0$

$K = \frac{0}{-12}$

$K = 0$

Q.6 Use distance formula to verify that the points $A(0,7), B(3,-5), C(-2,15)$ are Collinear.

Solution:

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{|3-0|^2 + |-5-7|^2}$

$|AB| = \sqrt{(3)^2 + (-12)^2}$

$|AB| = \sqrt{9+144}$

$|AB| = \sqrt{153}$

$|AB| = \sqrt{9 \times 17}$

$52 = 18 + 10$
$52 = 28$
Since $52 \neq 28$
So it not right angle triangle.

Q.4 Use distance formula to prove whether or not the points $(1,1), (-2,-8)$ and $(4,10)$ lie on a straight line?

Solution:

$A(1,1), B(-2,-8), C(4,10)$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|AB| = \sqrt{|-2-1|^2 + |-8-1|^2}$$

$$|AB| = \sqrt{(-3)^2 + (-9)^2}$$

$$|AB| = \sqrt{9+81}$$

$$|AB| = \sqrt{90}$$

$$|AB| = \sqrt{9 \times 10}$$

$$|AB| = 3\sqrt{10}$$

$$|BC| = \sqrt{|4-(-2)|^2 + |10-(-8)|^2}$$

$$|BC| = \sqrt{(4+2)^2 + (10+8)^2}$$

$$|BC| = \sqrt{(6)^2 + (18)^2}$$

$$|BC| = \sqrt{36+324}$$

$$|BC| = \sqrt{360}$$

$$|BC| = \sqrt{36 \times 10}$$

$$|BC| = 6\sqrt{10}$$

$$|AC| = \sqrt{|4-1|^2 + |10-1|^2}$$

$$|AC| = \sqrt{(3)^2 + (9)^2}$$

$$|AC| = \sqrt{9+81}$$

$$|AC| = \sqrt{90}$$

$$|AC| = \sqrt{9 \times 10}$$

$$|AC| = 3\sqrt{10}$$

$$|AC| + |AB| = |BC|$$

$$3\sqrt{10} + 3\sqrt{10} = 6\sqrt{10}$$

$$6\sqrt{10} = 6\sqrt{10}$$

It means that they lie on same line so they are collinear.

Q.5 Find K given that point (2, K) is equidistance from (3, 7) and (9,1)

Solution: $M(2,K), A(3,7)$ and $B(9,1)$

(3,7) A ——— (2,K) M ——— (9,1) B

$$|\overline{AM}| = |\overline{BM}|$$

$$\sqrt{|2-3|^2 + |K-7|^2} = \sqrt{|9-2|^2 + |1-K|^2}$$

$$\sqrt{(-1)^2 + (K-7)^2} = \sqrt{(7)^2 + (1-K)^2}$$

Taking square on both Side

$$\left(\sqrt{1+K^2+49-14K}\right)^2 = \left(\sqrt{49+1+K^2-2K}\right)^2$$

$$K^2 - 14K + 50 = 50 + K^2 - 2K$$

$$\cancel{K^2} - 14K \cancel{+50} \cancel{-50} \cancel{-K^2} + 2K = 0$$

$$-12K = 0$$

$$K = \frac{0}{-12}$$

$$K = 0$$

Q.6 Use distance formula to verify that the points $A(0,7), B(3,-5), C(-2,15)$ are Collinear.

Solution:

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|AB| = \sqrt{|3-0|^2 + |-5-7|^2}$$

$$|AB| = \sqrt{(3)^2 + (-12)^2}$$

$$|AB| = \sqrt{9+144}$$

$$|AB| = \sqrt{153}$$

$$|AB| = \sqrt{9 \times 17}$$

$$|AB| = 3\sqrt{17}$$

$$|BC| = \sqrt{|-2-3|^2 + |15-(-5)|^2}$$

$$|BC| = \sqrt{(-5)^2 + (15+5)^2}$$

$$|BC| = \sqrt{25 + (20)^2}$$

$$|BC| = \sqrt{25 + 400}$$

$$|BC| = \sqrt{425}$$

$$|BC| = \sqrt{25 \times 17}$$

$$|BC| = 5\sqrt{17}$$

$$|AC| = \sqrt{|-2-0|^2 + |15-7|^2}$$

$$|AC| = \sqrt{(-2)^2 + (8)^2}$$

$$|AC| = \sqrt{4 + 64}$$

$$|AC| = \sqrt{68}$$

$$|AC| = \sqrt{4 \times 17}$$

$$|AC| = 2\sqrt{17}$$

$$|AB| + |AC| = |BC|$$

$$3\sqrt{17} + 2\sqrt{17} = 5\sqrt{17}$$

$$5\sqrt{17} = 5\sqrt{17}$$

L.H.S = R.H.S So

They lie on same line and they are collinear.

Q.7 Verify whether or not the points $O(0,0)$ $A(\sqrt{3},1), B(\sqrt{3},-1)$ are the vertices of an equilateral triangle

Solution:

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|OA| = \sqrt{|\sqrt{3}-0|^2 + |0-1|^2}$$

$$|OA| = \sqrt{(\sqrt{3})^2 + (-1)^2}$$

$$|OA| = \sqrt{3+1}$$

$$|OA| = \sqrt{4}$$

$$|OA| = 2$$

$$|OB| = \sqrt{|\sqrt{3}-0|^2 + |-1-0|^2}$$

$$|OB| = \sqrt{(\sqrt{3})^2 + (-1)^2}$$

$$|OB| = \sqrt{3+1}$$

$$|OB| = \sqrt{4}$$

$$|OB| = 2$$

$$|AB| = \sqrt{|\sqrt{3} - \sqrt{3}|^2 + |-1-1|^2}$$

$$|AB| = \sqrt{0 + (-2)^2}$$

$$|AB| = \sqrt{4}$$

$$|AB| = 2$$

All the sides are same in length so it is equilateral triangle

Q.8 Show that the points $A(-6,-5), B(5,-5), C(5,-8)$ and $D(-6,-8)$ are the vertices of a rectangle find the length of its diagonals are equal

Solution:

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$A(-6,-5), B(5,-5)$$

$$|AB| = 3\sqrt{17}$$

$$|BC| = \sqrt{|-2-3|^2 + |15-(-5)|^2}$$

$$|BC| = \sqrt{(-5)^2 + (15+5)^2}$$

$$|BC| = \sqrt{25 + (20)^2}$$

$$|BC| = \sqrt{25+400}$$

$$|BC| = \sqrt{425}$$

$$|BC| = \sqrt{25\times17}$$

$$|BC| = 5\sqrt{17}$$

$$|AC| = \sqrt{|-2-0|^2 + |15-7|^2}$$

$$|AC| = \sqrt{(-2)^2 + (8)^2}$$

$$|AC| = \sqrt{4+64}$$

$$|AC| = \sqrt{68}$$

$$|AC| = \sqrt{4\times17}$$

$$|AC| = 2\sqrt{17}$$

$$|AB| + |AC| = |BC|$$

$$3\sqrt{17} + 2\sqrt{17} = 5\sqrt{17}$$

$$5\sqrt{17} = 5\sqrt{17}$$

L.H.S = R.H.S So

They lie on same line and they are collinear.

Q.7 Verify whether or not the points $O(0,0)$ $A(\sqrt{3},1), B(\sqrt{3},-1)$ are the vertices of an equilateral triangle

Solution:

$$d = \sqrt{|x_2-x_1|^2 + |y_2-y_1|^2}$$

$$|OA| = \sqrt{|\sqrt{3}-0|^2 + |0-1|^2}$$

$$|OA| = \sqrt{(\sqrt{3})^2 + (-1)^2}$$

$$|OA| = \sqrt{3+1}$$

$$|OA| = \sqrt{4}$$

$$|OA| = 2$$

$$|OB| = \sqrt{|\sqrt{3}-0|^2 + |-1-0|^2}$$

$$|OB| = \sqrt{(\sqrt{3})^2 + (-1)^2}$$

$$|OB| = \sqrt{3+1}$$

$$|OB| = \sqrt{4}$$

$$|OB| = 2$$

$$|AB| = \sqrt{|\sqrt{3}-\sqrt{3}|^2 + |-1-1|^2}$$

$$|AB| = \sqrt{0+(-2)^2}$$

$$|AB| = \sqrt{4}$$

$$|AB| = 2$$

All the sides are same in length so it is equilateral triangle

Q.8 Show that the points $A(-6,-5), B(5,-5), C(5,-8)$ and $D(-6,-8)$ are the vertices of a rectangle find the length of its diagonals are equal

Solution:

$$d = \sqrt{|x_2-x_1|^2 + |y_2-y_1|^2}$$

$$A(-6,-5), B(5,-5)$$

Recognition of the midpoint formula for any two points in the plane

Let $P_1(x, y)$ and $P_2(x_2, y_2)$ be any two points in the plane and $R(x, y)$ be midpoint of point P_1 and P_2 on the line segment P_1P_2 as shown in the figure.

If the line segment MN, parallel to *x-axis* has its midpoint $R(x, y)$,

then, $x_2 - x = x - x_1$

$$x_2 + x_1 = x + x$$

$$2x = x_1 + x_2 \Rightarrow x = \frac{x_1 + x_2}{2}$$

Similarly, $\mathrm{y} = \dfrac{\mathrm{y}_1 + \mathrm{y}_2}{2}$

Thus the point R (x, y)

$= R\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ is the midpoint of the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$

Verification of the midpoint formula

$$|P_1R| = \sqrt{\left(\frac{x_1 + x_2}{2} - x_1\right)^2 + \left(\frac{y_1 + y_2}{2} - y_1\right)^2}$$

$$|P_1R| = \sqrt{\left(\frac{x_1 + x_2 - 2x_1}{2}\right)^2 + \left(\frac{y_1 + y_2 - 2y_1}{2}\right)^2}$$

$$|P_1R| = \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2}$$

$$|P_1R| = \sqrt{\frac{(x_2 - x_1)^2}{4} + \frac{(y_2 - y_1)^2}{4}}$$

$$|P_1R| = \sqrt{\frac{(x_2 - x_1)^2 + (y_2 - y_1)^2}{4}}$$

$$|P_1R| = \sqrt{\frac{(x_2 - x_1)^2 + (y_2 - y_1)^2}{2}}$$

OR

$$|P_1R| = \frac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \frac{1}{2}|P_1P_2|$$

and $|P_2R| = \sqrt{\left(\dfrac{x_1 + x_2}{2} - x_2\right)^2 + \left(\dfrac{y_1 + y_2}{2} - y_2\right)^2}$

$$|P_2R| = \sqrt{\left(\frac{x_1 + x_2 + 2x_2}{2}\right)^2 + \left(\frac{y_1 + y_2 - 2y_2}{2}\right)^2}$$

$$|P_2R| = \sqrt{\left(\frac{x_1 - x_2}{2}\right)^2 + \left(\frac{y_1 - y_2}{2}\right)^2}$$

$$|P_2R| = \sqrt{\frac{(x_1 - x_2)^2}{4} + \frac{(y_1 - y_2)^2}{4}}$$

$$|P_2R| = \sqrt{\frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{4}}$$

$$|P_2R| = \frac{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{2}$$

OR

$$|P_2R| = \frac{1}{2}\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$\Rightarrow |P_2R| = |P_1R| = \frac{1}{2}|P_1R_2|$$

Thus it verifies that $R\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ is the midpoint of the line segment P_1RP_2 which lies on the line segment since $|P_1R| + |P_2R| = |P_1P_2|$

Exercise 9.3

Q.1 Find the midpoint of the line Segments joining each of the following pairs of points

Solution:

(a) $A(9,2), B(7,2)$

Let $M(x,y)$ the midpoint of AB

$(x,\text{y})=\left(\frac{x_1+x_2}{2},\frac{\text{y}_1+\text{y}_2}{2}\right)$

Midpoint formula

$M(x,y)=M\left(\frac{9+7}{2},\frac{2+2}{2}\right)$

$=M\left(\frac{\cancel{16}^{8}}{\cancel{2}},\frac{\cancel{4}^{2}}{\cancel{2}}\right)$

$=M(8,2)$

(b) $A(2,-6), B(3,-6)$

Let $M(x,y)$ the point of AB

$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

Midpoint formula

$M(x,y)=M\left(\frac{2+3}{2},\frac{-6-6}{2}\right)$

$M(x,y)=M\left(\frac{5}{2},\frac{-\cancel{12}^{6}}{\cancel{2}}\right)$

$M(x,y)=M(2.5,-6)$

(c) $A(-8,1), B(6\ ,1)$

Let $M(x,y)$ midpoint of AB

$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

Formula

$M(x,y)=M\left(\frac{-8+6}{2},\frac{1+1}{2}\right)$

$M(x,y)=M\left(\frac{-\cancel{2}}{\cancel{2}},\frac{\cancel{2}}{\cancel{2}}\right)$

$M(x,y)=M(-1,1)$

(d) A (-4 , 9) , B (-4 , -3)

Let $M(x,y)$ midpoint of AB

$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ Formula

$M(x,y)=M\left(\frac{-4-4}{2},\frac{9-3}{2}\right)$

$M(x,y)=M\left(\frac{-\cancel{8}^{4}}{\cancel{2}},\frac{\cancel{6}^{3}}{\cancel{2}}\right)$

$M(x,y)=M(-4,3)$

(e) $A(3,11), B(3,-4)$

Let $M(x,y)$ is the midpoint of AB

$M(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$M(x,y)=M\left(\frac{3+3}{2},\frac{-11-4}{2}\right)$

$M(x,y)=M\left(\frac{6}{2},\frac{-15}{2}\right)$

$M(x,y)=M(3,-7.5)$

(f) A (0, 0) , B (0, -5)

Let $M(x,y)$ is the midpoint of AB

$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$M(x,y)=M\left(\frac{0+0}{2},\frac{0-5}{2}\right)$

$M(x,y)=M\left(\frac{0}{2},\frac{-5}{2}\right)$

$=M(0,-2.5)$

Q.2 The end point of line segment PQ is $(-3,6)$ and its midpoint is $(5,8)$ find the coordinates of the end point Q

Solution:

P (-3,6) ———— M (5,8) ———— Q (x,y)

Let Q be the point (x,y), $M(5,8)$ is the midpoint of PQ

$M(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$$x=\frac{x_1+x_2}{2}$$
$$5=\frac{-3+x}{2}$$
$$5\times 2=-3+x$$
$$10+3=x$$
$$x=13$$
$$y=\frac{y_1+y_2}{2}$$
$$8=\frac{6+y}{2}$$
$$2\times 8=6+y$$
$$16-6=y$$
$$y=10$$
Hence point Q is $(13,10)$

Q.3 Prove that midpoint of the hypotenuse of a right triangle is equidistance from it three vertices $P(-2,5), Q(1,3)$ and $R(-1,0)$

Solution:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$
$$d=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$$
$$P(-2,5),Q(1,3)$$
$$|PQ|=\sqrt{|-2-1|^2+|5-3|^2}$$
$$|PQ|=\sqrt{(-3)^2+(2)^2}$$
$$|PQ|=\sqrt{9+4}$$
$$|PQ|=\sqrt{13}$$
$$Q(1,3),R(-1,0)$$
$$|QR|=\sqrt{|1-(-1)|^2+|3-0|^2}$$
$$|QR|=\sqrt{(1+1)^2+(3)^2}$$
$$|QR|=\sqrt{(2)^2+9}=\sqrt{4+9}$$
$$|QR|=\sqrt{13}$$
$$P(-2,5),R(-1,0)$$
$$|PR|=\sqrt{|-2-(-1)|^2+|5-0|^2}$$
$$|PR|=\sqrt{|-2+1|^2+|5|^2}$$
$$|PR|=\sqrt{(-1)^2+(5)^2}=\sqrt{1+25}$$
$$|PR|=\sqrt{26}$$

To find the length of hypotenuse and whether it is right angle triangle we use the Pythagoras theorem

$$(PR)^2=(PQ)^2+(QR)^2$$
$$\left(\sqrt{26}\right)^2=\left(\sqrt{13}\right)^2+\left(\sqrt{13}\right)^2$$
$$26 = 13 + 13$$
$$26 = 26$$

It is a right angle triangle and PR is hypotenuse

$$P(-2,5),R(-1,0)$$

Midpoint of PR

$$M(x,y)=\left(\frac{-2-1}{2},\frac{5+0}{2}\right)$$
$$M(x,y)=\left(\frac{-3}{2},\frac{5}{2}\right)$$
$$MP=MR$$
$$M\left(\frac{-3}{2},\frac{5}{2}\right),P(-2,5),R(-1,0)$$
$$|MP|=|MR|$$

(i)

$$|MP|=\sqrt{\left|\frac{-3}{2}-(-2)\right|^2+\left|\frac{5}{2}-5\right|^2}$$
$$=\sqrt{\left(\frac{-3}{2}+2\right)^2+\left(\frac{5-10}{2}\right)^2}$$
$$|MP|=\sqrt{\left(\frac{-3+4}{2}\right)^2+\left(\frac{-5}{2}\right)^2}$$
$$=\sqrt{\left(\frac{1}{2}\right)^2+\frac{25}{4}}$$
$$|MP|=\sqrt{\frac{1}{4}+\frac{25}{4}}=\sqrt{\frac{1+25}{4}}$$
$$|MP|=\sqrt{\frac{26}{4}}$$
$$|MP|=\frac{\sqrt{26}}{2}$$

(ii) $M\left(\frac{-3}{2},\frac{5}{2}\right),R(-1,0)$

$$|MR|=\sqrt{\left|\frac{-3}{2}-(-1)\right|^2+\left|\frac{5}{2}-0\right|^2}$$

$$|M\ R| = \sqrt{\left(\frac{-3}{2}+1\right)^2 + \left(\frac{5}{2}\right)^2}$$

$$|M\ R| = \sqrt{\left(\frac{-3+2}{2}\right)^2 + \frac{25}{4}}$$

$$= \sqrt{\left(\frac{-1}{2}\right)^2 + \frac{25}{4}}$$

$$|M\ R| = \sqrt{\frac{1}{4}+\frac{25}{4}}$$

$$|M\ R| = \sqrt{\frac{1+25}{4}} = \sqrt{\frac{26}{4}}$$

$$|M\ R| = \frac{\sqrt{26}}{2}$$

(iii) $M\left(\frac{-3}{2}, \frac{5}{2}\right)$

$Q(1,3)$

$$|MQ| = \sqrt{\left(\frac{-3}{2}-1\right)^2 + \left(\frac{5}{2}-3\right)^2}$$

$$= \sqrt{\left(\frac{-3-2}{2}\right)^2 + \left(\frac{5-6}{2}\right)^2}$$

$$= \sqrt{\left(\frac{-5}{2}\right)^2 + \left(\frac{-1}{2}\right)^2}$$

$$= \sqrt{\frac{25}{4}+\frac{1}{4}} = \sqrt{\frac{26}{4}}$$

Hence proved $MP = MR = |MQ|$

Q.4 If $O(0,0)$, $A(3,0)$ and $B(3,5)$ are three points in the plane find M_1 and M_2 as the midpoint of the line segments AB and OB respectively find $|M_1M_2|$

Solution:

M_1 is the midpoint of AB

$$M_1(x,y) = M_1\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$A(3,0), B(3,5)$

$$M_1\left(\frac{3+3}{2}, \frac{0+5}{2}\right)$$

$$M_1\left(\frac{6}{2}, \frac{5}{2}\right)$$

$$M_1\left(3, \frac{5}{2}\right)$$

M_2 is the midpoint of OB

$$M_2\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$0(0,0), B(3,5)$

$$M_2\left(\frac{0+3}{2}, \frac{0+5}{2}\right)$$

$$M_2\left(\frac{3}{2}, \frac{5}{2}\right)$$

$$M_1\left(3, \frac{5}{2}\right) M_2\left(\frac{3}{2}, \frac{5}{2}\right)$$

$$|M_1M_2| = \sqrt{\left|\frac{3}{2}-3\right|^2 + \left|\frac{5}{2}-\frac{5}{2}\right|^2}$$

$$|M_1M_2| = \sqrt{\left(\frac{3-6}{2}\right)^2 + (0)^2}$$

$$= \sqrt{\left(\frac{-3}{2}\right)^2 + 0}$$

$$|M_1M_2| = \sqrt{\frac{9}{4}}$$

$$|M_1M_2| = \frac{3}{2}$$

Q.5 Show that the diagonals of the parallelogram having vertices $A(1,2), B(4,2), C(-1,-3)$ and $D(-4,-3)$ bisect each other.

Solution:

$ABCD$ is parallelogram which vertices are

$A(1,2), B(4,2), C(-1,-3)\ D(-4,-3)$

Let $\overline{BD}$ and $\overline{AC}$ the diagonals of parallelogram they intersect at point M

$A(1,2), C(-1,-3)$ midpoint of AC

Midpoint formula

$$M_1(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M_1(x,y)=M_1\left(\frac{1-1}{2},\frac{2-3}{2}\right)$$

$$M_1(x,y)=M_1\left(\frac{0}{2},\frac{-1}{2}\right)=\left(0,\frac{-1}{2}\right)$$

Midpoint of BD,

$$M_2(x,y)=M_2\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M_2(x,\ y)=M_2\left(\frac{4-4}{2},\frac{2-3}{2}\right)$$

$$M_2(x,y)=M_2\left(\frac{0}{2},\frac{-1}{2}\right)$$

$$M_2(x,y)=M_2\left(0,\frac{-1}{2}\right)$$

As M_1 and M_2 Coincide the diagonals of the parallelogram bisect each other.

Q.6 The vertices of a triangle are $P(4,6),Q(-2,-4)$ and $R(-8,2)$. Show that the length of the line segment joining the midpoints of the line segments $\overline{PR},\overline{QR}$ is $\frac{1}{2}\overline{PQ}$

Solution:

M_1 the midpoint of QR is

$Q(-2,-4),R(-8,2)$

$$M_1(x,y)=M_1\left(\frac{-2-8}{2},\frac{-4+2}{2}\right)$$

$$=M_1\left(\frac{-10}{2},\frac{-2}{2}\right)$$

$$=M_1(-5,-1)$$

$$M_1(-5,-1)$$

M_2 the midpoint of PR is

$P(4,6),Q(-8,+2)$

$$M_2(x,y)=M\left(\frac{4-8}{2},\frac{6+2}{2}\right)$$

$$M_2(x,y)=M_2\left(\frac{-4}{2},\frac{8}{2}\right)$$

$$M_2(x,y)=M_2(-2,4)$$

$$M_2(-2,4)$$

$$|M_1M_2|=\sqrt{|-5+2|^2+|4+1|^2}$$

$$|M_1M_2|=\sqrt{(-3)^2+(5)^2}$$

$$|M_1M_2|=\sqrt{9+25}$$

$$|M_1M_2|=\sqrt{34}$$

$$|P\ Q|=\sqrt{|4+2|^2+|6+4|^2}$$

$$|P\ Q|=\sqrt{(6)^2+(10)^2}=\sqrt{36+100}$$

$$|P\ Q|=\sqrt{136}$$

$$|PQ| = \sqrt{4 \times 34}$$

$$|PQ| = 2\sqrt{34}$$

$$\frac{|PQ|}{2} = \sqrt{34}$$

OR

$$\frac{1}{2}|PQ| = \sqrt{34}$$

Hence we proved that

$$|M_1M_2| = \frac{1}{2}|PQ|$$

Review Exercise 9

Q.1 Choose the Correct answer

(i) Distance between point (0 , 0) and (1, 1) is

(a) 0 **(b)** 1

(c) 2 **(d)** $\sqrt{2}$

(ii) Distance between the point (1 , 0) and (0 ,1) is

(a) 0 **(b)** 1

(c) $\sqrt{2}$ **(d)** 2

(iii) Midpoint of the (2, 2) and (0, 0) is

(a) (1, 1) **(b)** (1, 0)

(c) (0, 1) **(d)** (-1, -1)

(iv) Midpoint of the points (2, -2) and (-2 , 2) is

(a) (2, 2) **(b)** (-2, -2)

(c) (0 , 0) **(d)** (1, 1)

(v) A triangle having all sides equal is called

(a) Isosceles **(b)** Scalene

(c) Equilateral **(d)** None of these

(vi) A triangle having all sides different is called

(a) Isosceles **(b)** Scalene

(c) Equilateral **(d)** None of these

ANSWER KEYS

i	ii	iii	iv	v	vi
d	c	a	c	c	b

Q.2 Answer the following which is true and which is false

(i) A line has two end points (False)

(ii) A line segment has one end point (False)

(iii) A triangle is formed by the three collinear points (False)

(iv) Each side of triangle has two collinear vertices. (True)

(v) The end points of each side of a rectangle are Collinear (True)

(vi) All the points that lie on the x-axis are Collinear (True)

(vii) Origin is the only point Collinear with the points of both axis separately (True)

Q.3 Find the distance between the following pairs of points

Solution:

(i) $(6,3)(3,-3)$

$A(6,3), B(3,-3)$

$d=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$

$|AB|=\sqrt{|3-6|^2+|-3-3|^2}$

$|AB|=\sqrt{(-3)^2+(-6)^2}$

$|AB|=\sqrt{9+36}$

$|AB|=\sqrt{45}$

$|AB|=\sqrt{9\times 5}$

$|AB|=3\sqrt{5}$

(ii) $(7,5),(1,-1)$

$A(7,5), B(1,-1)$

$d=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$

$|AB|=\sqrt{|7-1|^2+|5-(-1)|^2}$

$|AB|=\sqrt{(6)^2+(5+1)^2}$

$|AB|=\sqrt{36+(6)^2}=\sqrt{36+36}$

$|AB|=\sqrt{72}=\sqrt{36\times 2}$

$|AB|=6\sqrt{2}$

(iii) $(0,0),(-4,-3)$

$A(0,0), B(-4,-3)$

$d=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$

$|AB|=\sqrt{|0-4|^2+|0-(-3)|^2}$

$|AB|=\sqrt{(-4)^2+(3)^2}$

$|AB|=\sqrt{16+9}$

$|AB|=\sqrt{25}$

$|AB|=5$

Q.4 Find the midpoint between following pairs of points

Solution:

(i) $(6,6),(4,-2)$

$M(x,y)=M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$M(x,y)=M\left(\frac{6+4}{2},\frac{6-2}{2}\right)$

$M(x,y)=M\left(\frac{10}{2},\frac{4}{2}\right)$

$M(x,y)=M(5,2)$

(ii) $(-5,-7),(-7,-5)$

$M(x,y)=M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$M(x,y)=M\left(\frac{-5-7}{2},\frac{-7-5}{2}\right)$

$M(x,y)=M\left(\frac{-12}{2},\frac{-12}{2}\right)$

$M(x,y)=M(-6,-6)$

(iii) $(8,0),(0,-12)$

$M(x,y)=M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$M(x,y)=M\left(\frac{8+0}{2},\frac{0-12}{2}\right)$

$M(x,y)=M\left(\frac{8}{2},\frac{-12}{2}\right)$

$M(x,y)=M(4,-6)$

Q.5 Define the following

Solution:

(i) Co-ordinate Geometry:-

Co-ordinate geometry is the study of geometrical shapes in the Cartesian plane (or coordinate plane)

(ii) Collinear:-

Two or more than two points which lie on the same straight line are called collinear points with respect to that line.

(iii) Non- Collinear:-

The points which do not lie on the same straight line are called non-collinear.

(iv) Equilateral Triangle:-

If the length of all three sides of a triangle are same then the triangle is called an equilateral triangle.

ΔABC is an equilateral triangle.

(v) Scalene Triangle:-

A triangle is called a scalene triangle if measure of all sides are different.

ΔABC is a Scalene triangle.

(vi) Isosceles Triangle:-

An isosceles triangles is a triangle which has two of its sides with equal length while the third side has different length.

ΔABC is an isosceles triangle

(vii) Right Triangle:-

A triangle in which one of the angles has measure equal to 90° is called a right triangle.

ΔABC is a right angled triangle.

(viii) Square:-

A Square is closed figure formed by four non- collinear points such that lengths of all sides are equal and measure of each angles is 90°.

$ABCD$ is a square.

Unit 9: Introduction to Coordinate Geometry

Overview

Coordinate Geometry:

The study of geometrical shapes in a plane is called plane geometry. Coordinate geometry is the study of geometrical shapes in the Cartesian plane (coordinate plane).

Collinear Points:

Two or more than two points which lie on the same straight line are called collinear points with respect to that line.

Non-collinear points:

Tow or more points which does not lie on the same straight line are called non-collinear points.

R

P Q m

Equilateral Triangle:

If the lengths of all the three sides of a triangle are same, then the triangle is called an equilateral triangle.

An Isosceles Triangle:

An isosceles triangle PQR is a triangle which has two of its sides with equal length while the third side has a different length.

Right Angle Triangle

A triangle in which one of the angles has measure equal to 90° is called a right angle triangle.

Scalene Triangle:-

A triangle is called a scalene triangle if measure of all sides are different.

Square:-

A Square is closed figure formed by four non- collinear points such that lengths of all sides are equal and measure of each angles is 90°.

Rectangle

A figure formed in the plane by four non-collinear points is called a rectangle if,

(i) Its opposite sides are equal in length

(ii) The angle at each vertex is of measure 90°

Parallelogram

A figure formed by four non-collinear points in the plane is called a parallelogram if

(i) Its opposite sides are of equal length

(ii) Its opposite sides are parallel

(iii) Meausre of none of the angles is 90°.

Finding distance between two points.

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points in the coordinate plane where d is the length of the line segment PQ i,e, $|\overline{PQ}| = d$

The line segments MQ and LP parallel to y-axis meet x-axis at point M and L respectively with coordinates M(x_2 , o) and L (x_1 , o)

The line segment PN is parallel to x-axis

In the right triangle PNQ $\quad |\overline{NQ}| = |y_2 - y_1|$ and $|\overline{PN}| = |x_2 - x_1|$

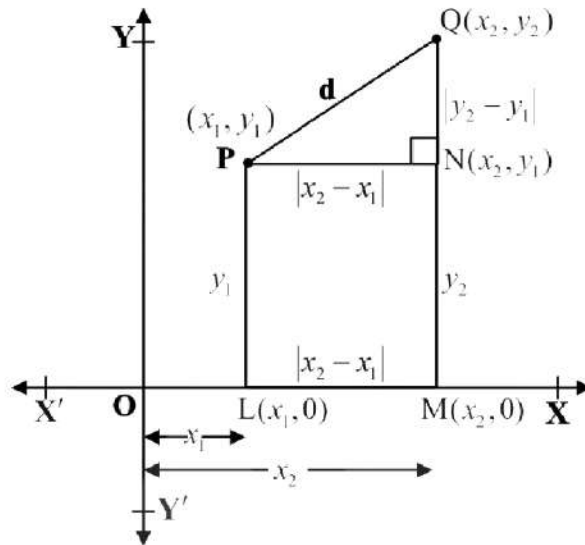
Using Pythagoras theorem

$$(\overline{PQ})^2 = (\overline{PN})^2 + (\overline{QN})^2$$

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

Taking under root on both side

$$\sqrt{d^2} = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

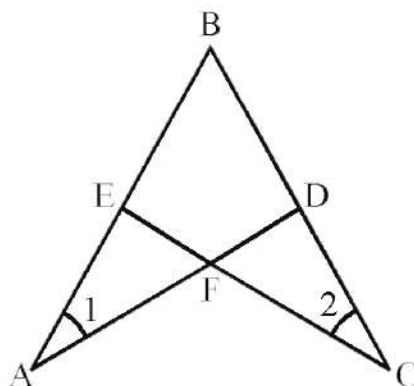
$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

Since d > 0 always

Exercise 10.1

Q.1 In the given figure

$\angle 1 \cong \angle 2$ and $\overline{AB} \cong \overline{CB}$

Prove that

$\Delta ABD \cong \Delta CBE$

Proof

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CBE$	
$\overline{AB} \cong \overline{CB}$	Given
$\angle BAD \cong \angle BCE$	Given $\angle 1 \cong \angle 2$
$\angle ABD \cong \angle CBE$	Common
$\Delta ABD \cong \Delta CBE$	S.A.A ≅ S.A.A

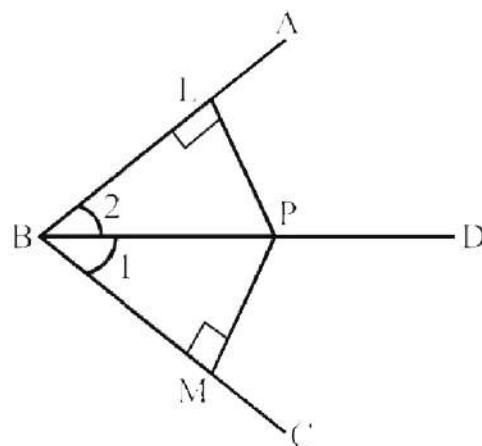
Q.2 From a point on the bisector of an angle, perpendiculars are drawn to the arms of the angle. Prove that these perpendiculars are equal in measure.

Given

$\overline{BD}$ is bisector of $\angle ABC$. P is point on $\overline{BD}$ and $\overline{PL}$ are $\overline{PM}$ are perpendicular to $\overline{AB}$ and $\overline{CB}$ respectively

To prove

$\overline{PL} \cong \overline{PM}$

Proof

Statements	Reasons
In $\Delta BLP \leftrightarrow \Delta BMP$	
$\overline{BP} \cong \overline{BP}$	Common
$\angle BLP \cong \angle BMP$	Each right angle (given)
$\angle LBP \cong \angle MBP$	Given $\overline{BD}$ is bisector of angle B
$\therefore\ \Delta BLP \cong \Delta BMP$	S.A.A ≅ S.A.A
So $\overline{PL} \cong \overline{PM}$	Corresponding sides of congruent triangles

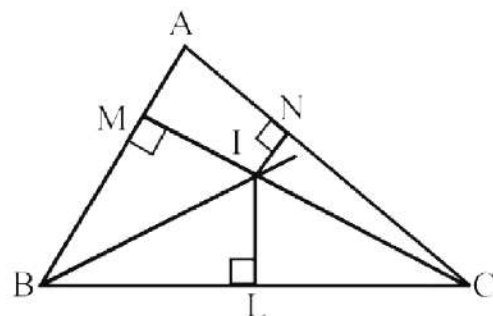
Q.3 In a triangle ABC, the bisects of $\angle B$ and $\angle C$ meet in point I prove that I is equidistant from the three sides by ΔABC

Given

In ΔABC, the bisector of $\angle B$ and $\angle C$ meet at I and $\overline{IL}$, $\overline{IM}$, and $\overline{IN}$ are perpendiculars to the three sides of ΔABC.

To prove

$\overline{IL} \cong \overline{IM} \cong \overline{IN}$

Proof

Statements	Reasons
In $\Delta ILB \leftrightarrow \Delta IMB$	
$\overline{BI} \cong \overline{BI}$	Common
$\angle IBL \cong \angle IBM$	Given BI is bisector of $\angle B$
$\angle ILB \cong \angle IMB$	Given each angle is rights angles
$\Delta ILB \cong \Delta IMB$	$SAA \cong S.A.A$
$\therefore$ $\overline{IL} \cong \overline{IM}$ _____ (i)	Corresponding sides of $\cong \Delta$'s
Similarly	
$\Delta IAC \cong \Delta INC$	
So $\overline{IL} \cong \overline{IN}$ _____ (ii)	Corresponding sides of $\cong \Delta$s
from (i) and (ii)	
$\overline{IL} \cong \overline{IM} \cong \overline{IN}$	
$\therefore$ I is equidistant from the three sides of ΔABC.	

Theorem 10.1.2

If two angles of a triangles are congruent then the sides opposite to them are also congruent

Given

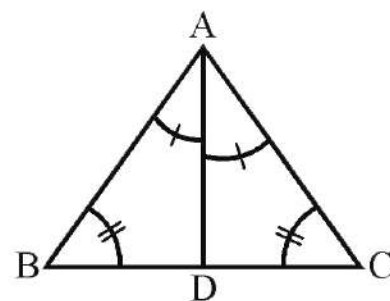
In ΔABC, $\angle B \cong \angle C$

To prove

$\overline{AB} \cong \overline{AC}$

Construction

Draw the bisector of $\angle A$, meeting $\overline{BC}$ at point D

Proof

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta ACD$	
$\overline{AD} \cong \overline{AD}$	Common
$\angle B \cong \angle C$	Given
$\angle BAD \cong \angle CAD$	Construction
$\Delta ABD \cong \Delta ACD$	$S.A.A \cong S.A.A$
Hence $\overline{AB} \cong \overline{AC}$	(Corresponding sides of congruent triangles)

Example 1

If one angle of a right triangle is of 30°, the hypotenuse is twice as long as the side opposite to the angle.

Given

In $\Delta ABC, m\angle B = 90^\circ$ and $m\angle C = 30^\circ$

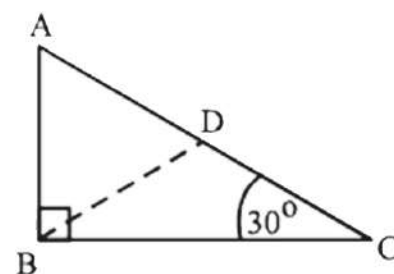
To prove

$m\overline{AC} = 2m\overline{AB}$

Constructions

At, B construct $\angle CBD$ of 30°

Let $\overline{BD}$ cut $\overline{AC}$ at the point D.

Proof

Statements	Reasons
In $\Delta ABD, m\angle A = 60^\circ$	$m\angle ABC = 90^\circ, m\angle C = 30^\circ$
$m\angle ABD = m\angle ABC, mCBD = 60^\circ$	$m\angle ABC = 90^\circ, m\angle CBD = 30^\circ$
$\therefore mADB = 60^\circ$	Sum of measures of $\angle$s of a Δ is 180°
$\therefore \Delta ABD$ is equilateral	Each of its angles is equal to 60°
$\therefore \overline{AB} \cong \overline{BD} \cong \overline{CD}$	Sides of equilateral Δ
In $\Delta BCD, \overline{BD} \cong \overline{CD}$	**$\angle C = \angle CBD$ (each of 30),**
Thus $m\overline{AC} = m\overline{AD} + m\overline{CD}$ $= m\overline{AB} + m\overline{AB}$ $= 2(m\overline{AB})$	$\overline{AD} \cong \overline{AB}$ and $\overline{CD} \cong \overline{BD} \cong \overline{AB}$

Example 2

If the bisector of an angle of a triangle bisects the side opposite to it, the triangle is isosceles.

Given

In $\Delta ABC, \overline{AD}$ bisect $\angle A$ and $\overline{BD} \cong \overline{CD}$

To prove

$\overline{AB} \cong \overline{AC}$

Construction

Produce $\overline{AD}$ to E, and take $\overline{ED} \cong \overline{AD}$

Joint C to E

Proof

Statements	Reasons
In $\Delta ADB \leftrightarrow EDC$	
$\overline{AD} \cong \overline{ED}$	Construction
$\angle ADB \cong \angle EDC$	Vertical angles
$\overline{BD} \cong \overline{CD}$	Given
$\therefore \Delta ADB \cong \Delta EDC$	S.A.S. Postulate
$\therefore \overline{AB} \cong \overline{EC}$ ...(i)	Corresponding sides
and $\angle BAD \cong \angle E$	Corresponding angles
But $\angle BAD \cong \angle CAD$	Given
$\therefore \angle E \cong \angle CAD$	Each $\cong \angle BAD$
In $\Delta ACE, \overline{AC} \cong \overline{EC}$ ...(ii)	$\angle E \cong \angle CAD$ (proved)
Hence $\overline{AB} \cong \overline{AC}$	From (i) and (ii)

Exercise 10.2

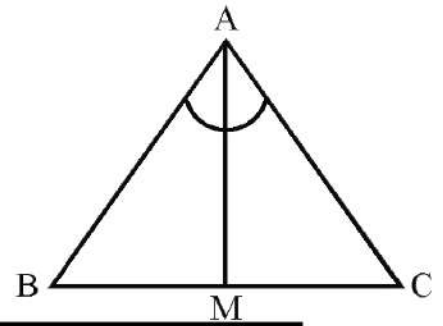
Q.1 Prove that any two medians of an equilateral triangle are equal in measure.

Given

In ΔABC, $\overline{AB} \cong \overline{AC}$ and M is midpoint of $\overline{BC}$

To prove

$\overline{AM}$ bisects $\angle A$ and $\overline{AM}$ is perpendicular to $\overline{BC}$

Proof

Statements	Reasons
In $\Delta ABM \leftrightarrow \Delta ACM$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{BM} \cong \overline{CM}$	Given M is midpoint of BC
$\overline{AM} \cong \overline{AM}$	Common
$\therefore \Delta ABM \cong \Delta ACM$	S.S.S $\cong$ S.S.S
So $\angle BAM \cong \angle CAM$	Corresponding angles of congruents triangle
$m\angle AMB + m\angle AMC = 180^o$	
$\therefore$ $m\angle AMB = m\angle AMC$	
i.e $\overline{AM}$ is perpendicular to $\overline{BC}$	

Q.2 Prove that a point which is equidistant from the end points of a line segment, is on the right bisector of line segment

Given

$\overline{AB}$ is line segment. The point C is such that $\overline{CA} \cong \overline{CB}$

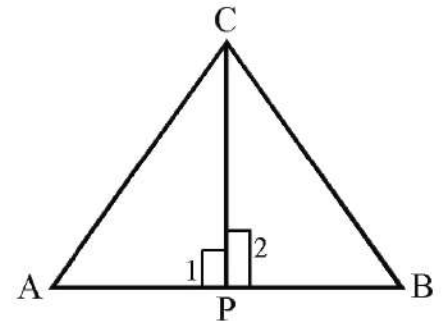
To prove

Point C lies on the right bisector of $\overline{AB}$

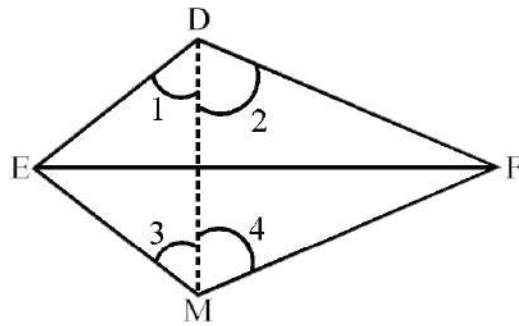
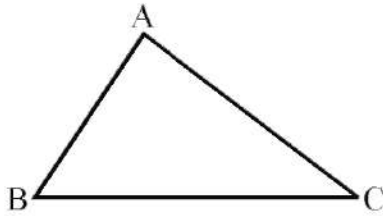
Construction

(i) Take P as midpoint of $\overline{AB}$ i.e. $\overline{AP} \cong \overline{BP}$

(ii) Joint point C to A, P, B

Proof:

Statements	Reasons
In ΔABC	
$\overline{CA} \cong \overline{CB}$	Given
$\angle A \cong \angle B$	Corresponding angles of congruent triangles
Δ CBP $\leftrightarrow \Delta CAP$	
$\overline{CB} \cong \overline{CA}$	
$\Delta CAP \cong \Delta CBP$	S.A.S $\cong$ S.A.S
$\therefore$ $\angle 1 \cong \angle 2$	
$m\angle 1 + m\angle 2 = 180^o$	Adjacent angles on one side of a line
Thus m $\angle 1 = m\angle 2 = 90$	
Hence $\overline{CP}$ is right bisector of $\overline{AB}$ and point C lies on $\overline{CB}$	

Theorem 10.1.3

In a correspondence of two triangles if three sides of one triangle are congruent to the corresponding three sides of the other. Then the two triangles are congruent (S.S.S $\cong$ S.S.S)

Given:

In $\Delta ABC \leftrightarrow \Delta DEF$

$\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}$ and $\overline{CA} \cong \overline{FD}$

To prove

$\Delta ABC \cong \Delta DEF$

Construction

Suppose that in ΔDEF the side $\overline{EF}$ is not smaller than any of the remaining two sides. On $\overline{EF}$ construct a ΔMEF in which, $\angle FEM \cong \angle B$ and $\overline{ME} \cong \overline{AB}$. Join D and M. as shown in the above figures we label some of the angles as 1, 2, 3, and 4

Proof:

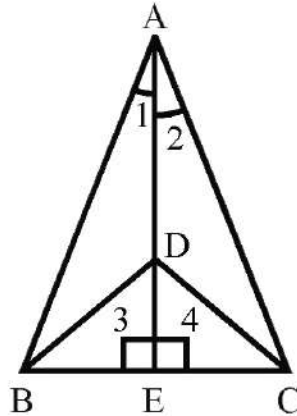
Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta MEF$	
$\overline{BC} \cong \overline{EF}$	Given
$\angle B \cong \angle FEM$	Construction
$\overline{AB} \cong \overline{ME}$	Construction
$\therefore\ \Delta ABC \cong \Delta MEF$	S.A.S Postulate
and $\overline{CA} \cong \overline{FM}$ ____ (i)	(Corresponding sides of congruent triangles)
also $\overline{CA} \cong \overline{FD}$ ___ (ii)	Given
$\therefore\ \overline{FM} \cong \overline{FD}$	{ From (i) and (ii) }
In ΔFDM	
$\angle 2 \cong \angle 4$ _____ (iii)	$\overline{FM} \cong \overline{FD}$ (proved)
Similarly $\angle 1 \cong \angle 3$ ______ (iv)	
$\therefore\ m\angle 2 + m\angle 1 = m\angle 4 + m\angle 3$	{ from (iii) and iv }
$\therefore\ m\angle EDF = m\angle EMF$	
Now in $\Delta DEF \leftrightarrow \Delta MEF$	
$\overline{FD} \cong \overline{FM}$	Proved
and $m\angle EDF \cong \angle EMF$	Proved
$\overline{DE} \cong \overline{ME}$	Each one $\cong \overline{AB}$
$\therefore\ \Delta DEF \cong \Delta MEF$	S.A.S postulates
also $\Delta ABC \cong \Delta MEF$	Proved
Hence $\Delta ABC \cong \Delta DEF$	Each $\Delta \cong \Delta MEF$ (proved)

Example 1

If two isosceles triangles are formed on the same side of their common base, the line through their vertices would be the right bisector of their common base.

Given

ΔABC and ΔDBC formed on the same side of $\overline{BA}$ such that

$\overline{BA} \cong \overline{AC}, \overline{DB} \cong \overline{DC}, \overline{AD}$ meets $\overline{BC}$ at E.

To prove

$\overline{BE} \cong \overline{CE}. \overline{AE} \perp \overline{BC}$

Proof

Statements	Reasons
In $\Delta ADB \leftrightarrow \Delta ADC$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{DB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{AD}$	Common
$\therefore \Delta ADB \cong \Delta ADC$	S.S.S ≅ S.S.S
$\therefore \angle 1 \cong \angle 2$	Corresponding angles of ≅ Δs
In $\Delta ABE \leftrightarrow \Delta ACE$	
$\overline{AB} \cong \overline{AC}$	Given
$\angle 1 \cong \angle 2$	Proved
$\Delta ABE \cong \Delta ACE$	S.A.S postulate
$\overline{AE} \cong \overline{AE}$	Common
$\therefore \overline{BE} \cong \overline{CE}$	Corresponding sides of ≅ Δs
$\angle 3 \cong \angle 4$	Corresponding angles of ≅ Δs
$m\angle 3 + m\angle 4 = 180°$	Supplementary angles postulate
$m\angle 3 = m\angle 4 = 90°$	From I and II
Hence $\overline{AE} \perp \overline{BC}$	

Exercise 10.3

Q.1 In the figure, $\overline{AB} \cong \overline{DC}, \overline{AD} \cong \overline{BC}$ prove that $\angle A = \angle C$, $\angle ABC \cong \angle ADC$

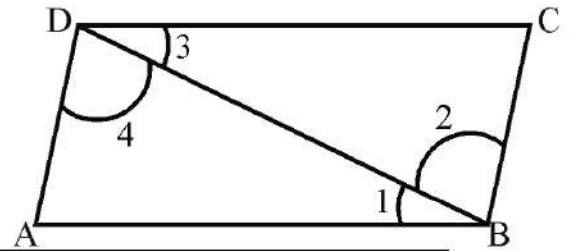
Given

In the figure $\overline{AB} \cong \overline{DC}, \overline{AD} \cong \overline{BC}$

To prove

$\angle A \cong \angle C$

$\angle ABC \cong \angle ADC$

Proof

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CDB$	
$\overline{AB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{BC}$	Given
$\overline{BD} \cong \overline{BD}$	Common
$\Delta ABD \cong \Delta CDB$	S.S.S $\cong$ S.S.S
$\therefore$ Hence $\angle A \cong \angle C$	Corresponding angles of congruent triangles
$\angle 1 \cong \angle 3$	Corresponding angles of congruent triangles
$\angle 2 \cong \angle 4$	Corresponding angles of congruent triangles
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	
or $m\angle ABC = m\angle ADC$	
$\angle ABC \cong \angle ADC$	

Q.2 In the figure $\overline{LN} \cong \overline{MP}$, $\overline{MN} \cong \overline{LP}$ prove that $\angle N \cong \angle P$, $\angle NML \cong \angle PLM$

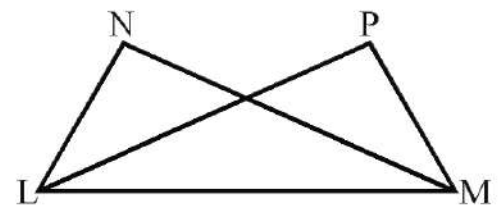
Given

In the figure

$\overline{LN} \cong \overline{MP}$ and $\overline{LP} \cong \overline{MN}$

To prove

$\angle N \cong \angle P$ and $\angle NML \cong \angle PLM$

Proof

Statements	Reasons
$\Delta LMN \leftrightarrow \Delta MLP$	
$\overline{LN} \cong \overline{MP}$	Given
$\overline{LP} \cong \overline{MN}$	Given
$\overline{LM} \cong \overline{ML}$	Common
$\Delta LMN \cong \Delta MLP$	S.S.S $\cong$ S.S.S
$\angle N \cong \angle P$	Corresponding angles of congruent triangles
$\angle NML \cong \angle PLM$	Corresponding angles of congruent triangles

Q.3 Prove that median bisecting the base of an isosceles triangle bisects the vertex angle and it is perpendicular to the base

Given

ΔABC

(i) $\overline{AB} \cong \overline{AC}$

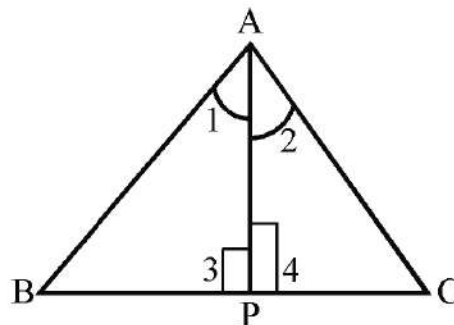
(ii) Point P is mid point of $\overline{BC}$ i.e: $\overline{BP} = \overline{CP}$

P is joined to A, i.e. $\overline{AP}$ is median

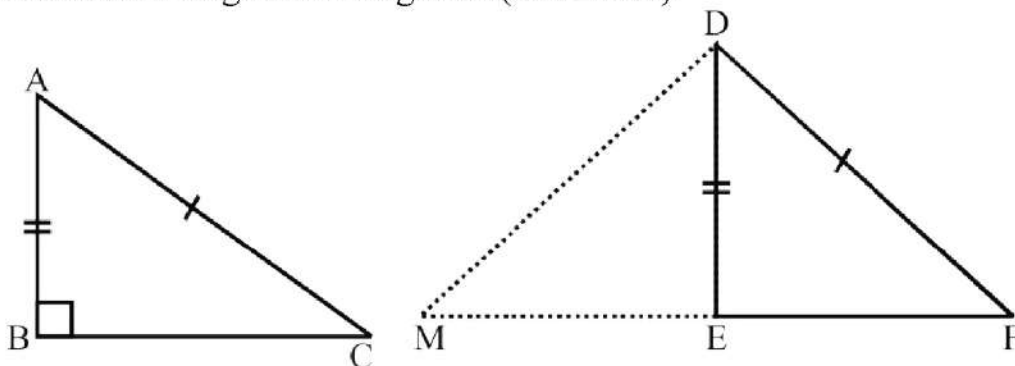
To prove

$\angle 1 \cong \angle 2$

$\overline{AP} \perp \overline{BC}$

Proof

Statements	Reasons
$\Delta ABP \leftrightarrow \Delta ACP$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{BP} \cong \overline{CP}$	Given
$\overline{AP} \cong \overline{AP}$	Common
$\Delta ABP \cong \Delta ACP$	S.S.S ≅ S.S.S
$\angle 1 \cong \angle 2$	Corresponding angles of congruent triangles
$\angle 3 \cong \angle 4$ ______(i)	
$m\angle 3 + m\angle 4 = 180^\circ$ ______(ii)	Corresponding angles of congruent triangles
Thus $m\angle 3 = m\angle 4 = 90$	
$\because \overline{AP} \perp \overline{BC}$	From equation (i) and (ii)

Theorem 10.1.4

If in the corresponding of the two right angled triangles, the hypotenuse and one side of one triangle are congruent to the hypotenuse and the corresponding side of the other them the triangles are congruent (H.S ≅ H.S)

Given

$\Delta ABC \leftrightarrow \Delta DEF$

$\angle B \cong \angle E$ (right angles)

$\overline{CA} \cong \overline{FD}, \overline{AB} \cong \overline{DE}$

To Prove

$\Delta ABC \cong \Delta DEF$

Construction

Prove $\overline{FE}$ to a point M such that $\overline{EM} \cong \overline{BC}$ and join the point D and M

Proof

Statements	Reasons
$m\angle DEF + \angle DEM = 180^o$ ____(i)	Supplementary angles
Now $m\angle DEF = 90^o$ ________(ii)	Given
$\therefore m\angle DEM = 90^o$	{ from (i) and (ii) }
In $\Delta ABC \leftrightarrow \Delta DEM$	
$\overline{BC} \cong \overline{EM}$	Construction
$\angle ABC \cong \angle DEM$	(Each angle equal to 90^o)
$\overline{AB} \cong \overline{DE}$	Given
$\Delta ABC \cong \Delta DEM$	SAS postulate
ad $\angle C = \angle M$	Corresponding angles of congruent triangles
$\overline{CA} \cong \overline{MD}$	Corresponding sides of congruent triangles
But $\overline{CA} \cong \overline{FD}$	Given
$\overline{MD} \cong \overline{FD}$	Each is congruent to $\overline{CA}$
In DMF	
$\angle F \cong \angle M$	$\overline{MD} \cong \overline{FD}$ (proved)
But $\angle C \cong \angle M$	(Proved)
$\angle C \cong \angle F$	Each is congruent to $\angle M$
	Given
$\angle ABC \cong \angle DEF$	Given
$\overline{AB} \cong \overline{DE}$	(Proved)
Hence $\Delta ABC \cong \Delta DEF$	(S.A.A $\cong$ S.A.A)

Example

If perpendiculars from two vertices of a triangle to the opposite sides are congruent, then the triangle is isosceles.

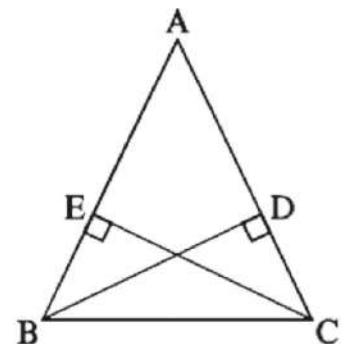
Given

In $\Delta ABC, \overline{BD} \perp \overline{AC}, \overline{CE} \perp \overline{AB}$

Such that $\overline{BD} \cong \overline{CE}$

To prove

$\overline{AB} \cong \overline{AC}$

Proof

Statements	Reasons
In $\Delta BCD \leftrightarrow \Delta CBC$	
$\angle BDC \cong \angle BEC$	$\overline{BD} \perp \overline{AC}, \overline{CE} \perp \overline{AB}$ given $\Rightarrow$ each angle $= 90^\circ$
$\overline{BC} \cong \overline{BC}$	Common hypotenuse
$\overline{BD} \cong \overline{CE}$	Given
$\Delta BCD \cong \Delta CBE$	H.S $\cong$ H.S
$\angle BCA \cong \angle CBE$	Corresponding angles Δ s
Thus $\angle BCA \cong \angle CBA$	
Hence $\overline{AB} \cong \overline{AC}$	In $\Delta ABC, \angle BCA \cong \angle CBA$

Exercise 10.4

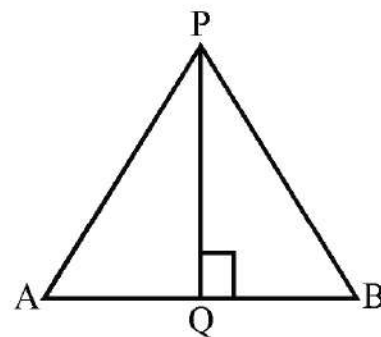
Q.1 In ΔPAB of figure $\overline{PQ} \perp \overline{AB}$ and $\overline{PA} \cong \overline{PB}$ prove that $\overline{AQ} \cong \overline{BQ}$ and ∠APQ ≅∠BPQ

Given:

In ΔPAB

$\overline{PQ} \perp \overline{AB}$ and $\overline{PA} \cong \overline{PB}$

To prove

$\overline{AQ} \cong \overline{BQ}$ and $\angle APQ \cong \angle BPQ$

Proof

Statements	Reasons
In ΔAPQ ↔ ΔBPQ	
$\overline{PA} \cong \overline{PB}$	Given
∠AQP ≅ ∠BQP	Given $\overline{PQ} \perp \overline{AB}$
$\overline{PQ} \cong \overline{PQ}$	Common
∴ ΔAPQ ≅ ΔBPQ	H.S ≅ H.S
So $\overline{AQ} \cong \overline{BQ}$	Corresponding sides of congruent triangles
and ∠APQ ≅ ∠BPQ	Corresponding angles of congruent triangles

Q.2 In the figure m∠C ≅ m∠D = 90° and $\overline{BC} \cong \overline{AD}$ prove that $\overline{AC} \cong \overline{BD}$ and ∠BAC ≅ ∠ABD

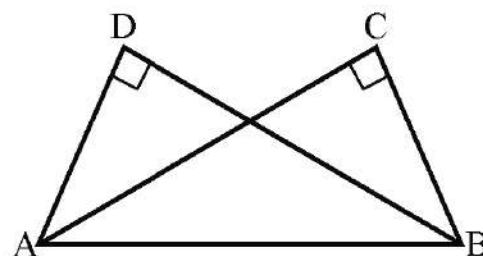
Given

In the figure given m∠C = m∠D = 90°

$\overline{BC} \cong \overline{AD}$

To Prove

$\overline{AC} \cong \overline{BD}$

∠BAC ≅ ∠ABD

Proof

Statements	Reasons
In ΔABD ↔ ΔBAC	
$\overline{AD} \cong \overline{BC}$	Given
∠D ≅ ∠C	Each 90°
$\overline{AB} \cong \overline{BA}$	Common
Thus ΔABD ≅ ΔBAC	H-S ≅ H-S
∴ $\overline{AC} \cong \overline{BD}$	Corresponding sides of congruent triangles
∴ ∠BAC ≅ ∠ABD	Corresponding angles of congruent triangles

Q.3 In the figure, $m\angle B = m\angle D = 90^o$ and $\overline{AD} \cong \overline{BC}$ prove that ABCD is a rectangle

Given

In the figure

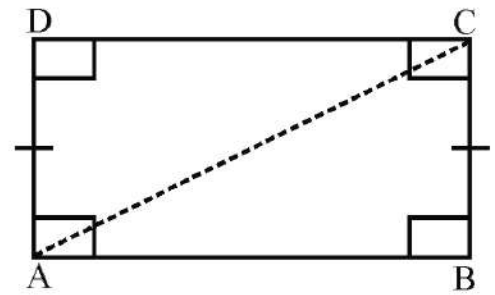
$m\angle B = m\angle D$ 90^o and $\overline{AD} \cong \overline{BC}$

To prove

$ABCD$ is a rectangle

Construction

Join A to C

Proof

Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta CDA$	
$\angle B \cong \angle D$	Given each angle = 90^o
$\overline{AC} \cong \overline{CA}$	Common
$\overline{BC} \cong \overline{DA}$	Given
$\therefore \Delta ABC \cong \Delta CDA$	H-S $\cong$ H-S
$\overline{AB} \cong \overline{CD}$	Corresponding sides of congruent triangles
and $\angle ACB \cong \angle CAD$	Corresponding angles of congruent triangles
Hence ABCD is a rectangle	

Review Exercise 10

Q.1 Which of the following are true and which are false.

(i) A ray has two end points. (False)

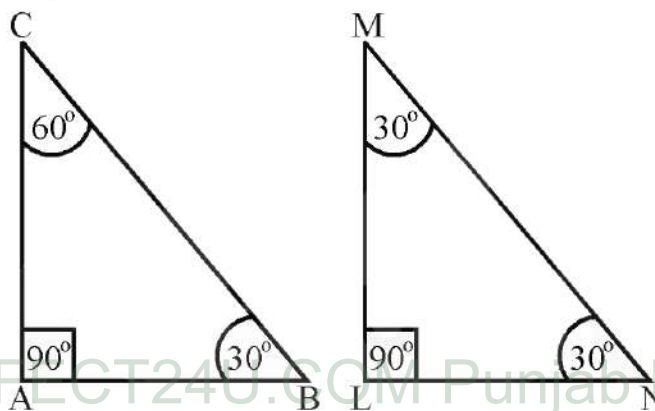
(ii) In a triangle there can be only are right angle. (True)

(iii) Three points are said to be collinear if they lie on same line. (True)

(iv) Two parallel lines intersect at a point. (False)

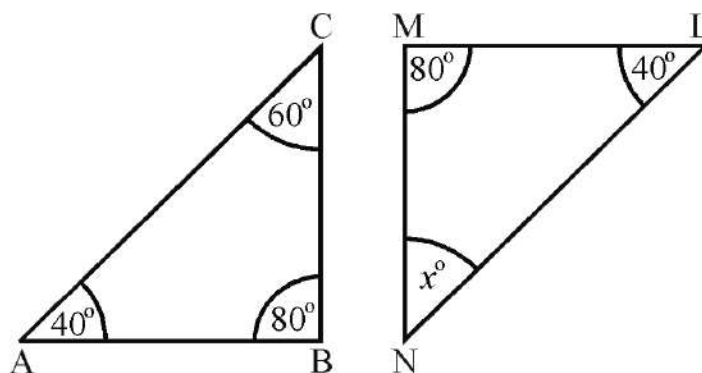
(v) Two line can intersect only one point. (True)

(vi) A triangle of congruent sides has non-congruent angles. (False)

Q.2 In $\Delta ABC \cong \Delta LMN$, then

(i) $m\angle M \cong \underline{m\angle B} = 30^\circ$

(ii) $m\angle N \cong \underline{m\angle C} = 60^\circ$

(iii) $m\angle A \cong \underline{m\angle L} = 90^\circ$

Q.3 If $\Delta ABC \cong \Delta \angle MN$ then find the value of x

$m\angle N = m\angle C = 60^\circ$

$m\angle N = x = 60^\circ$

Sum of three angle in a triangle is 180

So $x + 80 + 40 = 180$

$x + 120 = 180$

$x = 180 - 120$

$x = 60^o$

Q.4 Find the value of unknowns for the given congruent triangles.

It is an isosceles triangle

$m\overline{AB} = m\overline{AC}$

and $m\angle B = m\angle C$

when we draw a perpendicular from point A to BC it Bisect

So $\overline{BD} \cong \overline{DC}$

$5m - 3 = 2m + 6$

$5m - 2m = 6+3$

$3m = 9$

$m = \frac{9}{3}$

$m = 3$

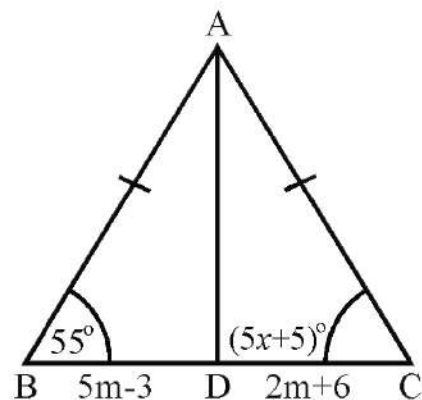
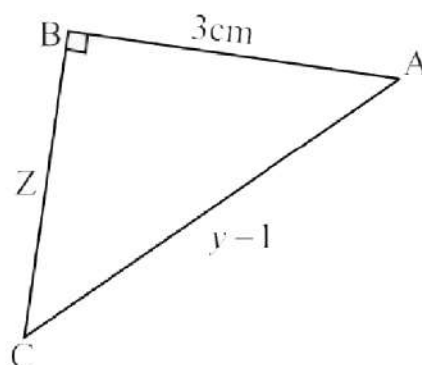
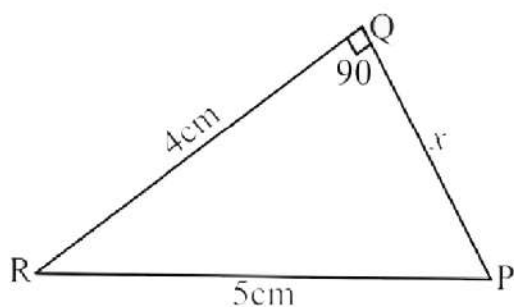
opposite angle are congruent

$\therefore \angle B = \angle C$

$55 = 5x + 5$

$55 - 5 = 5x$

$\frac{50}{5} = x$

$x = 10$

Q.5 If ΔPQR = ΔABC, the find the unknowns

By using definition of congruent triangles.

$\overline{RP} = \overline{AC}$

$5 = y - 1$

$5 + 1 = y$

$y = 6cm$

$\overline{AB} = \overline{QP}$

$3cm = x$

Or

$x = 3$cm

$\overline{BC} = \overline{QR}$

$Z = 4cm$

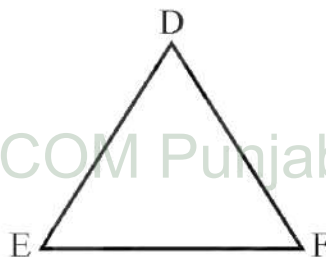
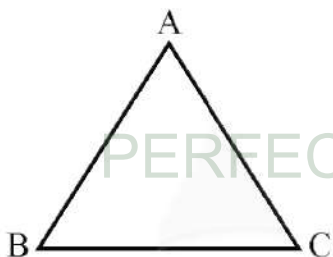
Unit 10: Congruent Triangle

Overview

Congruency of Triangles:

Tow triangles are said to be congruent written symbolically as $\cong$,if there exists a correspondence between them such that all the corresponding sides and angles are congruent.

i.e. if $\begin{cases} \overline{AB} \cong \overline{DE} \\ \overline{BC} \cong \overline{EF} \\ \overline{CA} \cong \overline{FD} \end{cases}$ and $\begin{cases} \angle A \cong \angle D \\ \angle B \cong \angle E \\ \angle C \cong \angle F \end{cases}$

then $\Delta ABC \cong \Delta DEF$

A.S.A postulate:

In any correspondence of two triangles, if one side and any two angles of one triangle are congruent to the corresponding side and angles of the other then the triangles are congruent this postulate is called A.S.A. postulate.

A.S.A postulate:

In any correspondence of two triangles, if one side and any two angles of one triangle are congruent to the corresponding side and angles of the other, the two triangles, are congruent. This postulate is called A.S.A postulate.

S.S.S postulate:

In a correspondence of two triangles, if three sides of one triangle are congruent to the corresponding three sides of the other, then the two triangles are congruent this postulate is called S.S.S postulate.

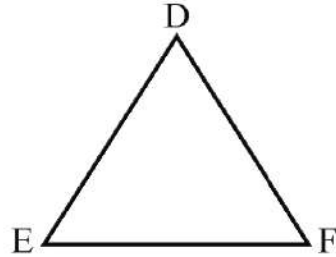
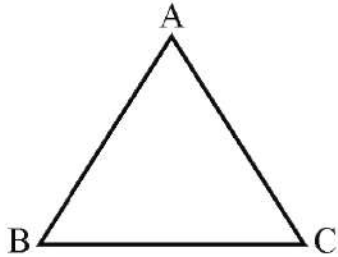
H.S postulate:

If in the correspondence of the two right-angled triangles, the hypotenuse and one side of one triangle are congruent ot the hypotenuse and the corresponding side of the other, then the triangles, are congruent this postulate is called H.S postulate.

Introduction:

Two triangles are said to be congruent if at least one $(1-1)$ correspondence can be established between them in which the angles and sides are congruent.

For example

If in the corresponding $\Delta ABC \leftrightarrow \Delta DEF$

(i) $\angle A \longleftrightarrow \angle D$ ($\angle A$ corresponds to $\angle D$)

(ii) $\angle B \longleftrightarrow \angle E$ ($\angle B$ corresponds to $\angle E$)

(iii) $\angle C \longleftrightarrow \angle F$ ($\angle C$ corresponds to $\angle F$)

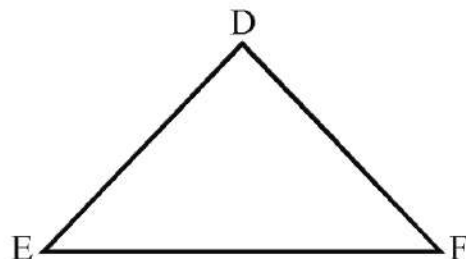
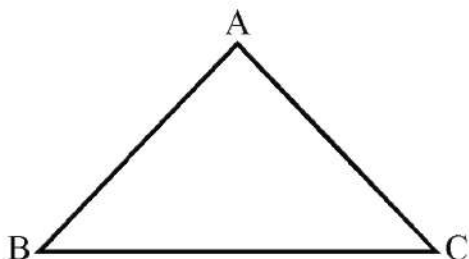
(iv) $\overline{AB} \longleftrightarrow \overline{DE}$ ($\overline{AB}$ corresponds to $\overline{DE}$)

(v) $\overline{BC} \longleftrightarrow \overline{EF}$ ($\overline{BC}$ corresponds to $\overline{EF}$)

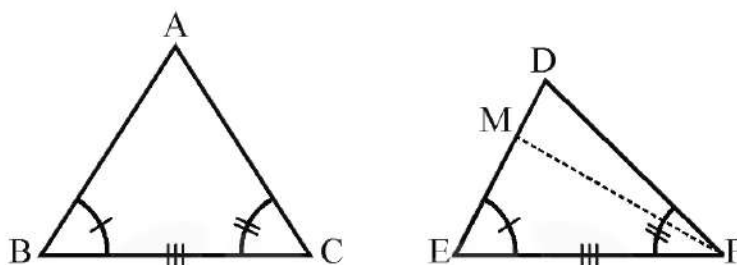
(vi) $\overline{CA} \longleftrightarrow \overline{FD}$ ($\overline{CA}$ corresponds to $\overline{FD}$)

Congruency of Triangles:

The two triangles are said to be congruent written as $\cong$ if there exists a correspondence between them such that all the corresponding sides and angles are congruent.

Then $\Delta ABC \cong \Delta DEF$

If $\begin{cases} \overline{AB} \cong \overline{DE} \\ \overline{BC} \cong \overline{EF} \\ \overline{AC} \cong \overline{DF} \end{cases}$ and $\begin{cases} \angle A \cong \angle D \\ \angle B \cong \angle E \\ \angle C \cong \angle F \end{cases}$

Theorem 10.1.1

In any correspondence of two triangles, if one side and any two angles of one triangle are congruent to the corresponding side and angles of the other then the triangles are congruent.(A.S.A $\cong$ A.S.A.)

Given

In $\Delta ABC \leftrightarrow \Delta DEF$

$\angle B \cong \angle E$, $\overline{BC} \cong \overline{EF}$, $\angle C \cong \angle F$

To prove

$\Delta ABC \cong \Delta DEF$

Construction

Suppose $\overline{AB} \not\cong \overline{DE}$. Take a point M on $\overline{DE}$ such that $\overline{AB} \cong \overline{ME}$. Join M to F

Proof

Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta MEF$	
$\overline{AB} \cong \overline{ME}$ ____ (i)	Construction
$\overline{BC} \cong \overline{EF}$ ____ (ii)	Given
$\angle B \cong \angle E$ ____ (iii)	Given
$\Delta ABC \cong \Delta MEF$	S.A.S postulate
So, $\angle C \cong \angle MFE$	(Corresponding angles of congruent triangles)
But $\angle C \cong \angle DFE$	Given
$\therefore$ $\angle DFE \cong \angle MFE$	Both congruent to $\angle C$
This is possible only if D and M are the same points and $\overline{ME} \cong DE$	

So $\overline{AB} \cong \overline{DE}$ ____(iv)	$\overline{AB} \cong \overline{ME}$ (construction) and $\overline{ME} \cong \overline{DE}$ (proved)
Thus from (ii), (iii) and (iv), we have $\Delta ABC \cong \Delta DEF$	S.A.S postulates

Example

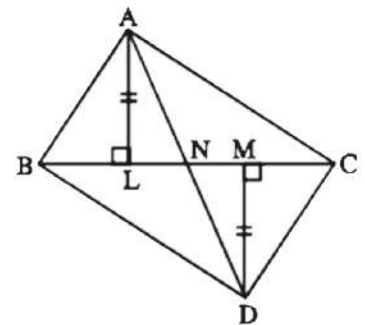
If ΔABC and ΔDCB are on the opposite sides of common base $\overline{BC}$ such that $\overline{AL} \perp \overline{BC}, \overline{DM} \perp \overline{BC}$ and $\overline{AL} \cong \overline{DM}$, then $\overline{BC}$ bisects $\overline{AD}$.

Given

ΔABC and ΔDCB are on the opposite sides of $\overline{BC}$ such that $\overline{AL} \perp \overline{BC}, \overline{DM} \perp \overline{BC}, \overline{AL} \cong \overline{DM}$, and $\overline{AD}$ is cut by $\overline{BC}$ at N.

To prove

$\overline{AN} \cong \overline{DN}$

Proof

Statements	Reasons
In $\Delta ALN \leftrightarrow \Delta DMN$	
$\overline{AL} \cong \overline{DM}$	Given
$\angle ALN \cong \angle \overline{DMN}$	Each angle is right angle
$\angle ALN \cong \angle \overline{DMN}$	Vertical angels
$\angle ALN \cong \angle \overline{DMN}$	SAA ≅ SAA
$\overline{AN} \cong \overline{DN}$	Corresponding sides of ≅ Δs.

Exercise 11.1

Q.1 One angle of a parallelogram in 130°. Find the measures of its remaining angles.

In parallelogram

$m\angle B = 130^\circ$

$\angle D = \angle B$ (Opposite angles of a parallelogram)

$m\angle D = m\angle B = 130^\circ$

We know that

$\angle A + \angle B = 180$

$\angle A + 130 = 180$ (sum of int. $\angle$s on same side of a parallelogram is 180°)

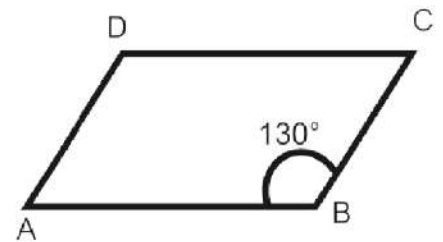
$\angle A = 180 - 130$

$\angle A = 50^\circ$

If $\angle D = \angle B$

Then

$\angle C = \angle A$

$\angle C = 50^\circ$

Q.2 One exterior angle formed on producing one side of a parallelogram is 40°. Find the measures of its interior angles.

$ABCD$ is a parallelogram. $\overline{BA}$ is produced towards A.

$m\angle DAM = 40^\circ$

$m\angle DAB = ?$

$m\angle D = ?$

$m\angle B = ?$

$m\angle C = ?$

$\angle DAM + \angle DAB = 180^\circ$

$40^\circ + \angle DAB = 180^\circ$

$\angle DAB = 180^\circ - 40^\circ$

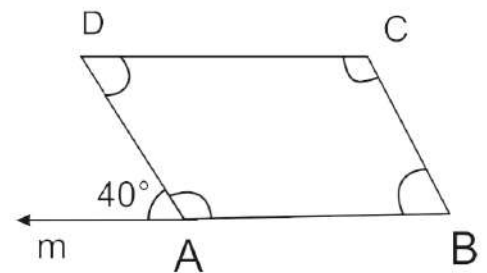
$\angle DAB = 140^\circ$

$\angle DAB + \angle B = 180^\circ$

$140^\circ + \angle B = 180^\circ$

$\angle B = 180^\circ - 140^\circ$

$\angle B = 40^\circ$

$\angle D = \angle B = 40^\circ$

$\angle D = 40°$

$\angle C = \angle DAB$

$\angle C = 140°$

Theorem 11.1.2

Statement: If two opposite sides of quadrilateral are congruent and parallel, it is a parallelogram

Given

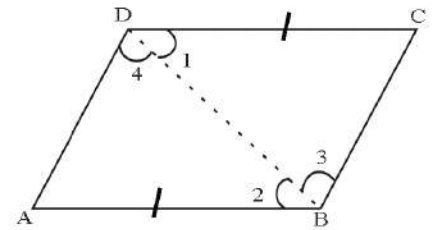
In quadrilateral $ABCD$,

$\overline{AB} \cong \overline{DC}$ *and* $\overline{AB} \| \overline{DC}$

To prove

ABCD is a parallelogram

Construction

Join the point B to D and in the figure name the angles as

Proof

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CDB$	
$\overline{AB} \cong \overline{DC}$	Given
$\angle 2 \cong \angle 1$	Alternate angles
$\overline{BD} \cong \overline{BD}$	Common
$\therefore \Delta ABD \cong \Delta CDB$	SAS postulate
Now $\angle 4 \cong \angle 3$............(i)	(Corresponding angles of congruent triangles)
$\therefore \overline{AD} \| \overline{BC}$...............(ii)	from (i)
and $\overline{AD} = \overline{BC}$...........(iii)	corresponding of sides of congruent triangles
Also $\overline{AB} \| \overline{DC}$...........(iv)	Given
Hence ABCD is a parallelogram	From (ii)-(iv)

Exercise 11.2

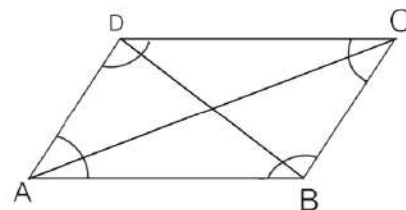
Q.1 Prove that a quadrilateral is a parallelogram if its
(a) Opposite angles are congruent
(b) Diagonals bisects each other

(a) Given

In quadrilateral ABCD

$m\angle A = m\angle C, m\angle B = m\angle D$

To Prove

$ABCD$ is a parallelogram

Statements	Reasons
$m\angle A = m\angle C \ldots (\text{i})$	Given
$m\angle B = m\angle D \ldots (\text{ii})$	Given
$m\angle A + m\angle B + m\angle C + m\angle D = 360^\circ$	Angles of quadrilateral
$m\angle A + m\angle B = 180^\circ$	
$m\angle C + m\angle D = 180^\circ$	
$\overline{AD} \parallel \overline{BC}$	
Similarity it can be proved that $\overline{AB} \parallel \overline{DC}$	
Hence ABCD is a parallelogram	

(b) Given

In quadrilateral ABCD, diagonals $\overline{AC}$ and $\overline{BD}$ bisect each other.

i.e. $\overline{OA} = \overline{OC}, \overline{OB} = \overline{OD}$

To prove ABCD is a parallelogram

Proof

Statements	Reasons
In ΔABO $\leftrightarrow$ ΔCDO	
$\overline{OA} \cong \overline{OC}$	Given
$\overline{OB} \cong \overline{OD}$	Given
$\angle AOB \cong \angle COD$	Vertical opposite angles
$\therefore \angle 1 \cong \angle 2$	Corresponding angles of congruent triangles
$\Delta ABO \cong \angle CDO$	$S.A.S \cong S.A.S$
Hence, $\overline{AB} \parallel \overline{CD} \ldots (\text{i})$	$\angle 1 \cong \angle 2$
By taking BOC and is ΔAOD it can be prove that	
$\overline{AD} \parallel \overline{BC} \ldots (\text{ii})$	From (i) and (ii)
Hence ABCD is a parallelogram	

Q.2 Prove that a quadrilateral is a parallelogram if its opposite sides are congruent

Given

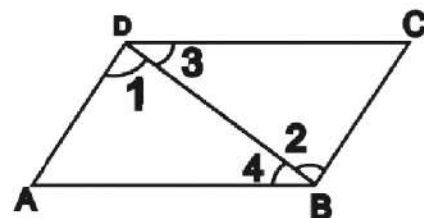
In quadrilateral $ABCD$

(i) $\overline{AB} \cong \overline{DC}$

(ii) $\overline{AD} \cong \overline{BC}$

To prove

$ABCD$ is a parallelogram i.e. $\overline{AD} \| \overline{BC}$

Prove

Statements	Reasons
$\Delta CDB \leftrightarrow \Delta ABD$	
$\overline{AB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{BC}$	Given
$\overline{BD} \cong \overline{BD}$	Common
$\Delta ABD \cong \Delta CDB$	$S.S.S \cong S.S.S$
Thus, $\angle 1 \cong \angle 2$	Corresponding angles of congruent triangles
$\angle 4 \cong \angle 3$	Corresponding angles of congruent triangles
(i) $\overline{AD} \| \overline{BC}$	Alternate angles are congruent
$\overline{AB} \| \overline{DC}$	Alternate angles are congruent
$\therefore$ ABCD is a parallelogram	

Example

The line segments, joining the mid-points of the sides of a quadrilateral, taken in order, form a parallelogram.

Given

A quadrilateral $ABCD$, in which P is the mid-point of $\overline{AB}$ Q is the mid-point of $\overline{BC}$ R is the mid-point of $\overline{CD}$ S is the mid-point of $\overline{DA}$

P is joined to Q, Q is joined to R.

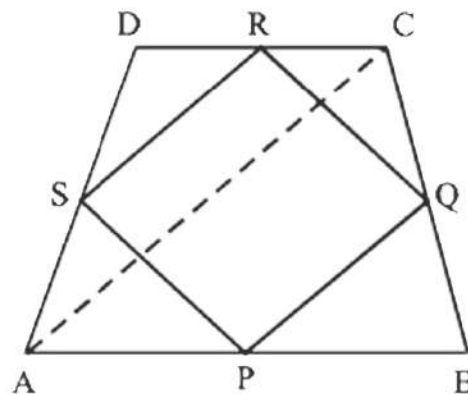
R is joined to S and S is joined to P.

To prove

PQRS is a parallelogram.

Construction

Join A to C.

Proof

Statements	Reasons
In ΔDAC,	
$\overline{SR} \| \overline{AC}$	S is the midpoint of $\overline{DA}$
$m\overline{SR} = \frac{1}{2} m\overline{AC}$	R is the midpoint of $\overline{CD}$

In ΔBAC,	
$\overline{PQ} \parallel \overline{AC}$	P is the midpoint of $\overline{AB}$
$m\overline{PQ} = \frac{1}{2} m\overline{AC}$	Q is the midpoint of $\overline{BC}$
$\overline{SR} \parallel \overline{PQ}$	Each $\parallel \overline{AC}$
$m\overline{SR} = m\overline{PQ}$	Each $= \frac{1}{2}\overline{AC}$
Thus $PQRS$ is a parallelogram	$\overline{SR} \parallel \overline{PQ}, m\overline{SR} = m\overline{PQ}$ (proved)

Theorem 11.1.3

The line segment, joining the midpoint of two sides of triangle, is parallel to the third side and is equal to one half of its length.

Given

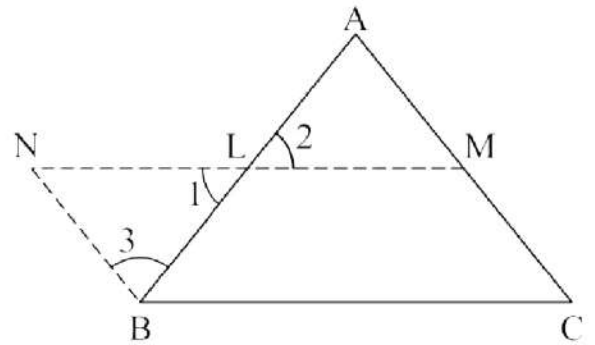
In ΔABC, the mid-point of $\overline{AB}$ and $\overline{AC}$ are L and M respectively

To prove

$\overline{LM} \parallel \overline{BC}$ and $m\overline{LM} = \frac{1}{2} m\overline{BC}$

Construction

Join M to L and produce $\overline{ML}$ to N such that $\overline{ML} \cong \overline{LN}$

Join N to B and in the figure, name the angles $\angle 1$, $\angle 2$ and $\angle 3$ as shown.

Proof

Statements	Reasons
In Δ BLN $\leftrightarrow$ Δ ALM	
$\overline{BL} \cong \overline{AL}$	Given
$\angle 1 \cong \angle 2$	Vertical angles
$\overline{NL} \cong \overline{ML}$	Construction
$\therefore$ ΔBLN $\cong$ ΔALM	S.A.S postulate
$\therefore \angle A \cong \angle 3$...(i)	(Corresponding angles of congruent triangles)
And $\overline{NB} \cong \overline{AM}$...(ii)	(Corresponding sides of congruent triangles)
But $\overline{NB} \parallel \overline{AM}$	from (i), alternative <s
Thus	
$\overline{NB} \parallel \overline{MC}$ ....(iii)	(M is a point of $\overline{AC}$)

$\overline{MC} \cong AM$ ....(iv)	Given
$\overline{NB} \cong \overline{MC}$ ....(v)	from (ii) and (iv)
$BC\,MN$ is a parallelogram	From (iii) and (v)
$\therefore \overline{BC} \parallel \overline{LM}$ or $\overline{BC} \parallel \overline{NL}$	(Opposite sides of a parallelogram BCMN) (Opposite sides of a parallelogram)
$\overline{BC} \cong \overline{NM}$ ....(vi)	Construction.
$m\overline{LM} = \frac{1}{2} m\overline{NM}$ ....(vii)	
Hence, $m\overline{LM} = \frac{1}{2} m\overline{BC}$	from (vi) and (vii)

Exercise 11.3

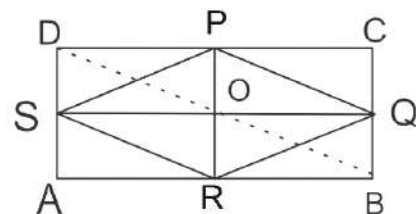
Q.1 Prove that the line segments joining the midpoint of the opposite side of a quadrilateral bisect each other.

Given

$ABCD$ is quadrilaterals point $QRSP$ are the mid point of the sides $\overline{RP}$ and $\overline{SQ}$ are joined they meet at O.

$\overline{OP} \cong \overline{OR}$ $\overline{OQ} \cong \overline{OS}$

Construction

Join P,Q,R and S in order join C to A or A to C

Proof

Statements	Reasons
$SP \parallel AC$ … (i)	In $\Delta ADC, S, P$ are mid point of AD, DC
$m\overline{SP} = \frac{1}{2} m\overline{AC}$...(ii)	
$\overline{AC} \parallel \overline{RQ}$...(iii)	In ΔABC, Q,R are midpoint of $\overline{BC}$, $\overline{AB}$
$m\overline{RQ} = \frac{1}{2}\overline{AC}$...(iv)	
$m\overline{SP} \parallel \overline{RQ}$...(v)	
and $\overline{RQ} = \overline{SP}$...(vi)	From (ii) and (iv)
Now $\overline{RP}$ *and* $\overline{QS}$ diagonals of parallelogram PQRS intersect at O.	
$\therefore$ $\overline{OP} \cong \overline{OR}$ $\overline{OS} \cong \overline{OQ}$	Diagonals of a parallelogram bisects each other.

Q.2 Prove that the line segments joining the midpoint of the opposite sides of a rectangle are the right bisectors of each other.
[Hint: Diagonals of a rectangle are congruent]

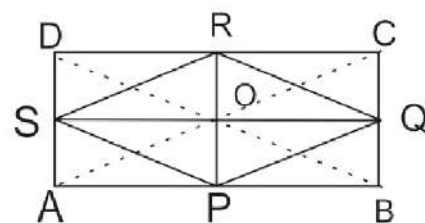
Given

(i) $ABCD$ is a rectangle

(ii) $P,Q.R.S$ are the midpoints of $\overline{AB}$, $\overline{CD}$ and $\overline{DA}$

(iii) $\overline{SQ}$ and $\overline{RP}$ cut each other at point O

$\overline{OS} \cong \overline{OQ}$

$\overline{OP} \cong \overline{OR}$

Construction

Join P to Q and Q to R and R to S and S to P

Join A to C and B to D

Proof

Statements	Reasons
Midpoint of $\overline{BC}$ is Q	Given
Midpoint of $\overline{AB}$ is P	Given
$\therefore \overline{AC} \parallel \overline{PQ}$.................(i)	
$\frac{1}{2}\overline{AC} = \overline{PQ}$............(ii)	
In ΔADC	
$\overline{AC} \parallel \overline{SR}$................(iii)	
$\frac{1}{2}\overline{AC} = \overline{SR}$............(iv)	
$\overline{PQ} = \overline{SR}$	From equation (i) and (ii) each are parallel to
$\overline{SP} = \overline{RQ}$	$\overline{AC}$ each are half of $\overline{DB}$
By joined B to D we can prove	
$\overline{RQ} \parallel \overline{SP}$	
$m\overline{SR} \parallel m\overline{PQ}$	Each of them $= \frac{1}{2}\overline{AC}$
$m\overline{AC} \parallel m\overline{BD}$	
PQRS is a parallelogram all it sides are equal	
$\overline{OP} \cong \overline{OR}$	
$\overline{OS} \cong \overline{OQ}$	
$\Delta OQR \leftrightarrow \Delta OQP$	
$\overline{OR} \cong \overline{OP}$	Proved
$\overline{OQ} \cong \overline{OQ}$	Common
$\overline{RQ} \cong \overline{PQ}$	Adjacent
$\therefore \Delta OQR \cong \Delta OQP$	
$\angle ROQ \cong \angle POQ$...............(vii)	
$\angle ROQ + \angle POQ = 180$........(viii)	Supplementary angle
$\angle ROQ = \angle POQ = 90°$	From (vii) and (viii)
Thus $\overline{PR} \perp \overline{QS}$	

Q.3 Prove that line segment passing the midpoint of one side and parallel to other side of a triangle also bisects the third side.

Given

In $\Delta ABC, R$ is the midpoint of $\overline{AB}, \overline{RQ} \parallel \overline{BC}$

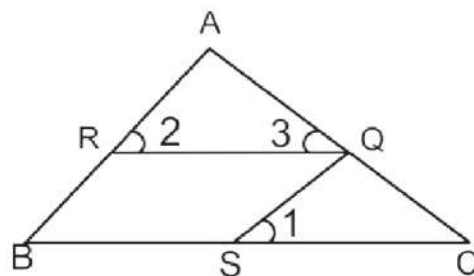
$\overline{RQ} \parallel \overline{BS}$

To prove

$\overline{AQ} = \overline{QC}$

Construction

$\overline{QS} \parallel \overline{AB}$

Proof

Statements	Reasons
$\overline{RQ} \parallel \overline{BS}$	Given
$\overline{QS} \parallel \overline{BR}$	Construction
$RBSQ$ is a Parallelogram	
$\overline{QS} \cong \overline{BR}$...(i)	Opposite side
$\overline{AR} \cong \overline{RB}$...(ii)	Given
$\overline{QS} \cong \overline{AR}$...(iii)	From (i) and (ii)
$\angle 1 \cong \angle B$ and $\angle 1 \cong \angle 2$...(iv)	
$\Delta ARQ \leftrightarrow \Delta QSC$	
$\angle 2 \cong \angle 1$	From (iv)
$\angle 3 \cong \angle C$	
$\overline{AR} \cong SQ$	From (iii)
Hence, $\Delta ARQ \cong \Delta QSC$	$A.A.S \cong A.A.S$
$\overline{AQ} \cong \overline{QC}$	Corresponding sides

Theorem: 11.1.4

Statement: The median of triangle are concurrent and their point of concurrency is the point of trisection of each median.

Given ΔABC

To prove

The medians of the ΔABC are concurrent and the point of concurrency is the point of trisection of each median

Construction

Draw two medians $\overline{BE}$ and $\overline{CF}$ of the ΔABC which intersect each other at point G. Join A to G and produce it to the point H such that $AG \simeq \overline{GH}$ Join H to the points B and C $\overline{AH}$ Intersects $\overline{BC}$ at the point D.

Proof

Statements	Reasons
In Δ ACH,	
$\overline{GE} \parallel \overline{HC}$	G and E are mid-points of sides $\overline{AH}$ and $\overline{AC}$ respectively
Or $\overline{BE} \parallel \overline{HC}$ ....(i)	G is point of $\overline{BE}$ diagonals $\overline{BC}$
Similarly $\overline{CF} \parallel \overline{HB}$...(ii)	
$\therefore$ BHCG is a parallelogram	From (i) and (ii)
And	
$m\overline{GD} = \frac{1}{2} m\overline{GH}$...(iii)	Diagonals $\overline{BC}$ and $\overline{GH}$ of a parallelogram $BHCG$ intersect each other at point D.
$\overline{BD} = \overline{CD}$	
$\overline{AD}$ is a median of ΔABC medians $\overline{AD}, \overline{BE}$ and $\overline{CF}$ pass through the point G	G is the interesting point of $\overline{BE}$, $\overline{CF}$ and $\overline{AD}$ pass through it.
Now $\overline{GH} \cong \overline{AG}$...(iv)	Construction
$m\overline{GD} = \frac{1}{2} m\overline{AG}$	From (iii) and (iv)
and G is the point of trisection of $\overline{AD}$...(v)	
similarly it can be proved that G is also the point of trisection of $\overline{CF}$ and $\overline{BE}$	

Exercise 11.4

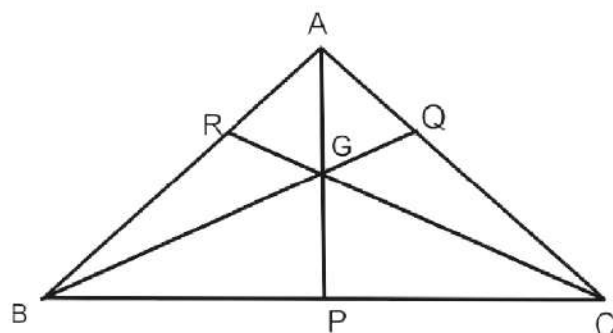
Q.1 The distance of the point of concurrency of the medians of a triangle from its vertices are respectively 1.2 cm. 1.4 cm and 1.6 cm. Find the length of its medians.

Let ΔABC with the point of concurrency of medians at G

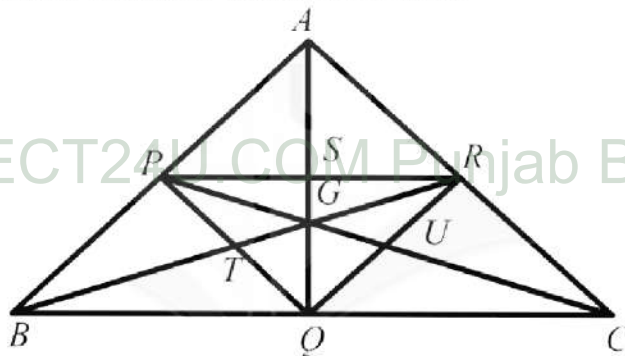
$\overline{AG}=1.2cm,\ \overline{BG}=1.4cm\ and\ \overline{CG}=1.6cm$

$\overline{AP}=\frac{3}{2}\overline{AG}=\frac{3}{2}\times 1.2=1.8cm$

$\overline{BQ}=\frac{3}{2}\overline{BG}=\frac{3}{2}\times 1.4=2.1cm$

$\overline{CR}=\frac{3}{2}\overline{CG}=\frac{3}{2}\times 1.6=2.4cm$

Q.2 Prove that the point of concurrency of the medians of a triangle and the triangle which is made by joining the midpoint of its sides to the same.

Given

In ΔABC, AQ, CP, BR are medians which meet at G.

To prove

G is the point of concurrency of the medians of ΔABC and ΔPQR

Proof

Statements	Reasons
$\overline{PR}\parallel\overline{BC}$	P, R are midpoint of $\overline{AB},\overline{AC}$
$\overline{BQ}\parallel\overline{PR}$	
Similarly $\overline{QR}\parallel\overline{BP}$	
$\therefore PBQR$ is a parallelogram it diagonals $\overline{BR}$ and $\overline{PQ}$ bisector each other at T	
Similarly U is the midpoint of QR and S is midpoint of $\overline{PR}$	
$\therefore \overline{PU},\overline{QS},\overline{RT}$ are medians of ΔPQR	
(i) $\overline{AQ}$ and $\overline{SQ}$ pass through G	
(ii) $\overline{BR}$ and $\overline{TR}$ pass through G	
(iii) $\overline{UP}$ and $\overline{CP}$ pass through G	
Hence G is point of concurrency of medians of ΔPQR and ΔABC	

Example

A line, through the mid-point of one side, parallel to another side of a triangle, bisects the third side.

Given

In $\Delta ABC, D$ is the mid-point of $\overline{AB}$.

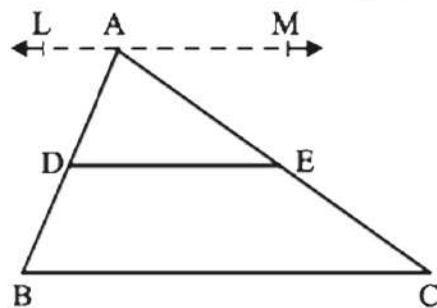
$\overline{DE} \parallel \overline{BC}$ which cuts $\overline{AC}$ at E.

To prove

$\overline{AE} \cong \overline{BC}$

Construction

Through A, draw $\overleftrightarrow{LM} \parallel \overline{BC}$.

Proof

Statements	Reasons
Intercepts cut by $\overleftrightarrow{LM}, \overline{DE}, \overline{BC}$ on $\overline{AC}$ are congruent. i.e., $\overline{AE} \cong \overline{EC}$.	Intercepts cut by parallels $\overleftrightarrow{LM}, \overline{DE}$. $\overline{BC}$ on $\overline{AB}$ are congruent (given)

Theorem 11.1.5

Statement: In three or more parallel lines make congruent segments on a traversal they also intercept congruent segments on any other line that cuts them.

Given

$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel$ EF

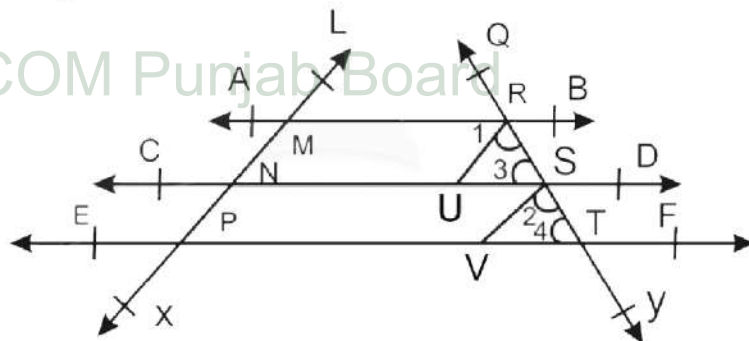
The transversal $\overleftrightarrow{LX}$ intersects $\overleftrightarrow{AB}, \overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ at the points M, N and P respectively, such that $\overline{MN} \cong \overline{NP}$. The transversal $\overleftrightarrow{QY}$ intersects them at point R, S and T respectively.

Prove

$\overline{RS} \cong \overline{ST}$

Construction

From R, draw $\overline{RU} \parallel \overline{LX}$, which meets $\overline{CD}$ at U, from S draw $\overline{SV} \parallel \overline{LX}$ which meets $\overline{EF}$ at V. as shown in the figure let the angles be labeled as $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$.

Proof

Statements	Reasons
$MNUR$ is parallelogram	$\overline{RU} \parallel \overline{LX}$ (Construction) $\overline{AB} \parallel \overline{CO}$ (given)
$\therefore \overline{MN} \cong \overline{RU}$ (i)	(Opposite side of parallelogram).
Similarly.	
$\overline{NP} \cong \overline{SV}$ (ii)	
But $\overline{MN} \cong \overline{NP}$ (iii)	Given
$\therefore \overline{RU} \cong \overline{SV}$	{from (i) (ii) and (iii)} each is $\parallel$ $\overline{LX}$ (construction)

Also $\overline{RU} \parallel \overline{SV}$	
$\therefore \angle 1 \cong \angle 2$	Corresponding angles
and $\angle 3 \cong \angle 4$	Corresponding angles
In $\Delta RUS \leftrightarrow \Delta SVT$	
$\overline{RU} \cong \overline{SV}$	Proved
$\angle 1 \cong \angle 2$	Proved
$\angle 3 \cong \angle 4$	Proved
$\therefore \Delta RUS \cong \Delta SVT$	$S.A.A \cong S.A.A$
Hence $\overline{RS} \cong \overline{ST}$	(Corresponding sides of congruent triangles)

Exercise 11.5

Q.1 **In the given figure**

$\overleftrightarrow{AX} \parallel \overleftrightarrow{BY} \parallel \overleftrightarrow{CZ} \parallel \overleftrightarrow{DU} \parallel \overleftrightarrow{EV}$ and $\overline{AB} = \overline{BC} = \overline{CD} = \overline{DE}$

If $\overline{MN} = 1cm$ then find the length of $\overline{LN}$ and $\overline{LQ}$

$\therefore \overline{PQ} \cong \overline{NP} \cong \overline{MN} \cong \overline{LM}$

$MN = 1cm$

Given

$\overline{AP} \cong \overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{ST}$

Therefore, $\overline{LN} = \overline{LM} + \overline{MN}$

$\overline{LM} = \overline{MN}$

so, $\overline{LN} = \overline{MN} + \overline{MN}$

$\overline{LN} = 1 + 1$

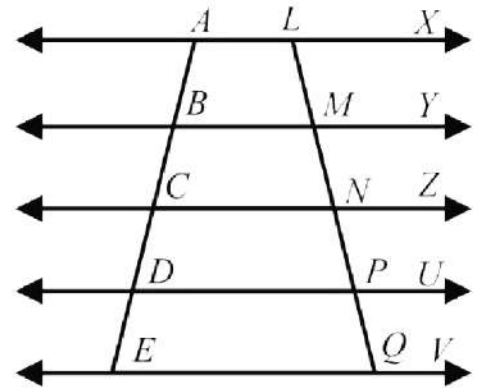
$\overline{LN} = 2cm$

$\overline{LM} = \overline{NP} = \overline{PQ} = \overline{MN} = 1cm$

$So, \overline{LM} = 1cm, \overline{NP} = 1cm, \overline{PQ} = 1cm$

$\overline{LQ} = \overline{LM} + \overline{MN} + \overline{NP} + \overline{PQ}$

$\overline{LQ} = 1 + 1 + 1 + 1$

$\overline{LQ} = 4cm$

Q.2 **Take a line segment of length** $5.5cm$ **and divide it into five congruent parts [Hint: draw an acute angle** $\angle B$ AX**. On** $\overrightarrow{AX}$ *take* $\overline{AP} \cong \overline{PQ} \cong \overline{RS} \cong \overline{ST}$ **join** T **to** B **draw lines parallel to** $\overline{TB}$ **from the point** P, Q R **and** S.

Proof

Construction:

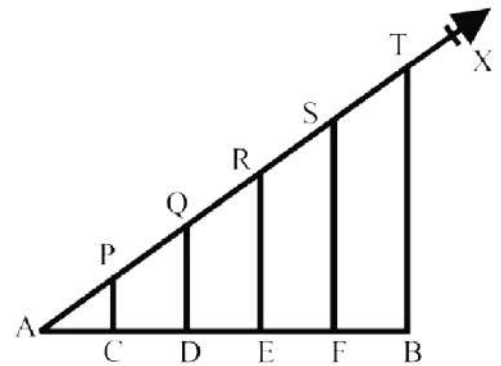
(i) Take a line segment $AB = 5.5cm$

(ii) Draw any acute angle $\angle BAX$

(iii) Draw arcs on $\overrightarrow{AX}$ which are $\overline{AP} \cong \overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{ST}$

(iv) Join T to B

(v) Draw lines $\overline{SF}, \overline{RE}, \overline{QD}, \& \overline{PC}$ Parallel to $\overline{TB}$.

Result line segment $\overline{AB}$ is divided into congruent line segments $\overline{AC} \cong \overline{CD} \cong \overline{DE} \cong \overline{EF} \cong \overline{FB}$.

Review Exercise 11

Q.1 Fill in the blanks

(i) In a parallelogram opposite side are ……

Ans: Congruent

(ii) In a parallelogram opposite angles are …….

Ans: Congruent

(iii) Diagonals of a parallelogram …….. each other at a point.

Ans: Bisects

(iv) Medians of a triangle are ……….

Ans: Concurrent

(v) Diagonals of a parallelogram divide the parallelogram into two ……. Triangles

Ans: Congruent

Q.2 In parallelogram *ABCD*

(i) $m\overline{AB} =$ ....

Ans: $m\overline{AB} = m\overline{DC}$

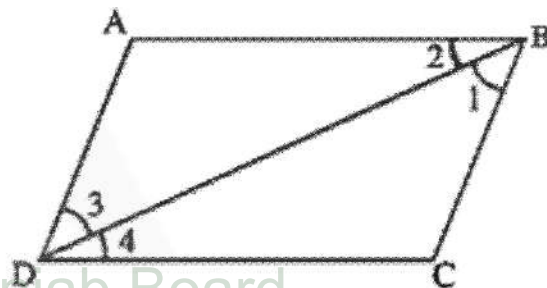
(ii) $m\overline{BC}$ ....

Ans: $m\overline{BC} = m\overline{AD}$

(iii) $m\angle 1 \cong$ ....

Ans: $m\angle 1 = m\angle 3$

(iv) $m\angle 2 =$ ....

Ans: $m\angle 2 = m\angle 4$

Q.3 Find the unknown in the figure given

Solution

$n° = 75$

$y° = n°$

Substituting the value of $n°$

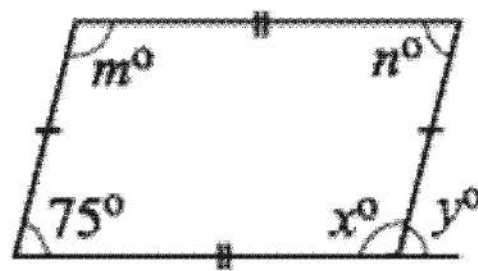
$y° = 75°$

$x° + 75 = 180$ Adjacent and supplementary

$x° = 180 - 75$

$x° = 105°$

$m° = x°$

$m° = 105°$

Q.4 If the given figure ABCD is a parallelogram then find *x*, *m*

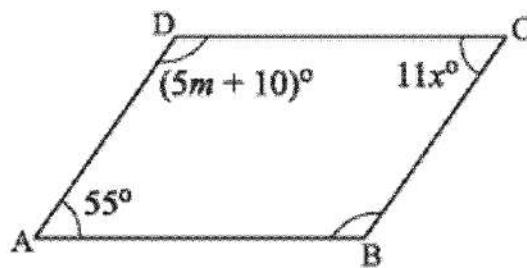
$11x° = 55°$

$x° = \frac{\cancel{55}^{5}}{\cancel{11}}$

$x° = 5°$

$\angle A + \angle B = 180°$

$\angle B = 180° - \angle A$

$\angle B = 180° - 55 = 125°$

$\angle B = 130°$
$\angle D + \angle C = 180°$
$5m + 10° + 55° = 180°$
$5m + 65° = 180°$
$5m = 180° - 65°$
$5m = 115°$

$m = \frac{115°}{5°}$

$m = 23°$

Q.5 The given figure ∠MNP is a parallelogram finds the value of m, n

4m + n = 10………. (i)

In parallelogram opposite sides are congruent 8m – 4n = 8 … (ii)

Multiply 4 with equation

$4(4m + n) = 4 \times 10$

$16m + 4n = 40$ … (iii)

Adding equation (ii) and (iv)

$8m - \cancel{4n} = 8$

$16m + \cancel{4n} = 40$

$24m = 48$

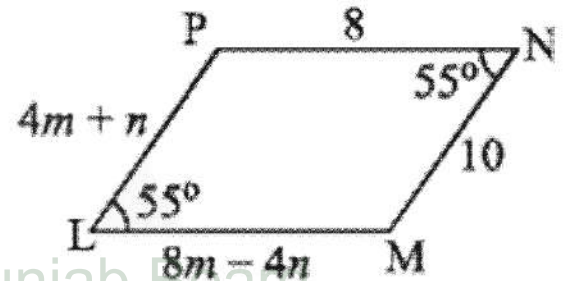
$m = \frac{48}{24}$

$m = 2$

Putting the value of m in equation (i) 4(2) + n = 10

8 + n = 10

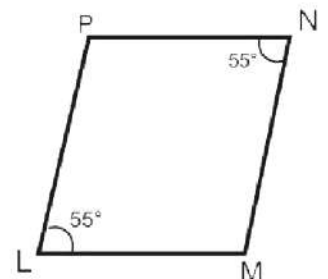
n = 10 – 8

n = 2

Q.6 In the equation 5, sum of the opposite angles of the parallelogram in 110°

∠L + ∠M = 180

55°+∠M=180°

∠M=180°-55°

∠M = 125°

$\angle P = \angle M$ opposite angles are congruent in parallelogram

$\angle P = 125^\circ$

Unit 11: Parallelograms and Triangles

Overview

Parallelogram:

If two opposite sides of a quadrilateral are congruent and parallel, it is a parallelogram.

Medians

A line segment joining a vertex of a triangle to the mid-point of the opposite side is called median of the triangle.

Trisection

The process to divide a line segment into three equal parts.

Theorem 11.11

In a parallelograms
(i) Opposite sides are congruent
(ii) Opposite angles are congruent
(iii)The diagonals bisect each other

Given

In a quadrilateral $ABCD$, $\overline{AB} \parallel \overline{DC}, \overline{BC} \parallel \overline{AD}$ and the diagonals $\overline{AC}, \overline{BD}$ meet each other at point O.

D C 1 2 6 O 5 3 4 A B

To Prove

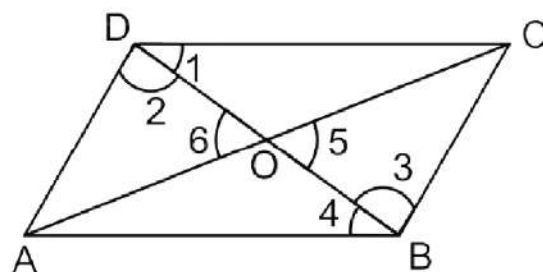
(i) $\overline{AB} \cong \overline{DC}, \overline{AD} \cong \overline{BC}$

(ii) $\angle ADC \cong \angle ABC,\ \angle BAD \cong \angle BCD$

(iii) $\overline{OA} \cong \overline{OC},\ \overline{OB} \cong \overline{OD}$

Construction

In the figure as shown, we label the angles as $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$.

Proof

Statements	Reasons
(i) In $\Delta ABD \leftrightarrow \Delta CDB$	
$\angle 4 \cong \angle 1$	Alternate angles
$\overline{BD} \cong \overline{BD}$	Common
$\angle 2 \cong \angle 3$	Alternate angles
$\therefore \Delta ABD \cong \Delta CDB$	$A.S.A \cong A.S.A$
So, $\overline{AB}, \overline{DC}, \overline{AD} \cong \overline{BC}$	(Corresponding sides of congruent triangles)
and $\angle A \cong \angle C$	(Corresponding angles of congruent triangles)
(ii) Since	
and $\angle 1 \cong \angle 4$..............(a)	Proved

$\angle 2 \cong \angle 3 ....(b)$	Proved
$\therefore m\angle 1 + m\angle 2 = m\angle 4 + m\angle 3$ or $m\angle ADC = m\angle ABC$ or $\angle ADC \cong \angle ABC$	From (a) and (b)
and $\angle BAD \cong m\angle BCD$	Proved in (i)
(iii) In $\Delta BOC \leftrightarrow \Delta DOA$	
$\overline{BC} \cong \overline{AD}$	Proved in (i)
$\angle 5 \cong \angle 6$	Vertical angles
$\angle 3 \cong \angle 2$	Proved
$\because \Delta BOC \cong \Delta DOA$	$(A.A.S \cong A.A.S)$
Hence $\overline{OC} \cong \overline{OA}, \overline{OB} \cong \overline{OD}$	(Corresponding sides of congruent triangles)

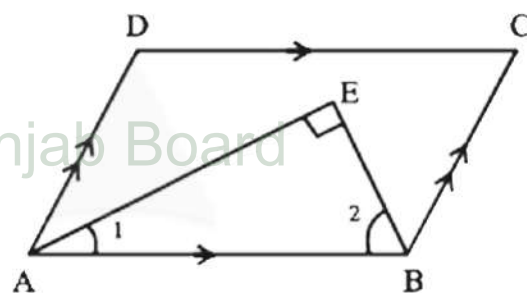
Example

The bisectors of two angles on the same side of a parallelogram cut each other at right angles.

Given

A parallelogram ABCD, in which

$\overline{AB} \parallel \overline{DC}, \overline{AD} \parallel \overline{BC}$

The bisectors of EA and EB cut each other at E.

To Prove

$m\angle E = 90°$

Construction:

Name the angles $\angle 1$ and $\angle 2$ as shown in the figure.

Proof

Statements	Reasons
$m\angle 1 + m\angle 2$ $= \frac{1}{2}(m\angle BAD + m\angle ABC)$	$m\angle 1 = \frac{1}{2} m\angle BAD$ $m\angle 2 = \frac{1}{2} m\angle ABC$
$= \frac{1}{2}(180°)$ $= 90°$	int. angles on the same side of $\overline{AB}$ which cuts $\parallel$ segments $\overline{AD}$ and $\overline{BC}$ are supplementary.
Hence in $\Delta ABE, m\angle E = 90°$	$m\angle 1 + m\angle 2 = 90°$ (proved)

Exercise 12.1

Q.1 Prove that the centre of a circle is on the right bisectors of each of its chords.

Given

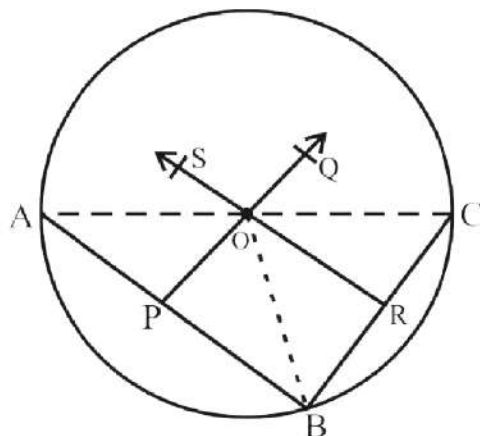
A, B, C are the three non-collinear points.

Required: To find the centre of the circle passing through A,B,C

Construction

Join B to C, A take $\overrightarrow{PQ}$ is right bisector of $\overline{AB}$ and $\overline{RS}$ right bisector of BC, they intersect at O.

Join O to A, O to B, O to C.

$\therefore$ O is the centre of circle.

Proof

Statements	Reasons
$\overline{OB} \cong \overline{OC}$ ___________ (i)	O is the right bisector of $\overline{BC}$
$\overline{OA} \cong \overline{OB}$ ___________ (ii)	O is the right bisector of $\overline{AB}$
$\overline{OA} = \overline{OB} = \overline{OC}$	From (i) and (ii)
Hence is equidistant from the A,B,C $\therefore O$ is center of circle which is required	

Q.2 Where will the center of a circle passing through three non-collinear points? And Why?

Given

A.B.C are three non collinear points and circle passing through these points.

To prove

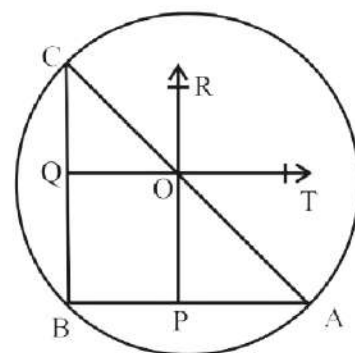
Find the center of the circle passing through vertices A, B and C.

Construction

(i) Join B to A and C.

(ii) Take $\overrightarrow{QT}$ right bisector of $\overline{BC}$ and take also $\overrightarrow{PR}$ right bisector of $\overline{AB}$.

$\overrightarrow{PR}$ and $\overrightarrow{QT}$ intersect at point O. joint O to A,B and C. O is the center of the circle.

Proof

Statements	Reasons
$\overline{QO}$ is right bisector $\overline{BC}$ $\overline{OB} \cong \overline{OC}$ ...(i) $\overline{PO}$ is right bisector of $\overline{AB}$ $\overline{OA} \cong \overline{OB}$ ...(ii) So $\overline{OA} \cong \overline{OC} \cong \overline{OB}$ $\therefore$ It is proved that O is the center of the circle.	From (i) and (ii)

Q.3 Three village P,Q and R are not on the same line. The people of these villages want to make a children park at such a place which is equidistant from these three villages. After fixing the place of children park prove that the park is equidistant from the three villages.

Given

P,Q,R are three villages not on the same straight line.

To prove

The point equidistant from P,R,Q.

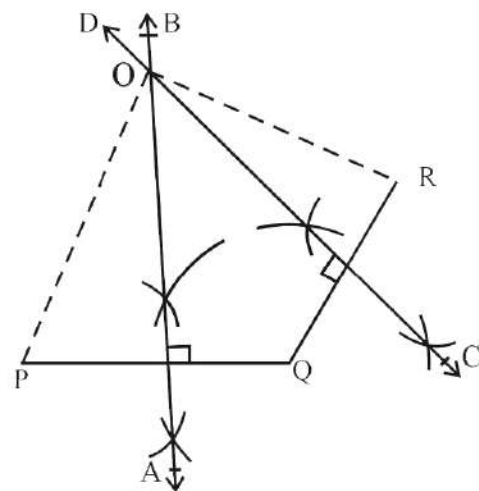
Construction

(i) Joint Q to P and R.

(ii) Take $\overleftrightarrow{AB}$ right bisector of $\overline{PQ}$ and $\overleftrightarrow{CD}$ right bisector of $\overline{QR}$. $\overline{AB}$ and $\overleftrightarrow{CD}$ intersect at O.

(iii) Join 0 to P, Q, R

The place of children part at point O.

Proof

Statements	Reasons
$\overline{OQ} \cong \overline{OR}$ ________ (i)	O is on the right bisector of $\overline{QR}$
$\overline{OP} \cong \overline{OQ}$ ________ (ii)	O is on the right bisector of $\overline{PQ}$
$\overline{OP} \cong \overline{OQ} \cong \overline{OR}$ ________ (iii)	From (i) and (ii)
$\therefore O$ is on the bisector of $\angle P$	
Hence $\overline{PO}$ is bisector of $\angle P$	

O is equidistant from P,Q and R

Theorem 12.1.3

The right bisectors of the sides of a triangle are concurrent.

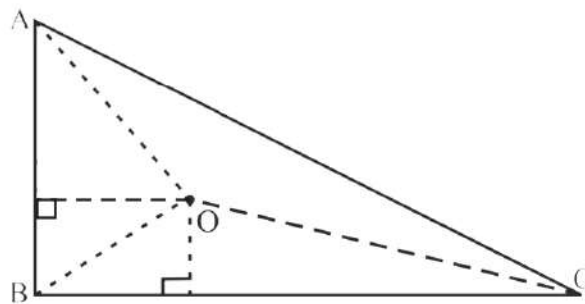
Given

ΔABC

To prove

The right bisectors of $\overline{AB}, \overline{BC}$ and $\overline{CA}$ are concurrent.

Construction

Draw the right bisectors of $\overline{AB}$ and $\overline{BC}$ which meet each other at the point O. Join O to A, B and C.

Proof

Statements	Reasons
$\overline{OA} \cong \overline{OB}$ ⟶ (i)	(Each point on right bisector of a segment is equidistant from its end points)
$\overline{OB} \cong \overline{OC}$ ⟶ (ii)	As in (i)
$\overline{OA} \cong \overline{OC}$	from (i) and (ii)
$\therefore$ Point O is on the right bisector of $\overline{CA}$ → (iv)	(O is equidistant from A and C)
But point O is on the right bisector of $\overline{AB}$ and of $\overline{BC}$ ⟶ (v)	**Construction**
Hence the right bisectors of the three sides of triangle are concurrent at O	{from (iv) and (v)}

Theorem 12.1.4

Any point on the bisector of an angle is equidistant from its arms.

Given

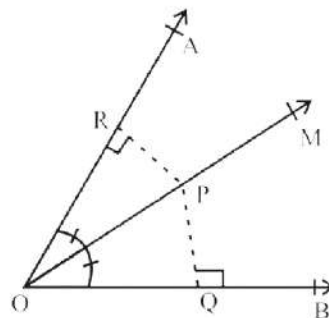
A point P is on $\overrightarrow{OM}$, the bisector of $\angle AOB$

To Prove

$\overline{PQ} \cong \overline{PR}$ i.e P is equidistant from $\overrightarrow{OA}$ and $\overrightarrow{OB}$

Construction

Draw $\overline{PR} \perp \overrightarrow{OA}$ and $\overline{PQ} \perp \overrightarrow{OB}$

Proof

Statements	Reasons
In $\Delta POQ \leftrightarrow \Delta POR$	
$\overline{OP} \cong \overline{OP}$	Common
$\angle PQO \cong \angle PRO$	Construction
$\angle POQ \cong \angle POR$	Given
$\therefore \Delta POQ \cong \Delta POR$	$S.A.A \cong S.A.A$
Hence $\overline{PQ} \cong \overline{PR}$	(Corresponding sides of congruent triangles)

Theorem 12.1.5 (Converse of Theorem 12.1.4)

Any point inside an angle, equidistant from its arms, is on the bisector of it.

Given

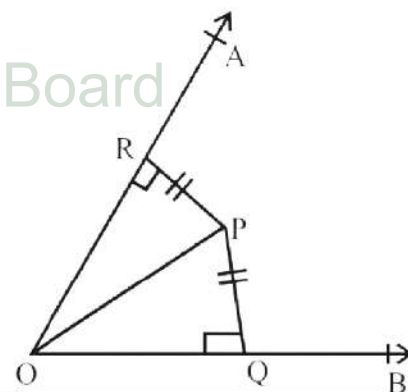
Any point P lies inside $\angle AOB$, such that $\overline{PQ} \cong \overline{PR}$, where $\overline{PQ} \perp \overrightarrow{OB}$ and $\overline{PR} \perp \overrightarrow{OA}$

To prove

Point P is on the bisector of $\angle AOB$

Construction

Join P to O

Proof

Statements	Reasons
In $\Delta POQ \leftrightarrow \Delta POR$	
$\angle PQO \cong \angle PRO$	Given (Right angles)
$\overline{PO} \cong \overline{PO}$	Common
$\overline{PQ} \cong \overline{PR}$	Given
$\therefore \Delta POQ \cong \Delta POR$	$H.S \cong H.S$
Hence $\angle POQ \cong \angle POR$	(Corresponding angles of congruent triangles)
i.e, P is on the bisector of $\angle AOB$	

Exercise 12.2

Q.1 In a quadrilateral ABCD $\overline{AB} \cong \overline{BC}$ and the right bisectors of $\overline{AD}, \overline{CD}$ meet each other at point N. Prove that $\overline{BN}$ is a bisector of $\angle ABC$

Given

In the quadrilateral ABCD

$\overline{AB} \cong \overline{BC}$

$\overline{NM}$ is right bisector of $\overline{CD}$

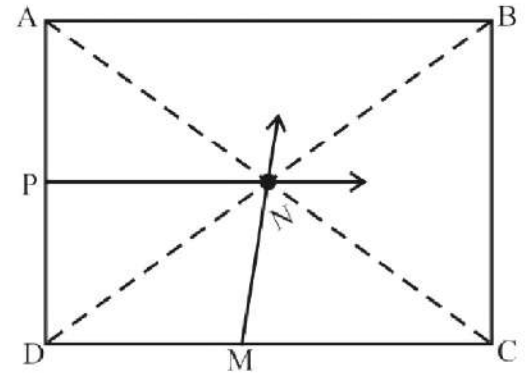
$\overline{PN}$ is right bisector of $\overline{AD}$

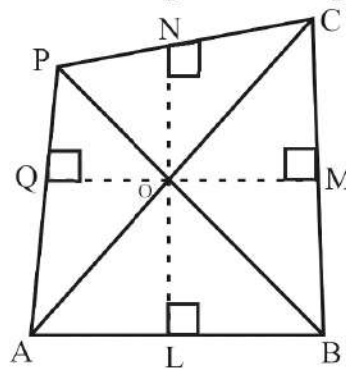
They meet at N

To prove

$\overline{BN}$ is the bisector of angle ABC

Construction join N to A,B,C,D

Proof

Statements	Reasons
$\overline{ND} \cong \overline{NA}$ ________ (i)	N is an right bisector of $\overline{AD}$
$\overline{ND} \cong \overline{NC}$ ________ (ii)	N is on right bisector of $\overline{DC}$
$\overline{NA} = \overline{NC}$ ________ (iii)	from (i) and (ii)
$\Delta BNC \leftrightarrow \Delta ANB$	
$\overline{NC} = \overline{NA}$	Already proved (from iii)
$\overline{AB} \cong \overline{CB}$	Given
$\overline{BN} \cong \overline{BN}$	Common
$\therefore \Delta BNA \cong \Delta BNC$	$S.S.S \cong S.S.S$
Hence $\angle ABN \cong \angle NBC$	Corresponding angles of congruent triangles
Hence $\overline{BN}$ is the bisector of $\angle ABC$	

Q.2 The bisectors of $\angle A, \angle B$ and $\angle C$ of a quadrilateral ABCP meet each other at point O. Prove that the bisector of $\angle P$ will also pass through the point O.

Given

ABCP is quadrilateral. $\overline{AO}, \overline{BO}, \overline{CO}$ are bisectors of $\angle A, \angle B$ and $\angle C$ meet at point O.

To prove

$\overline{PO}$ is bisector of $\angle P$

Construction:

Join P to O.

Draw $\overline{OQ} \perp \overline{AP}$, $\overline{ON} \perp \overline{PC}$ and $\overline{OL} \perp \overline{AB}$, $\overline{OM} \perp \overline{BC}$

Proof:

Statements	Reasons
$\overline{OM} \cong \overline{ON}$ ________ (i)	O is on the bisector of $\angle C$
$\overline{OL} \cong \overline{OM}$ ________ (ii)	O is on the bisector of $\angle B$
$\overline{OL} \cong \overline{OQ}$ ________ (iii)	O is on the bisector of $\angle A$
$\overline{OQ} \cong \overline{ON}$	From i, ii, iii
Point O lines on the bisector of $\angle P$	
$\therefore \overline{OP}$ is the bisector of angle P	

Q.3 Prove that the right bisector of congruent sides of an isosceles triangle and its altitude are concurrent.

Given

ΔABC

$\overline{AB} \cong \overline{AC}$ due to isosceles triangle $\overline{PM}$ is right bisector of $\overline{AB}$

$\overline{QN}$ is right bisector of $\overline{AC}$

$\overrightarrow{PM}$ and $\overrightarrow{QN}$ intersect each other at point O

A

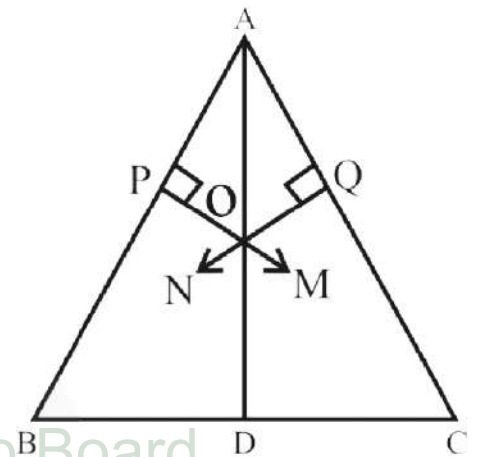
P O Q

N M

B D C

Required

The altitude of ΔABC lies at point O

Join A to O and extend it to cut $\overline{BC}$ at D.

Proof

Statements	Reasons
$m\overline{AB} \cong m\overline{AC}$	Given
$\frac{1}{2}m\overline{AB} = \frac{1}{2}m\overline{AC}$	Dividing both side by 2
$\overline{AQ} \cong \overline{AP}$	
In $\Delta AQO \leftrightarrow \Delta APO$	
$\angle APO \cong \angle AQO$	Each 90^o (Given)
$\overline{AQ} \cong \overline{AP}$	Already Proved
$\overline{AO} \cong \overline{AO}$	Common
$\Delta APO \cong \Delta AQO$	$H.S \cong H.S$
$\angle PAO \cong \angle QAO$ (i)	Corresponding angles of congruent triangles
$\Delta BAD \leftrightarrow \Delta CAD$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{AD} \cong \overline{AD}$	Common

$\angle BAD \cong \angle CAD$	Proved from (i)
$\Delta BAD \cong \Delta CAD$	$S.A.S \cong S.A.S$
$\angle ODB \cong \angle ODC$	Each angle is 90^o (Given)
$m\angle ODM + m\angle ODC = 180^o$	Supplementary angle
$\therefore \overline{AD} \perp \overline{BC}$	
Point 0 lies on altitude $\overline{AD}$	

Q.4 Prove that the altitudes of a triangle are concurrent.

Given

In ΔABC

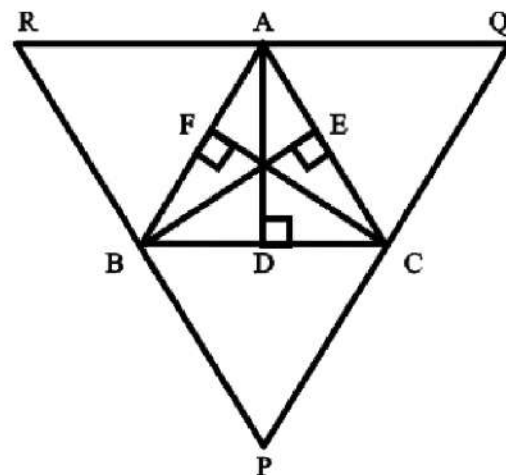
$\overline{AD}, \overline{BE}, \overline{CF}$ are its altitiudes

i.e $\overline{AD} \perp \overline{BC}, \overline{BE} \perp \overline{AC}, \overline{CF} \perp \overline{AB}$

Required $\overline{AD}, \overline{BE}$ and $\overline{CF}$ are concurrent

Construction:

Passing through A, B, C take

$\overline{RQ} \| \overline{BC}, \overline{RP} \| \overline{AC}$ and $\overline{QP} \| \overline{AB}$ respectively forming a ΔPQR

Proof

Statements	Reasons
$\overline{BC} \| \overline{AQ}$	Construction
$\overline{AB} \| \overline{QC}$	Construction
$\therefore$ ABCQ is a $\|^{gm}$	
Hence $\overline{AQ} \cong \overline{BC}$	
Similarly $\overline{AB} \cong \overline{QC}$	
Hence point A is midpoint RQ	
And $\overline{AD} \perp \overline{BC}$	Given
$\overline{BC} \| \overline{RQ}$	Opposite sides of parallelogram ABCQ
$\overline{AD} \| \overline{RQ}$	
Thus $\overline{AD} \perp$ is right bisector of $\overline{RQ}$	
similarly $\overline{BE}$ is a right bisector of $\overline{RP}$ and	
$\overline{CF}$ is right bisector of PQ	
$\therefore \perp^s$ $\overline{AD}, \overline{BE}, \overline{CF}$ are right bisector of sides of ΔPQR	
$\therefore \overline{AD}, \overline{BE}$ and $\overline{CF}$ are Concurrent	

Theorem12.1.6

The bisectors of the angles of a triangle are concurrent

Given

ΔABC

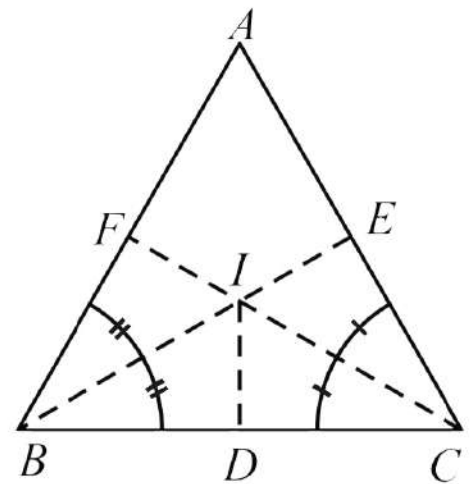
To Prove

The bisector of $\angle A$, $\angle B$, and $\angle C$ are concurrent

Construction:

Draw the bisectors of $\angle B$ and $\angle C$ which intersect at point I. From I, draw

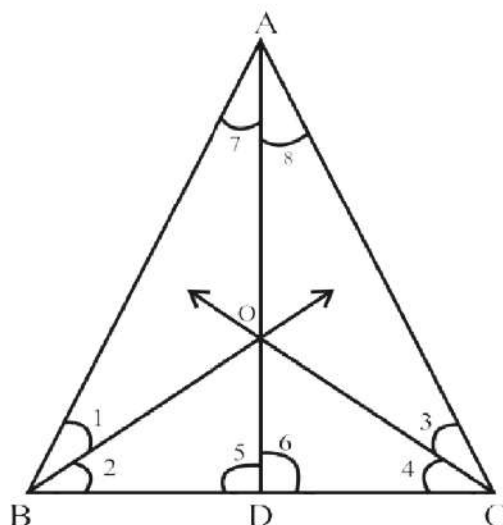
$\overline{IF} \perp \overline{AB}$, $\overline{ID} \perp \overline{BC}$ and $\overline{IE} \perp \overline{CA}$

Proof

Statements	Reasons
$\overline{ID} \cong \overline{IF}$	(Any point on bisector of an angle is equidistance from its arms.
Similarly $ID \cong IE$ $\therefore \overline{IE} \cong \overline{IF}$	Each $\cong$ ID
So the point I is on the bisector of $\angle A$ … (i)	
Also the point I is on the bisectors of $\angle ABC$ and $\angle BCA$ … (ii)	Construction
Thus the bisector of $\angle A$, $\angle B$ and $\angle C$ are concurrent at I	{From (i) and (ii)}

Exercise 12.3

Q.1 Prove that the bisectors of the angles of base of an isosceles triangle intersect each other on its altitude.

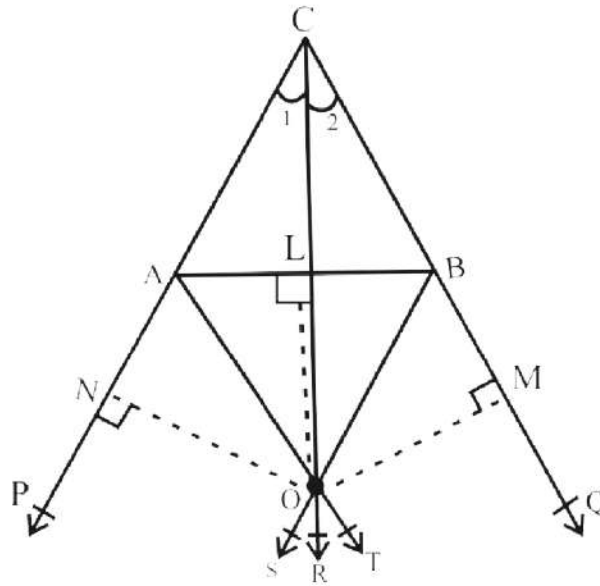
Given

ΔABC

$\overline{AB} = \overline{AC}$ Due to isosceles triangle

Bisect $\angle B$ and $\angle C$ to intersect at point O Join A to D and extend to BC at D $\overline{AD}$ is the altitude of ΔABC $\overline{AD} \perp \overline{BC}$

Proof

Statements	Reasons
In ΔABC	
$\overline{AB} \cong \overline{AC}$	Given
$\angle B \cong \angle C$	Due to isosceles triangle opposite angle are congruent
$\frac{1}{2} m\angle B = \frac{1}{2} m\angle C$	Dividing both side by 2
$\angle 1 \cong \angle 3$	
$\Delta ABO \leftrightarrow \Delta ACO$	
$\overline{AO} = \overline{AO}$	
$\overline{AB} = \overline{AC}$	
$\overline{BO} \cong \overline{CO}$	Given
$\Delta ABO \cong \Delta ACO$	Due to isosceles triangle
$\Delta ABD \leftrightarrow \Delta ACD$	
$\overline{AD} \cong \overline{AD}$	
$\angle 7 \cong \angle 8$	
$\overline{AB} \cong \overline{AC}$	
$\Delta ABD \cong \Delta ACD$	
$\angle 5 + \angle 6 = 180$	
$\angle 5 = \angle 6 = 90^{o}$	
So $\overline{AD} \perp \overline{BC}$	Supplementary angles
$\overline{AD}$ Passes from point O	

Q.2 Prove that the bisectors of two exterior and third interior angle of a triangle are concurrent

Given

ΔABC

Exterior angles are $\angle ABQ$ and $\angle BAP$ $\overrightarrow{AT}$ and $\overrightarrow{BS}$ intersect each other at point O therefore join O to C

Draw the angle bisecter of C

$\angle 1 \cong \angle 2$

Construction

$\overline{OM} \perp \overrightarrow{CQ}, \overline{OL} \perp \overline{AB}, \overline{ON} \perp \overrightarrow{CP}$

Proof

Statements	Reasons
$\overline{ON} \cong \overline{OM}$...........(i)	
$\overline{OL} \cong \overline{OM}$...........(ii)	
$\overline{ON} \cong \overline{OL}$	
Hence Angle Bisector of C	Comparing equation (i) and (ii)
i,e $\angle 1 \cong \angle 2$	

Review Exercise 12

Q.1 Which of the following are true and which are false?

(i) Bisection means to divide into two equal parts (True)

(ii) Right bisection of line segment means to draw perpendicular which passes through the midpoint of line segment (True)

(iii) Any point on the right bisector of a line segment is not equidistant from its end points (False)

(iv) Any point equidistant from the end points of a line segment is on the right bisector of it (True)

(v) The right bisectors of the sides of a triangle are not concurrent (False)

(vi) The bisectors of the angles of a triangle are concurrent (True)

(vii) Any point on the bisector of an angle is not equidistant from its arms (False)

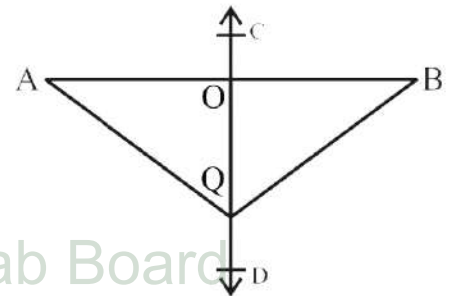
(viii) Any point inside an angle equidistant from its arms, is on the bisector of it (True)

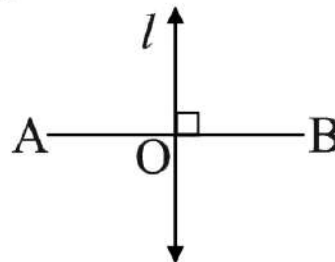
Q.2 If $\overleftrightarrow{CD}$ is right bisector of line segment $\overline{AB}$, then

(i) $m\overline{OA} =$ _______ (ii) $m\overline{AQ} =$ _______

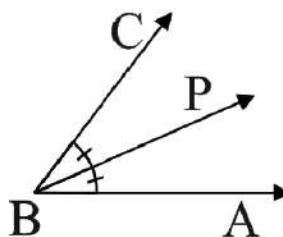
Solution

(i) $m\overline{OA} = m\overline{OB}$

(ii) $m\overline{AQ} = m\overline{BQ}$

Q.3 Define the following

(i) Right Bisector of a Line Segment

A line l is called a right bisector of a line segment if l is perpendicular to the line segment and passes through its midpoint.

(ii) Bisector of an Angle

A ray BP is called the bisector of $m\angle ABC$, if P is a point in the interior of the angle and $m\angle ABP = m\angle PBC$.

Q.4 The given triangle ABC is equilateral triangle and $\overline{AD}$ is bisector of angle A, then find, the values of unknown x^o, y^o and z^o.

Solution

In equilateral triangle all side are equal to each and there angle of the triangle equal to 60°.

So

$\angle B = z^o = 60^o$

$\overline{AD}$ is the bisector of $\angle A$

$\angle A = 60^o$

$\therefore$ When angle A is bisected

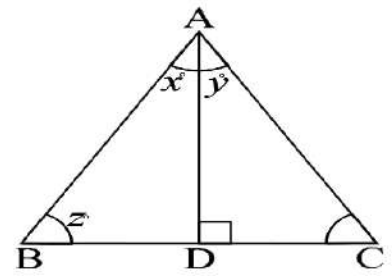
$x^\circ = y^\circ$

$x^\circ = \frac{1}{2} m\angle A$

$= \frac{1}{2} \times 60^\circ$

$x^\circ = 30^\circ$

$y^\circ = 30^\circ \qquad (\because x^\circ = y^\circ)$

So $x^o = y^o = 30^o$

Q.5 In the given congruent triangle LMO and LNO find the unknowns x and m given

$\Delta LMO \cong \Delta LNO$

$m\overline{LM} = m\overline{LN}$

$2x + 6 = 18$

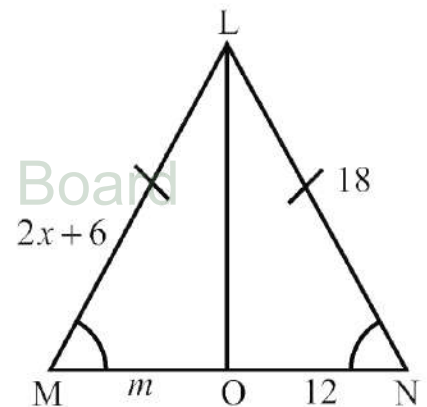
$2x = 18 - 6$

$2x = 12$

$x = \frac{12^6}{2}$

$x = 6$ Unit

$m\overline{MO} = m\overline{ON}$

$\therefore m = 12$ unit

Q.6 CD is right bisector of the line segment AB

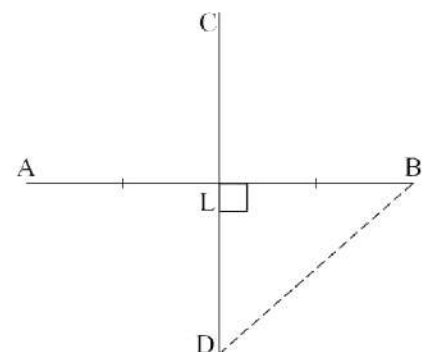
(i) If $m\overline{AB} = 6cm$ then find the $m\overline{AL}$ and $m\overline{LB}$

Solution

L is the midpoint of $\overline{AB}$

$\therefore m\overline{AL} = m\overline{LB}$

$m\overline{AL} = \frac{1}{2} mAB = \frac{1}{2} \times 6$

So $m\overline{AL} = 3cm$

$m\overline{LB} = 3cm \quad \left(\therefore m\overline{AL} = m\overline{LB}\right)$

(ii) **If** $m\overline{BD} = 4cm$ **then find m** $\overline{AD}$

$m\overline{AD} = m\overline{BD}$ (Any point on the right bisector of a line segment is equidistant from its end points.)

$m\overline{AD} = 4$

$m\overline{AD} = 4cm$

Unit 12: Line Bisectors and Angle Bisectors

Overview

Right Bisector of a line segment:

Right bisection of a line segment means to draw a perpendicular at the mid-point of line segment.

Bisector of an angle:

Bisection of an angle means to draw a ray to divide the given angle into two equal parts.

Theorem 12.1.1

Statement:

Any point on the right bisector of a line segment is equidistant from its end points.

Given

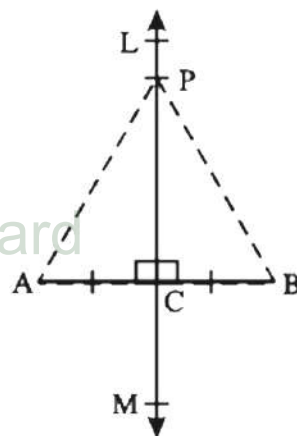
A line $\overleftrightarrow{LM}$ intersects the line segment AB at the point C Such that $\overleftrightarrow{LM} \perp \overline{AB}$ and $\overline{AC} \cong \overline{BC}$. P is a point on $\overleftrightarrow{LM}$

To prove

$\overline{PA} \cong \overline{PB}$

Construction

Join P to the point A and B

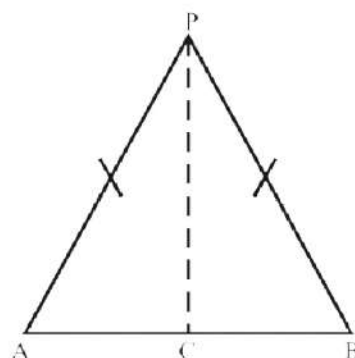
Proof

Statements	Reasons
In $\Delta ACP \leftrightarrow \Delta BCP$	
$\overline{AC} \cong \overline{BC}$	Given
$\angle ACP \cong \angle BCP$	Given $\overline{PC} \perp \overline{AB}$, so that each $\angle$ at $C = 90^o$
$\overline{PC} \cong \overline{PC}$	Common
$\therefore \Delta ACP \cong \Delta BCP$	S.A.S Postulate
Hence $\overline{PA} \cong \overline{PB}$	(Corresponding sides of congruent triangles)

Theorem 12.1.2

{Converse of Theorem 12.1.1}

Any point equidistant from the end points of a line segment is on the right bisector of it.

Given

$\overline{AB}$ is a line segment. Point P is such that $\overline{PA} \cong \overline{PB}$

To prove

The point P is the on the right bisector of $\overline{AB}$

Construction

Join P to C, the midpoint of $\overline{AB}$

Proof

Statements	Reasons
In $\Delta ACP \leftrightarrow \Delta BCP$	
$\overline{PA} \cong \overline{PB}$	Given
$\overline{PC} \cong \overline{PC}$	Common
$\overline{AC} \cong \overline{BC}$	Construction
$\therefore \Delta ACP \cong \Delta BCP$	$S.S.S \cong S.S.S$
$\angle ACP \cong \angle BCP$ ___________ (i)	Corresponding angles of congruent triangles
But $m\angle ACP + m\angle BCP = 180^o$ ___ (ii)	Supplementary angles
$\therefore m\angle ACP = m\angle BCP = 90^o$	From (i) and (ii)
i.e $\overline{PC} \perp \overline{AB}$ ___________ (iii)	$m\angle ACP = 90^o$ (Proved)
Also $\overline{CA} \cong \overline{CB}$ ___________ (iv)	**Construction**
$\therefore$ $\overline{PC}$ is a right bisector of $\overline{AB}$	from (iii) and (iv)
i.e. the point P is on the right bisector of $\overline{AB}$	

Exercise 13.1

Q.1 Two sides of a triangle measure 10cm and 15 cm which of the following measure is possible for the third side?

(a) $5cm$
(b) $20\ cm$
(c) $25\ cm$
(d) $30\ cm$

Solution

Lengths of two sides are 15 and 10 cm.
So, sum of two lengths of triangle = 10 + 15 = 25 m
$10 + 15 > 20$
$10 + 20 > 15$
$15 + 20 > 10$
$\therefore$ 20 cm is possible for third side
Or
Sum of length of two sides is always greater than the third sides of a triangle.
Given

Q.2 Point O is interior of ΔABC
Show that

$$m\overline{OA} + m\overline{OB} + m\overline{OC} > \frac{1}{2}\left(m\overline{AB} + m\overline{BC} + m\overline{CA}\right)$$

Given
Point O is interior of ΔABC
To prove:

$$m\overrightarrow{OA} + m\overline{OB} + m\overline{OC} < \frac{1}{2}\left(m\overline{AB} + m\overline{BC} + m\overline{AC}\right)$$

Construction
Join O with A, B and C.
So that we get three triangle ΔOAB, ΔOBC and ΔOAC
Proof

Statements	Reasons
In ΔOAB $m\overline{OA} + OB > m\overline{AB}$ _____(i)	In any triangle the sum of length of two sides is greater then the third sides.
In ΔOAC $m\overline{OC} + m\overline{OA} > m\overline{AC}$ _____(ii)	As in (i)
In ΔOBC $mOB + \overline{OC} > m\overline{BC}$ _____(iii)	As in (i)
Adding equation i, ii and iii $\overline{OA} + \overline{OC} + \overline{OA} + \overline{OB} + \overline{OB} + \overline{OC} > \overline{AC} + \overline{AB} + \overline{BC}$ $2\overline{OA} + 2\overline{OC} + 2\overline{OB} > \overline{AB} + \overline{BC} + \overline{CA}$ $2(OA + OC + OB) > \overline{AB} + \overline{BC} + \overline{CA}$	

$\frac{\not{2}(OA+OC+OB)}{\not{2}} > \frac{\overline{AB}+\overline{BC}+\overline{CA}}{2}$ $(OA+OC+OB) > \frac{1}{2}(\overline{AB}+\overline{BC}+\overline{CA})$	Dividing both sides by 2

Q.3 In the ΔABC m∠B = 70° and m∠C = 45° which of the sides of the triangle is longest and which is the shortest.

Solution

Sum of three angle in a triangle is 180°

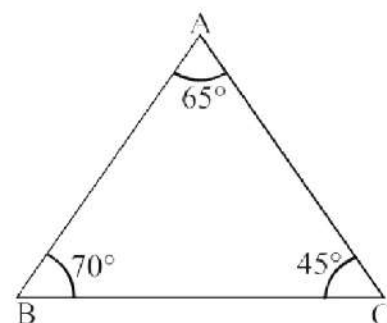
$\angle A + \angle B + \angle C = 180$

$\angle A + 70 + 45 = 180$

$\angle A + 115 = 180$

$\angle A = 180 - 115$

$\angle A = 65^\circ$

Sides of the triangle depend upon the angles largest angle has largest opposite side and smallest angle has smallest opposite side here $\angle B$ is largest so, $\overline{AC}$ is largest $\angle C$ is smallest, so $\overline{AB}$ is smallest side.

Q.4 Prove that in a right-angled triangle, the hypotenuse is longer than each of the other two sides.

Solution

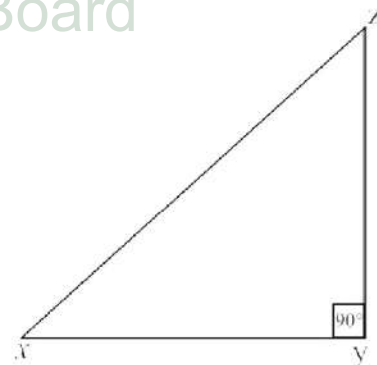
Sum of three angles in a triangle is equal to 180°. So in a triangle one angle will be equal to 90° and rest of two angles are acute angle (less than 90°)

$\therefore$ m∠y = 90

And m∠x + m∠z = 90

So m∠x and m∠z are acute angle

$\therefore$ Opposite to m∠y = 90° is hypotenuse

It is largest side.

Q.5 In the triangular figure $\overline{AB} > \overline{AC}$.$\overline{BD}$ and $\overline{CD}$ are the bisectors of ∠B and ∠C respectively prove that $\overline{BD} > \overline{DC}$

Given

InΔABC

$\overline{AB} > \overline{AC}$

$\overline{BD}$ and $\overline{CD}$ are the bisectors of $\angle B$ and $\angle C$

To prove

$\overline{BD} > \overline{CD}$

Construction

Label the angles $\angle 1, \angle 2, \angle 3$ and $\angle 4$

Proof

Statements	Reasons
In ΔABC	
$\overline{AB} > \overline{AC}$	Given
$\overline{BD}$ is the bisector of $\angle B$	
$\frac{1}{2}m\angle ACB > \frac{1}{2}m\angle ABC$	
$m\angle ABC$	
$m\angle 2 < m\angle 4$	
$\overline{CD}$ is the bisector of $\angle C$	Given
InΔBCD	
$\overline{BD} > \overline{DC}$	Side opposite to greater angle is greater

Theorem 13.1.4

From a point, out side a line, the perpendicular is the shortest distance from the point to the line.

Given:

A line AB and a point C

(Not lying on $\overleftrightarrow{AB}$)and a point D on $\overleftrightarrow{AB}$ such that

$\overline{CD} \perp \overleftrightarrow{AB}$

To prove

$m\overline{CD}$ is the shortest distance from the point C to $\overleftrightarrow{AB}$

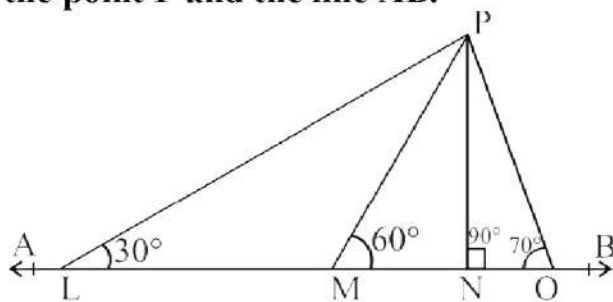
Construction

Take a point E on $\overleftrightarrow{AB}$ .Join C and E to form a ΔCDE

Proof

Statements	Reasons
In ΔCDE	
$m\angle CDB > m\angle CED$	(An exterior angle of a triangle is greater than non adjacent interior angle)
But $m\angle CDB = m\angle CDE$	Supplement of right angle
$\therefore$ $m\angle CDE > m\angle CED$	
Or $m\angle CED < m\angle CDE$	
Or $m\overline{CD} < m\overline{CE}$	Side opposite to greater angle is greater.
But E is any point on $\overleftrightarrow{AB}$	
Hence $m\overline{CD}$ is the shortest distance from C to $\overleftrightarrow{AB}$	

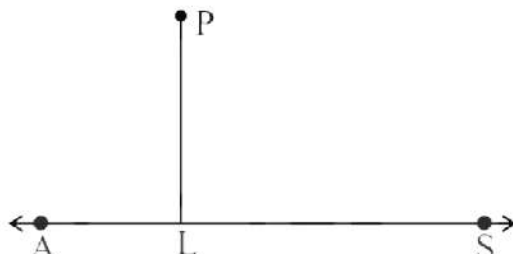
Exercise 13.2

Q.1 In the figure P is any point and AB is a line which of the following is the shortest distance between the point P and the line AB.

(a) $m\overline{PL}$ (b) $m\overline{PM}$ (c) $m\overline{PN}$ (d) $m\overline{PO}$

As we know that $\overline{PN} \perp \overleftrightarrow{AB}$

So $\overline{PN}$ is the shortest distance

Q.2 In the figure, P is any point lying away from the line $\overline{AB}$. Then $m\overline{PL}$ will be the shortest distance if

(a) $m\angle P \angle A = 80^o$ (b) $m\angle P\angle B = 100^o$ (c) $m\angle P\angle A = 90^o$

Solution:

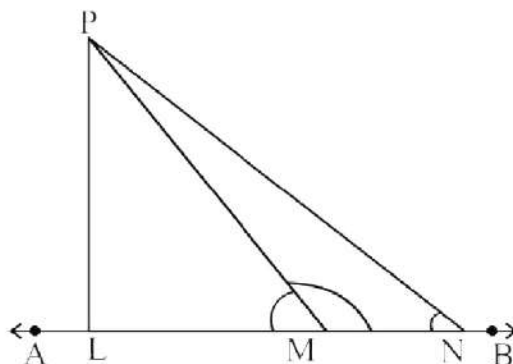
$m\angle PLA = 90^o$

$\overline{PL} \perp \overleftrightarrow{AS}$

PL is the shortest distance

So $\angle PLA$ or PLS equal to 90^o

Q.3 In the figure, $\overline{PL}$ is perpendicular to the line AB and $\overline{LN} > m\overline{LM}$. Prove that $m\overline{PN} > m\overline{PM}$.

Given

$\overline{PL} \perp \overleftrightarrow{AB}$

$m\overline{LN} > m\overline{LM}$

To proved:

$m\overline{PN} > m\overline{PM}$

Proof

Statements	Reasons
ΔPLM	
$\angle PLM = 90^o$	
$\therefore \angle PMN > \Delta PLM$	Exterior angle
$\angle PMN > 90^o$	
InΔPLN	
$\angle PLN = 90^o$	
$m\angle PNL < 90^o$	Acute angle
ΔPMN	
$m\angle PMN > m\angle PNL$	
$\therefore \overline{PN} > \overline{PM}$	

Review Exercise 13

Q.1 Which of the following are true and which are false?

(i)	The angle opposite to the longer side is greater.	(True)
(ii)	In a right-angled triangle greater angle is of 60°.	(False)
(iii)	In an isosceles right-angled triangle, angles other than right angle are each of 45°.	(True)
(iv)	A triangle having two congruent sides is called equilateral triangle.	(False)
(v)	A perpendicular from a point to line is shortest distance.	(True)
(vi)	Perpendicular to line forms an angle of 90°.	(True)
(vii)	A point out side the line is collinear.	(False)
(viii)	Sum of two sides' of a triangle is greater than the third.	(True)
(ix)	The distance between a line and a point on it is zero.	(True)
(x)	Triangle can be formed of length 2cm, 3cm and 5cm.	(False)

Q.2 What will be angle for shortest distance from an outside point to the line?

The angle for shortest distance from an outside point to the line is 90° angle.

Q.3 If 13cm, 12cm and 5cm are the length of a triangle, then verify that difference of measures of any two sides of a triangle is less than the third side.

$a = 13,\ b = 5,\ c = 12$ cm

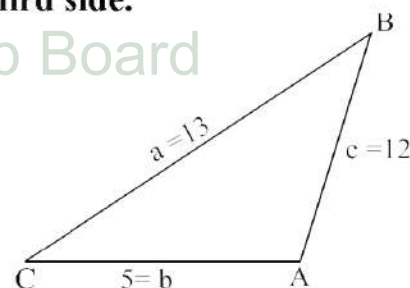
$a - b = 13 - 5 = 8$

$8 < c$

$c - b = 12 - 5 = 7$

$7 < a$

$a - c = 13 - 12 = 1$

$1 < b$

This is the process which show the difference of any two sides of a triangle is less then the measure of the third.

Q.4 If 10cm, 6cm and 8cm are the length of a triangle, then verify that sum of measures of two sides of a triangle is greater than the third side.

$a = 8$cm, $b = 10$cm, $c = 6$cm

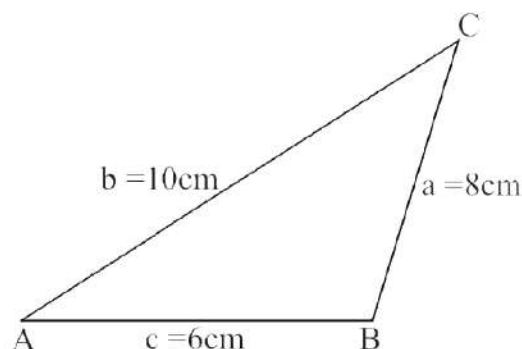
$8 + 10 = 18\text{cm} > 6\text{cm}$

$a + b > c$

$10 + 6 = 16\text{cm} > 8\text{cm}$

$b + c > a$

$6 + 8 = 14\text{cm} > 10\text{cm}$

$c + a > b$

$\therefore$ The sum of measures of two sides of a triangle is greater than the third side.

Q.5 3cm, 4cm and 7cm are not the length of the triangle. Give reasons.

a = 3cm b = 4cm c = 7cm

$3 + 4 = 7$

$a + b = c$

$b + c > a$

$4 + 7 > 3$

$c + a > b$

$7 + 3 > 4$

In a triangle sum of measures of two sides should be greater than the third sides.

Q.6 If 3cm and 4cm are the length of two sides of a right angle triangle than what should be the third length of the triangle.

If sum of the squares of two sides of a triangles is equal to the square of the third side then it is called right angled triangle.

So by Pythagoras theorem.

$(\overline{AC})^2 = (BC)^2 + (AB)^2$

$(\overline{AC})^2 = (4)^2 + (3)^2$

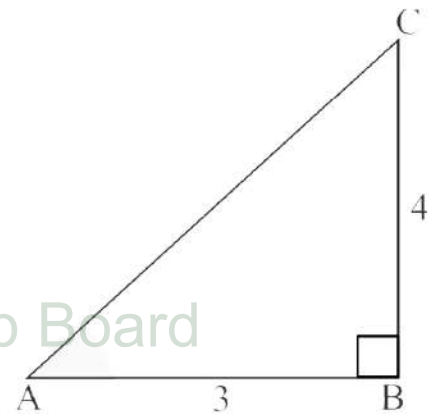
$(\overline{AC})^2 = 16 + 9$

$(\overline{AC})^2 = 25$

Taking square root on both sides

$\sqrt{(\overline{AC})^2} = \sqrt{25}$

$\overline{AC} = 5cm$

$\therefore$ Length of third side of right angled triangle is 5cm.

Unit 13: Sides and Angles of Triangles

Overview

Theorem 13.1.1

If two sides of a triangle are unequal in length, the longer side has an angle of greater measure opposite to it.

Given:

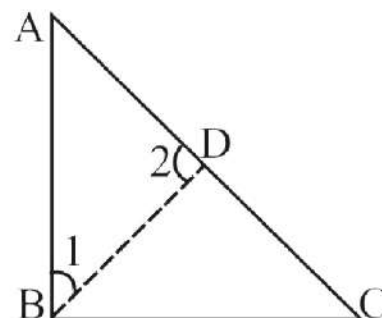
In ΔABC, $m\overline{AC} > m\overline{AB}$

To prove

$m\angle ABC > m\angle ACB$

Construction

On $\overline{AC}$ take a point D such that $\overline{AD} \cong \overline{AB}$. Join B to D so that ΔADB is an isosceles triangle. Label $\angle 1$ and $\angle 2$ as shown in the given figure.

Proof

Statements	Reasons
In ΔABD	
$m\angle 1 = m\angle 2$ ... (i)	Angles opposite to congruent sides (construction)
In ΔBCD, $m\angle ACB < m\angle 2$	
i.e. $m\angle 2 > m\angle ACB$ _____ (ii)	(An anterior angle of a triangle is greater than a non adjacent interior angle.)
$\therefore$ $m\angle 1 > m\angle ACB$ _____ (iii)	By (i) and (ii)
But $m\angle ABC = m\angle 1 + m\angle DBC$	Postulate of addition of angles
$\therefore$ $m\angle ABC > m\angle 1$ _____ (iv)	
$\therefore$ $m\angle ABC > m\angle 1 > m\angle ACB$	By (iii) and (iv)
Hence $m\angle ABC > m\angle ACB$	(Transitive property of inequality of real number)

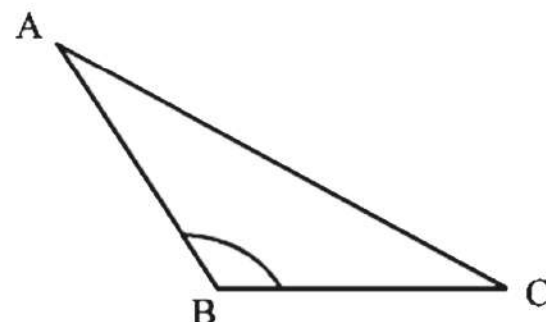
Example 1

Prove that in a scalene triangle, the angle opposite to the largest side is of measure greater than 60°. (i.e., two-third of a right-angle)

Given

In $\Delta ABC, m\overline{AC} > m\overline{AB}, m\overline{AC} > m\overline{BC}$.

To prove

$m\angle B > 60^\circ$

Proof

Statements	Reasons
In ΔABC	
$m\angle B > m\angle C$	$m\overline{AC} > m\overline{AB}$ (given)
$m\angle B > m\angle A$	$m\overline{AC} > m\overline{BC}$ (given)
But $m\angle A + m\angle B + m\angle C = 180°$	$\angle A, \angle B, \angle C$ are the angles of ΔABC
$\therefore m\angle B + m\angle B + m\angle B > 180°$	$m\angle B > m\angle C, m\angle B > m\angle A$ (proved)
Hence $m\angle B > 60°$	$\frac{180°}{3} = 60°$

Example 2

In a quadrilateral ABCD, $\overline{AB}$ is the longest side and $\overline{CD}$ is the shortest side. Prove that ΔBCD>mBAD

Given

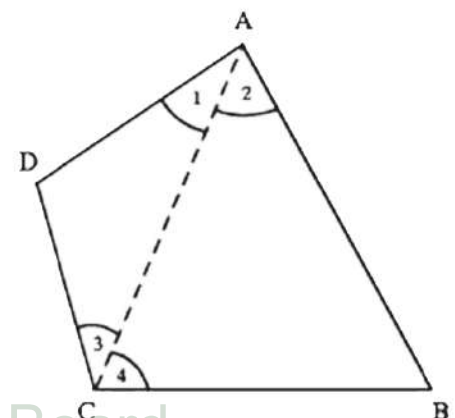
In quad. ABCD, $\overline{AB}$ is the longest side and $\overline{CD}$ is the shortest side.

To prove

m∠BCD>m∠BAD

Construction

Joint A to C.

Name the angles ∠1, ∠2, ∠3 and ∠4 as shown in the figure.

Proof

Statements	Reasons
In $\Delta ABC, m\angle 4 > \angle 2 \ldots (\text{i})$	$m\overline{AB} > m\overline{BC}$ (given)
In $\Delta ACD, m\angle 3 > \angle 1 \ldots (\text{ii})$	$m\overline{AD} > m\overline{CD}$ (given)
$\therefore m\angle 4 + m\angle 3 > m\angle 2 + m\angle 1$	From (i) and (ii)
Hence $m\angle BCD > m\angle BAD$	$\therefore \begin{cases} m\angle 4 + m\angle 3 = m\angle BCD \\ m\angle 2 + m\angle 1 = m\angle BAD \end{cases}$

Theorem 13.1.2 (Converse of theorem 13.1.1)

If two angles of a triangle are unequal in measure, the side opposite to the greater angle is longer than the side opposite to the smaller angle.

Given:

In ΔABC, m∠A > m∠B

To prove

$m\overline{BC} > m\overline{AC}$

Proof

Statements	Reasons
If $m\overline{BC} \not> m\overline{AC}$, then Either (i) $m\overline{BC} = m\overline{AC}$ Or (ii) $m\overline{BC} < m\overline{AC}$	(Trichotomy property of real numbers)
From (i) if $m\overline{BC} = m\overline{AC}$, then	
$m\angle A = m\angle B$	(Angles opposite to congruent sides are congruent)
Which in not possible From (ii) if $m\overline{BC} < m\overline{AC}$, then	
$m\angle A < m\angle B$ This is also not possible	(The angle opposite to longer side is greater than angle opposite to smaller side? Contrary to the given
$\therefore$ $m\overline{BC} \neq m\overline{AC}$	
And $m\overline{BC} \not< m\overline{AC}$ Thus $m\overline{BC} > m\overline{AC}$	Trichotomy property of real numbers.

Corollaries

(i) The hypotenuse of a right triangle is longer than each of the other two sides.

(ii) In an obtuse angled triangle, the side opposite to the obtuse angle is longer than each of the other two sides.

Example

ABC is an isosceles triangle with base $\overline{BC}$. On $\overrightarrow{BC}$ a point D is taken away from C. A line segment through D cuts $\overline{AC}$ at L and $\overline{AB}$ at M. prove that $m\overline{AL} > m\overline{AM}$.

Given

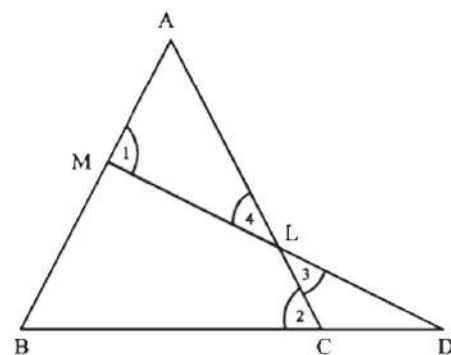
In ΔABC, $\overline{AB} \cong \overline{AC}$

D is a point on $\overrightarrow{BC}$ away from C

A line segment through D cuts $\overline{AC}$ at L and $\overline{AB}$ at M.

To Prove

$m\overline{AL} > m\overline{AM}$

Proof

Statements	Reasons
In ΔABC $\angle B \cong \angle 2$..(i)	$\overline{AB} \cong \overline{AC}$ (given)
In ΔMBD $m\angle 1 > m\angle B$...(ii)	($\angle 2$ is an ext. $\angle$ and $\angle 3$ is its internal opposite $\angle$)
$\therefore m\angle 1 > m\angle 2$...(iii)	From (i) and (ii)
In ΔLCD $m\angle 2 > m\angle 3$	($\angle 1$ is an ext. $\angle$ and $\angle 3$ is its internal opposite $\angle$)
$\therefore m\angle 1 > m\angle 3$...(v)	From (iii) and (iv)

But $m\angle 3 \cong m\angle 4 ...(vi)$	Vertical angles
$\therefore m\angle 1 > m\angle 4$	From (v) and (vi)
Hence $m\overline{AL} > m\overline{AM}$	In $\Delta ALM, m\angle 1 > m\angle 4$ (proved)

Theorem 13.1.3

The sum of the lengths of any two sides of a triangle is greater than the length of third side.

Given ΔABC

To prove

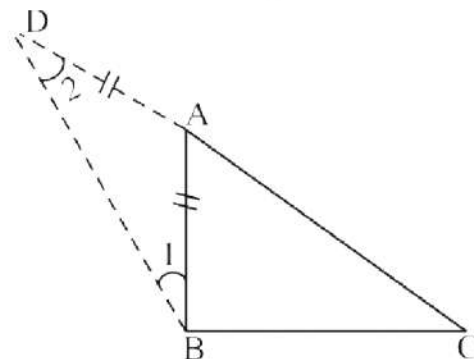
(i) $m\overline{AB} + m\overline{AC} > m\overline{BC}$

(ii) $m\overline{AB} + m\overline{BC} > m\overline{AC}$

(iii) $m\overline{BC} + m\overline{AC} > m\overline{AB}$

Construction

Take a point D on $\overrightarrow{CA}$ such that $\overline{AD} \cong \overline{AB}$ join B to D and name the angles $\angle 1$, $\angle 2$ as shown in the given figure.

Proof

Statements	Reasons
In ΔABD,	
$\angle 1 \cong \angle 2$ _____ (i)	$\overline{AD} \cong \overline{AB}$ (construction)
$m\angle DBC > m\angle 1$ _____ (ii)	$m\angle DBC = m\angle 1 + m\angle ABC$
$\because m\angle DBC > m\angle 2$ _____ (iii)	From (i) and (ii)
In ΔDBC	
$m\overline{CD} > m\overline{BC}$	By (iii)
i.e. $m\overline{AD} + m\overline{AC} > m\overline{BC}$	$m\overline{CD} = m\overline{AD} > m\overline{AC}$
Hence $m\overline{AB} + m\overline{AC} > m\overline{BC}$	$m\overline{AD} = m\overline{AB}$ (Construction)
Similarly	
$m\overline{AB} + m\overline{BC} > m\overline{AC}$	
And $m\overline{BC} + m\overline{CA} > m\overline{AB}$	

Example 1

Which of the following sets of lengths can be the lengths of the sides of a triangle?

(a) 2cm, 3cm, 5cm (b) 3cm, 4cm, 5cm, (c) 2cm, 4cm, 7cm,

(a) $\because 2 + 3 = 5$

$\therefore$ This set of lengths cannot be those of the sides of a triangle.

(b) $\because 3 + 4 > 5, 3 + 5 > 4, 4 + 5 > 3$

$\therefore$ This set can form a triangle

(c) $\because 2 + 4 < 7$

$\therefore$ This set of lengths cannot be the sides of a triangle.

Example 2

Prove that the sum of the measures of two sides of a triangle is greater than twice the measure of the median which bisects the third side.

Given

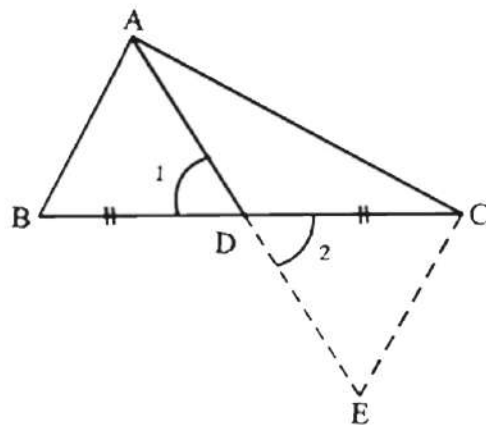
In ΔABC, median AD bisects side $\overline{BC}$ at D.

To prove

$m\overline{BC} + \overline{AC} > 2m\overline{AD}$.

Construction

On $\overrightarrow{AD}$, Take a point E, such that $\overline{DE} \cong \overline{AD}$.

Join C to E. Name the angles $\angle 1, \angle 2$ as shown in the ________ figure.

Proof

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta ECD$	
$\overline{BD} \cong \overline{CD}$	Given
$\angle 1 \cong \angle 2$	Vertical angles
$\overline{AD} \cong \overline{ED}$	Construction
$\Delta ABD \cong \Delta ECD$	S.A.S. Postulate
$\overline{AB} \cong \overline{EC}$...(i)	Corresponding sides of $\cong \Delta s$
$m\overline{AC} + m\overline{EC} > m\overline{AE}$...(ii)	ACE is a triangle
$m\overline{AC} + m\overline{AB} > m\overline{AE}$...(ii)	From (i) and (ii)
Hence $m\overline{AC} + m\overline{AB} > 2m\overline{AD}$	$m\overline{AE} = 2m\overline{AD}$ (Construction)

Example 3

Prove that the difference of measures of two sides of a triangle is less than the measure of the third side.

Given

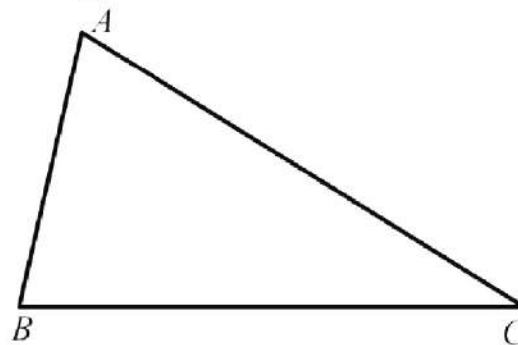
ΔABC

To Prove

$m\overline{AC} - m\overline{AB} < m\overline{BC}$

$m\overline{BC} - m\overline{AB} < m\overline{AC}$

$m\overline{BC} - m\overline{AC} < m\overline{AB}$

Proof

Statements	Reasons
$m\overline{AB} + m\overline{BC} > m\overline{AC}$	ABC is a triangle
$\left(\cancel{m\overline{AB}} + m\overline{BC} - \cancel{m\overline{AB}}\right) > \left(m\overline{AC} - m\overline{AB}\right)$	Subtracting $m\overline{AB}$ from both sides
$\therefore m\overline{BC} > \left(m\overline{AC} - m\overline{AB}\right)$	
or $m\overline{AC} - m\overline{AB} < m\overline{BC}$...(i)	$a > b \Rightarrow b < a$
Similarly	
$\left.\begin{array}{l} m\overline{BC} - m\overline{AB} < m\overline{AC} \\ m\overline{BC} - m\overline{AC} < m\overline{AB} \end{array}\right\}$	Reason similar to (i)

Exercise 14.1

Q.1 In ΔABC

$\overline{DE} \parallel \overline{BC}$

(i) **If** $\overline{AD} = 1.5\text{cm}$ $\overline{BD} = 3\text{cm}$

$\overline{AE} = 1.3\text{cm}$, then find $\overline{CE}$

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

By substituting the values of $\overline{AD}, \overline{BD}$ and $\overline{AE}$

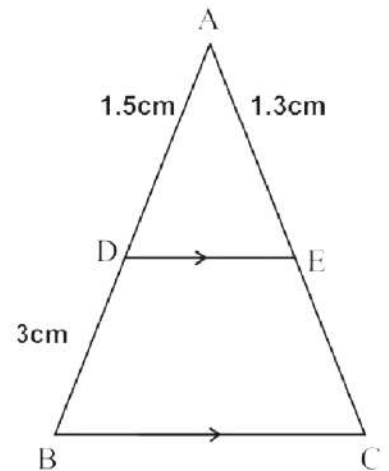
So

$$\frac{1.5}{3} = \frac{1.3}{\text{EC}}$$

$$\overline{EC}(1.5) = 1.3 \times 3$$

$$\overline{EC} = \frac{1.3 \times 3}{1.5}$$

$$\overline{EC} = \frac{3.9}{1.5}$$

$$\overline{EC} = 2.6\text{cm}$$

(ii) **If AD = 2.4cm** $\overline{AE} = 3.2\text{cm}$

$\overline{EC} = 4.8\text{cm}$ find AB

$$\frac{AD}{AB} = \frac{AE}{AC}$$

$$\overline{AC} = AE + EC$$

$$\overline{AC} = 3.2 + 4.8$$

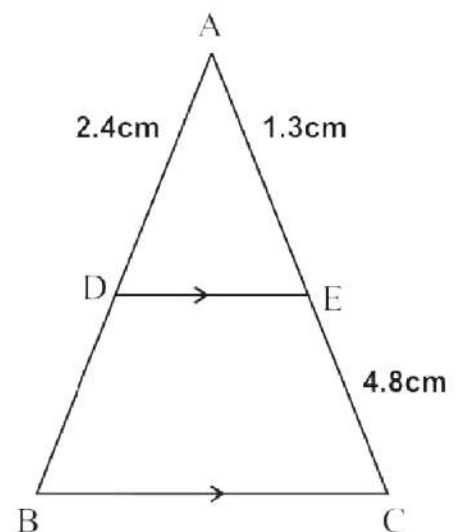
$$\overline{AC} = 8cm$$

$$\therefore \frac{\overline{AD}}{\overline{AB}} = \frac{\overline{AE}}{\overline{AC}}$$

$$\frac{2.4}{\text{AB}} = \frac{3.2}{8}$$

$$2.4 \times 8 = (3.2)\overline{AB}$$

$$\frac{19.2}{3.2} = \overline{AB}$$

$$\overline{AB} = 6\text{cm}$$

(iii) **If** $\frac{\overline{AD}}{\overline{BD}} = \frac{3}{5}\ \overline{AC} = 4.8cm$ **find** $\overline{AE}$

$\overline{AC} = \overline{AE} + \overline{EC}$

$\overline{AC} = \overline{EC} + \overline{AE}$

$\overline{AE} = 4.8 - \overline{EC}$

$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$

$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AC} - \overline{EC}}{\overline{EC}}$

$\frac{3}{5} = \frac{4.8 - EC}{EC}$

$3(\overline{EC}) = 5(4.8 - \overline{EC})$

$3(\overline{EC}) = 24 - 5(\overline{EC})$

$3(\overline{EC}) + 5(\overline{EC}) = 24$

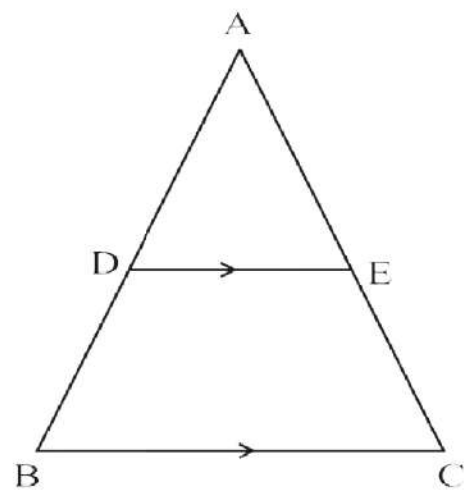
$8(\overline{EC}) = 24$

$(\overline{EC}) = \frac{24^{3}}{8}$

$\overline{EC} = 3cm$

$\overline{AE} = \overline{AC} - \overline{EC}$

$= 4.8 - 3$

$= 1.8cm$

(iv) **If** $\overline{AD} = 2.4cm\ \overline{AE} = 3.2cm\ \overline{DE} = 2cm\ \overline{BC} = 5cm$. **Find** $\overline{AB}, \overline{DB}, \overline{AC}, \overline{CE}$.

$\frac{\overline{AD}}{\overline{AB}} = \frac{\overline{AE}}{\overline{AC}} = \frac{\overline{DE}}{\overline{BC}}$

$\frac{2.4}{AB} = \frac{3.2}{AC} = \frac{2}{5}$

$\frac{2.4}{AB} = \frac{2}{5}$

$(2.4)5 = 2(AB)$

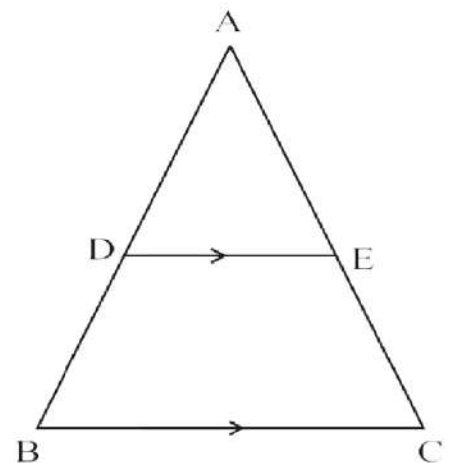
$\frac{12.0}{2} = AB$

$\overline{AB} = 6\text{ cm}$

$\frac{3.2}{AC} = \frac{2}{5}$

$16.0 = 2(AC)$

$\frac{16^{8}}{2} = AC$

$\overline{AC} = 8cm$

$\overline{DB} = \overline{AB} - \overline{AD}$

$\overline{DB} = 6 - 2.4$

$\overline{BD} = 3.6\,\text{cm}$

$$\frac{\overline{AD}}{\overline{AB}} = \frac{\overline{AE}}{\overline{AC}}$$

$$\frac{2.4}{6} = \frac{\overline{AE}}{8}$$

$$\overline{AE} = \frac{2.4}{6} \times 8$$

$$\overline{AE} = \frac{19.2}{6}$$

$\overline{AE} = 3.2cm$

$\overline{CE} = \overline{AC} - \overline{AE}$

$\overline{CE} = 8 - 3.2$

$\overline{CE} = 4.8cm$

If $\overline{AD} = 4x - 3$ $\overline{AE} = 8x - 7$

$\overline{BD} = 3x - 1$ and $CE = 5x - 3$ Find the value of x

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

By putting the value of $\overline{AD}, \overline{AE}, \overline{BD}$ and $\overline{CE}$

$$\frac{4x - 3}{3x - 1} = \frac{8x - 7}{5x - 3}$$

By cross multiplying

$(4x - 3)(5x - 3) = (8x - 7)(3x - 1)$

$20x^2 - 12x - 15x + 9 = 24x^2 - 8x - 21x + 7$

$20x^2 - 27x + 9 = 24x^2 - 29x + 7$

$0 = 24x^2 - 20x^2 - 29x + 27x + 7 - 9$

$4x^2 - 2x - 2 = 0$

$2(2x^2 - x - 1) = 0$

$$2x^2 - 2x + 1x - 1 = \frac{0}{2}$$

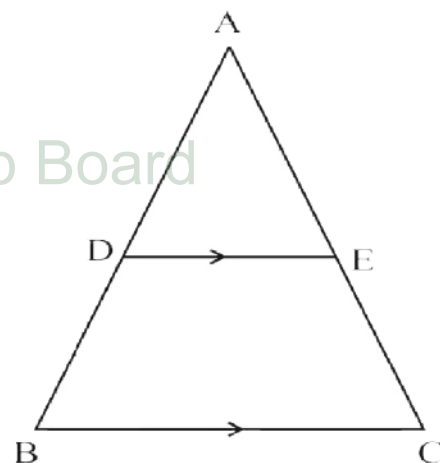
$2x(x - 1) + 1(x - 1) = 0$

$(x - 1)(2x + 1) = 0$

$x - 1 = 0$ $\quad\quad$ $2x + 1 = 0$

$x = 1$ $\quad\quad$ $2x = -1$

$$x = -\frac{1}{2}$$

Distance is not taken in negative it is always in positive so the value of x = 1.

Q.2 In ΔABC is an isosceles triangle ∠A is vertex angle and $\overline{DE}$ intersects the sides $\overline{AB}$ and $\overline{AC}$ as shown in the figure so that

$m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$

Prove that ΔADE is also an isosceles triangle.

Given:

ΔABC is an isosceles triangle, $\angle A$ is vertex and $\overline{DE}$ intersects the sides $\overline{AB}\ and\ \overline{AC}$.

$$\frac{m\overline{AD}}{m\overline{BD}} = \frac{m\overline{AE}}{m\overline{EC}}$$

To Prove

$$m\overline{AD} = m\overline{AE}$$

Proof

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

Or $\frac{\overline{BD}}{\overline{AD}} = \frac{\overline{EC}}{\overline{AE}}$

Or $\frac{\overline{AD}+\overline{BD}}{\overline{AD}} = \frac{\overline{AE}+\overline{EC}}{\overline{EC}}$

As we know

$$\overline{AB} = \overline{AD} + \overline{BD}$$

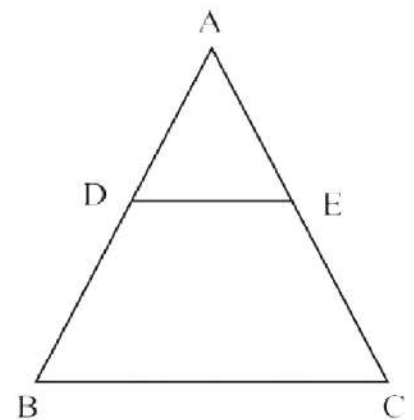
$$\overline{AC} = \overline{AE} + \overline{EC}$$

$$\frac{\overline{AB}}{\overline{AD}} = \frac{\overline{AC}}{\overline{AE}}$$

From this

$$\frac{\overline{AB}}{\overline{AD}} = \frac{\overline{AC}}{\overline{AE}}$$

$$\overline{AD} = \overline{AE}$$

$\overline{AB} = \overline{AC}$ **(Given)**

Q.3 In an equilateral triangle ABC shown in the figure $m\overline{AE} : m\overline{AC} = m\overline{AD} : m\overline{AB}$ find all the three angles of ΔADE and name it also.

Given

ΔABC is equilateral triangle

To prove

To find the angles of ΔADE

Solution:

$$\frac{m\overline{AE}}{m\overline{AC}} = \frac{m\overline{AD}}{m\overline{AB}}$$

All angles are equal as it is an equilateral triangle which are equal to 60° each

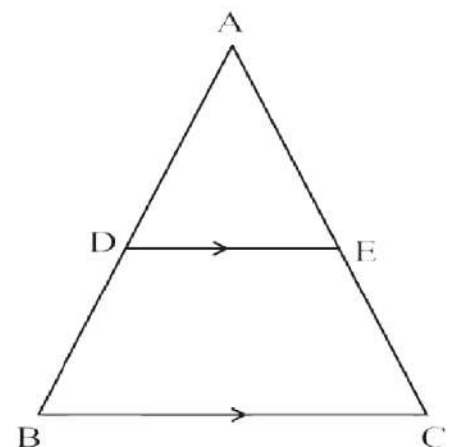
$\angle A = \angle B = \angle C$

$m\overline{BC} \parallel m\overline{DE}$

$\angle ADE = \angle ABC = 60^\circ$

$\angle AED = \angle ACB = 60^\circ$

$\angle A = 60^\circ$

ΔADE is an equilateral triangle

Q.4 Prove that line segment drawn through the midpoint of one side of a triangle and parallel to another side bisect the third side

Given

$\overline{AD} = \overline{BD}$

$\overline{DE} \| \overline{BC}$

To Prove

$\overline{AE} = \overline{EC}$

In ΔABC

$\overline{DE} \| \overline{BC}$

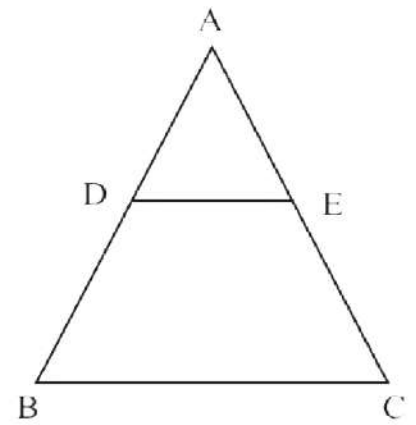
In theorem it is already discussed that

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

As we know $\overline{AD} = \overline{BD}$ or $\overline{BD} = \overline{AD}$

$$\frac{\cancel{\overline{AD}}}{\cancel{\overline{AD}}} = \frac{\overline{AE}}{\overline{EC}}$$

$$1 = \frac{\overline{AE}}{\overline{EC}}$$

$\overline{EC} = \overline{AE}$

Q.5 Prove that the line segment joining the midpoint of any two sides of a triangle is parallel to the third side

Given

ΔABC the midpoint of $\overline{AB}$ and $\overline{AC}$ are L and M respectively

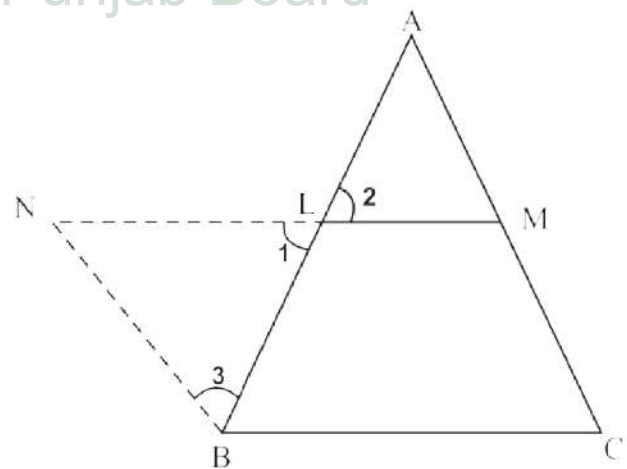
To Prove

$\overline{LM} \| \overline{BC}$ and $m\overline{LM} = \frac{1}{2}\overline{BC}$

Construction

Join M to L and produce $\overline{ML}$ to N such that $\overline{ML} \cong \overline{LN}$

Join N to B and in the figure name the angles $\angle 1$, $\angle 2$, and $\angle 3$

Proof

Statements	Reasons
$\Delta BLN \leftrightarrow \Delta ALM$	
$\overline{BL} \cong AL$	Given
$\angle 2 = \angle 1$ or $\angle 1 = \angle 2$	Vertical angles
$\overline{NL} = \overline{ML}$	Construction
$\therefore\ \Delta BLN \cong \Delta ALM$	Corresponding angle of congruent triangles Given
$\therefore \angle A = \angle 3$	
And $\overline{NB} \cong \overline{AM}$	
$\overline{NB} \| \overline{AM}$	

$\overline{ML} = \overline{AM}$	Given
$\overline{NB} \cong \overline{ML}$	
$\overline{BC}\overline{MN}$ is parallelogram	
$\therefore\ \overline{BC} \parallel \overline{LM}$ or $\overline{BC} \parallel \overline{NL}$	
$\overline{BC} \cong \overline{NM}$	(Opposite side of parallelogram BCMN)
$mLM = \frac{1}{2} m\overline{NM}$	(Opposite side of parallelogram)
Hence $m\overline{LM} = \frac{1}{2} m\overline{BC}$	

Theorem 14.1.3

The internal bisector of an angle of a triangle divides the sides opposite to it in the ratio of the lengths of the sides containing the angle.

Given

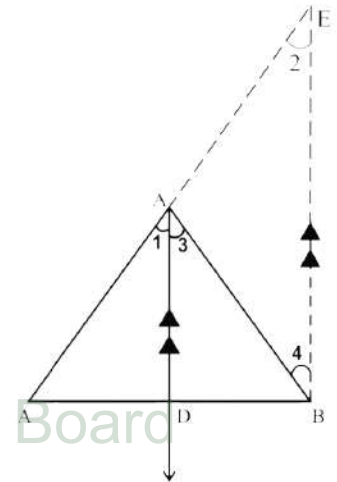
In ΔABC internal angle bisector of $\angle A$ meets $\overline{CB}$ at the points D.

To prove

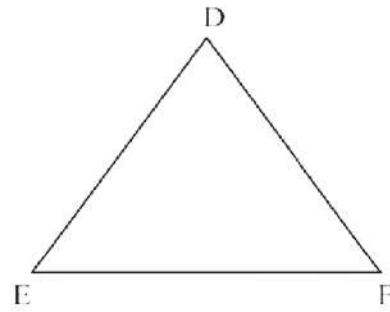
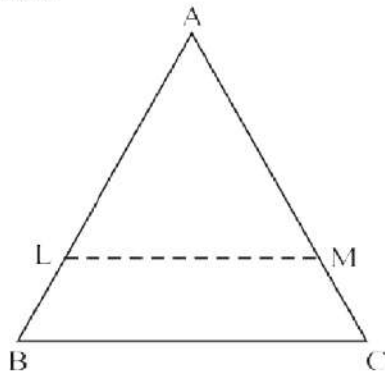
$m\overline{BD} : m\overline{DC} = m\overline{AB} : m\overline{AC}$

Construction

Draw a line segment $\overline{BE} \parallel \overline{DA}$ to meet $\overline{CA}$ Produced at E

Proof

Statements	Reasons
$\because \overline{AD} \parallel \overline{EB}$ and $\overline{EC}$ intersect them	Construction
$m\angle 1 = m\angle 2$..........(i)	Corresponding angles
Again $\overline{AD} \parallel \overline{EB}$ and $\overline{AB}$ intersects them	
$\therefore\ m\angle 3 = m\angle 4$ ....(ii)	Alternate angles
But $m\angle 1 = m\angle 3$	Given
$\therefore\ m\angle 2 = m\angle 4$	From (i) and (ii)
And $\overline{AB} \cong \overline{AE}$ or $\overline{AE} \cong \overline{AB}$	In a Δ, the sides opposite to congruent angles are also congruent
Now $\overline{AD} \parallel \overline{EB}$	Construction
$\therefore \frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{EA}}{m\overline{AC}}$	A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.
or $\frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{AB}}{m\overline{AC}}$	$m\overline{EA} = m\overline{AB}$ (proved)
Thus $m\overline{BD} : m\overline{DC} = m\overline{AB} : \overline{AC}$	

Theorem 14.1.4

If two triangles are similar, then the measures of their corresponding sides are proportional

Given

$\Delta ABC \sim \Delta DEF$

i.e $\angle A \cong \angle D$, $\angle B \cong \angle E$ and $\angle C \cong \angle F$

To Prove

$\dfrac{m\overline{AB}}{m\overline{DE}} = \dfrac{m\overline{AC}}{m\overline{DF}} = \dfrac{m\overline{BC}}{m\overline{EF}}$

Construction

(I) Suppose that $mAB > m\overline{DE}$

(II) $m\overline{AB} \le m\overline{DE}$

On $\overline{AB}$ take a point L such that $m\overline{AL} = m\overline{DE}$

On $\overline{AC}$ take a point M such that $m\overline{AM} = m\overline{DF}$

Join L and M by the line segment LM

Proof

Statements	Reasons
In $\Delta ALM \leftrightarrow \Delta DEF$	
$\angle A \cong \angle D$	Given
$\overline{AL} \cong \overline{DE}$	Construction
$\overline{AM} \cong \overline{DF}$	Construction
Thus $\Delta ALM \cong \Delta DEF$	S.A.S Postulate
And $\angle L \cong \angle E$, $\angle M \cong \angle F$	(Corresponding angles of congruent triangles)
Now $\angle E \cong \angle B$ and $\angle F \cong \angle C$	Given
$\therefore$ $\angle L \cong \angle B$, $\angle M \cong \angle C$	Transitivity of congruence
Thus $\overline{LM} \parallel \overline{BC}$	Corresponding angles are equal
Hence $\dfrac{m\overline{AL}}{m\overline{AB}} = \dfrac{m\overline{AM}}{m\overline{AC}}$	A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.
Or $\dfrac{m\overline{DE}}{m\overline{AB}} = \dfrac{m\overline{DF}}{m\overline{AC}}$ ....(i)	$m\overline{AL} = m\overline{DE}$ and $m\overline{AM} = m\overline{DF}$ (Construction)
Similarly by intercepting segments on $\overline{BA}$ and $\overline{BC}$, we can prove that $\dfrac{m\overline{DE}}{m\overline{AB}} = \dfrac{m\overline{EF}}{m\overline{BC}}$..........(ii)	

Thus $\frac{m\overline{DE}}{m\overline{AB}}=\frac{m\overline{DF}}{m\overline{AC}}=\frac{m\overline{EF}}{m\overline{BC}}$	By (i) and (ii)
Or $\frac{m\overline{AB}}{m\overline{DE}}=\frac{m\overline{AC}}{m\overline{DF}}=\frac{m\overline{BC}}{m\overline{EF}}$	By taking reciprocals
If $m\overline{AB}=m\overline{DE}$	
Then in $\Delta ABC \leftrightarrow \Delta DEF$	
(II) If $m\overline{AB}<m\overline{DE}$, it can similarly be proved by taking intercepts on the sides of ΔDEF	
$\angle A \cong \angle D$	
$\angle B \cong \angle E$	
And $\overline{AB}\cong\overline{DE}$	
So $\Delta\ ABC \cong \Delta DEF$	A.S.A ≅ A.S.A
Thus $\frac{m\overline{AB}}{m\overline{DE}}=\frac{m\overline{AC}}{m\overline{DF}}=\frac{m\overline{BC}}{m\overline{EF}}=1$	$\overline{AC}\cong\overline{DF}$, $\overline{BC}\cong\overline{EF}$
Hence the result is true for all the cases.	

Exercise 14.2

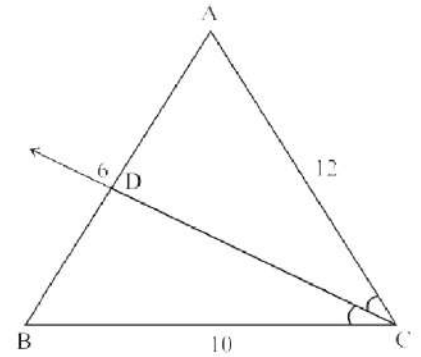
Q.1 In ΔABC as shown in the figure $\overrightarrow{CD}$ bisects $\angle C$ and meets $\overline{AB}$ at D. $m\overline{BD}$ is equal to

(a) 5 **(b)** 16 **(c)** 10 **(d)** 18

$$\frac{m\overline{BD}}{m\overline{DA}}=\frac{m\overline{BC}}{m\overline{CA}}$$

$$\frac{\overline{BD}}{6}=\frac{10}{12}$$

$$\overline{BD}=\frac{10\times 6}{12}\quad \text{or}\quad \overline{BD}=\frac{10\times6}{12}=\frac{60}{12}$$

$$\overline{BD}=5$$

Q.2 In ΔABC shown in the figure $\overrightarrow{CD}$ bisects $\angle C$. If $m\overline{AC}=3$, $\overline{CB}=6$ and $m\overline{AB}=7$ then find $m\overline{AD}$ and $\overline{DB}$

$$\overline{AB}=\overline{AD}+\overline{BD}$$

$$\overline{AD}=\overline{AB}-\overline{BD}$$

$$\overline{AD}=7-x$$

$$\frac{m\overline{AD}}{m\overline{BD}}=\frac{m\overline{AC}}{m\overline{CB}}$$

$$\frac{x}{7-x}=\frac{3}{6}$$

$$\frac{x}{7-x}=\frac{1}{2}$$

$2x = 7-x$

$2x + x = 7$

$3x = 7$

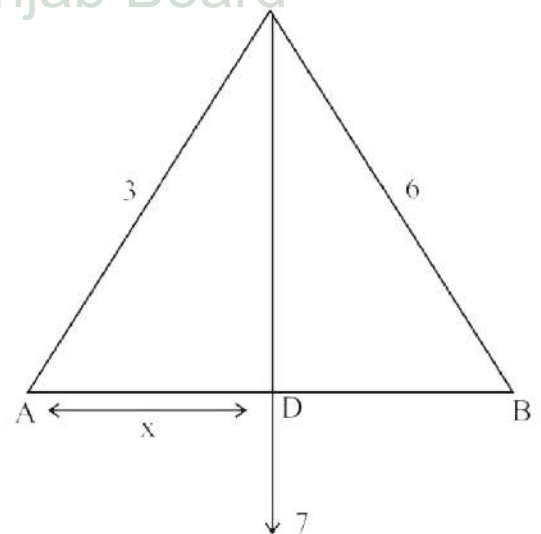
$$x=\frac{7}{3}\quad \text{or}\quad \overline{AD}=\frac{7}{3}$$

$$\overline{AB}=\overline{AD}+\overline{BD}$$

$$7=\frac{7}{3}+\overline{BD}$$

$$7-\frac{7}{3}=\overline{BD}$$

$$\frac{21-7}{3}=\overline{BD}$$

$$\overline{BD}=\frac{14}{3}$$

Q.3 **Show that in any corresponding of two triangles if two angles of one triangle are congruent to the corresponding angles of the other, then the triangle are similar**

Given

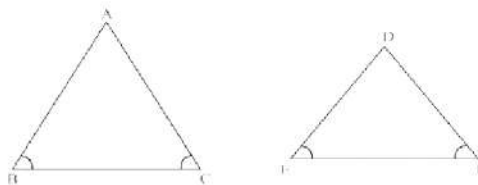
ΔABC and Δ DEF

$\angle B \cong \angle E$

$\angle C \cong \angle F$

To Prove

$\Delta ABC \cong \Delta DEF$

Proof

Statements	Reasons
$\angle A + \angle B + \angle C = 180^\circ$ $\angle D + \angle E + \angle F = 180$ $\angle A \cong \angle D$ $\angle B = \angle E$ $\angle C = \angle F$ Hence Δ ABC $\cong \Delta DEF$	Sume of three angles of a triangle = 180°

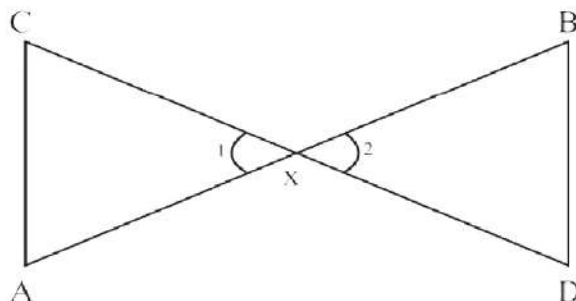
Q.4 **If line segment $\overline{AB}$ and $\overline{CD}$ are intersecting at point *X* and $\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$ then show that ΔAXC and ΔBXD are similar**

Given

Line segment $\overline{AB}$ and $\overline{CD}$ intersect at X

$\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$

To Prove

ΔCXA and ΔDXB are similar

Proof

Statements	Reasons
$\frac{\overline{AX}}{\overline{XB}} = \frac{\overline{CX}}{\overline{XD}}$	Given
$\angle 1 \cong \angle 2$	
$\overline{AC} \parallel \overline{BD}$	Vertical angles
$\angle A = m\angle B$	
$m\angle C = m\angle D$	Alternate angles
Hence proved the triangle are similar	

Review Exercise 14

Q.1 Which of the following are true which are false?

(i)	Congruent triangles are of same size and shape.	(True)
(ii)	Similar triangles are of same shape but different sizes.	(True)
(iii)	Symbol used for congruent is '$\sim$'	(False)
(iv)	Symbol used for similarity is '$\cong$'	(False)
(v)	Congruent triangle are similar	(True)
(vi)	Similar triangles are congruent	(False)
(vii)	A line segment has only one midpoint	(True)
(viii)	One and only one line can be drawn through two points	(True)
(ix)	Proportion is non equality of two ratio	(False)
(x)	Ratio has no unit	(True)

Q.2 Define the following

(i) Ratio

The ratio between two a like quantities is defined as $a:b=\frac{a}{b}$ where a and are the elements of the ratio.

(ii) Proportion

Proportion is defined as the equality of two ratio i,e a : b = c : d

(iii) Congruent Triangles

Two triangles are said to be congruent (symbols $\cong$) if there emits a corresponding betweet them such that all the corresponding sides and angles are congruent.

(iv) Similar Triangles

If two triangles are similar then the measures of their corresponding sides are proportional.

Q.3 In ΔLMN shown in the figure $\overline{MN} \parallel \overline{PQ}$

(i) If $m\overline{LM} = 5\text{cm}$, $m\overline{LP} = 2.5\text{cm}$

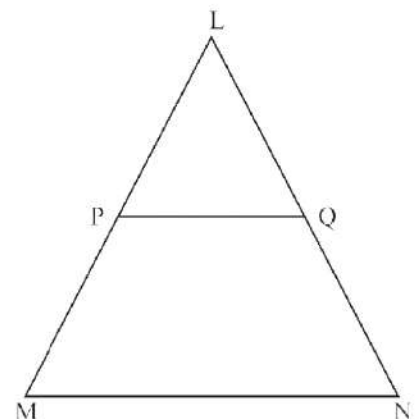
$m\overline{LQ} = 2.3$ cm then find LN

$$\frac{m\overline{LP}}{m\overline{LM}} = \frac{m\overline{LQ}}{m\overline{LN}}$$

$$\frac{2.5}{5} = \frac{2.3}{\overline{LN}}$$

$$(2.5)\ \overline{LN} = 5 \times 2.3$$

$$\overline{LN} = \frac{11.5}{2.5}$$

$$\overline{LN} = 4.6\text{cm}$$

(ii) If mLM = 6cm , mLQ = 2.5cm
mQN = 5cm then find
mLP

$$\frac{m\overline{LP}}{m\overline{LM}}=\frac{m\overline{LQ}}{m\overline{LN}}$$

$$\frac{LP}{6}=\frac{2.5}{LN}$$

$$\overline{LN}=\overline{LQ}+\overline{QN}$$

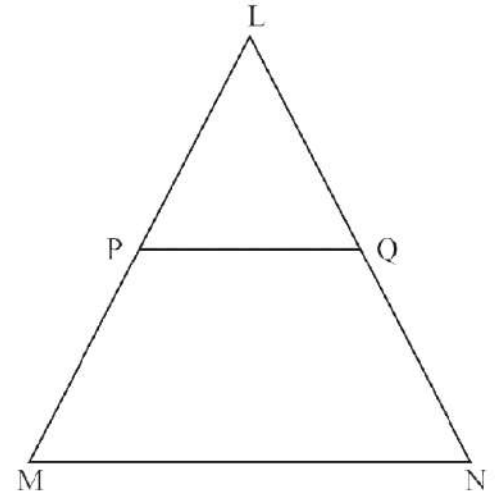
$$\overline{LN}=2.5+5$$

$$\overline{LN}=7.5\text{cm}$$

$$\frac{\overline{LP}}{6}=\frac{2.5}{7.5}$$

$$\overline{LP}=\frac{2.5\times 6}{7.5}$$

$$\overline{LP}=\frac{15}{7.5}$$

$$\overline{LP}=2\text{cm}$$

Q.4 **In the show figure let mPA = 8x – 7 mPB = 4x – 3** $m\overline{AQ}=5x-3$
$m\overline{BR}=3x-1$ **find the value of x if** $\overline{AB}\,||\,\overline{QR}$

$$\frac{mPA}{mAQ}=\frac{mBP}{mBR}$$

$$\frac{8x-7}{5x-3}=\frac{4x-3}{3x-1}$$

By cross multiplying

$$(8x-7)(3x-1)=(4x-3)(5x-3)$$

$$24x^2-8x-21x+7=20x^2-12x-15x+9$$

$$24x^2-29x+7=20x^2-27x+9$$

$$24x^2-20x^2-29x+27x+7-9=0$$

$$4x^2-2x-2=0$$

$$4x^2-4x+2x-2=0$$

$$4x(x-1)+2(x-1)=0$$

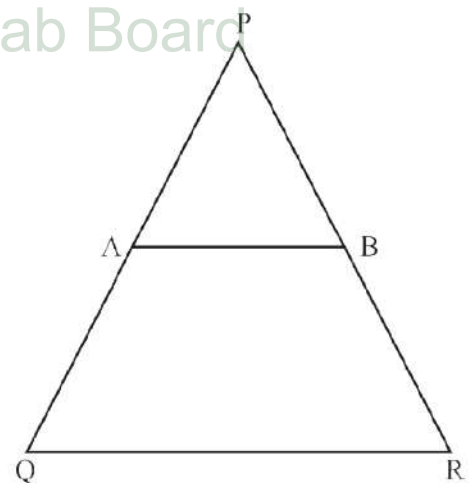
$$(x-1)(4x+2)=0$$

$$x-1=0$$

$$x=1$$

$$4x+2=0$$

$$4x=-2$$

$$x=\frac{-\cancel{2}^{\,1}}{\cancel{4}_{\,2}}$$

$x = \frac{-1}{2}$

Length is always taken as positive not negative so value of $x = 1$

Q.5 In ΔLMN Shown in figure $\overrightarrow{LA}$ bisects ∠L. If $m\overline{LN} = 4m$ $m\overline{LM} = 6cm$ $m\overline{MN} = 8$ then find $m\overline{MA}$ and $m\overline{AN}$

$\frac{m\overline{MA}}{m\overline{AN}} = \frac{m\overline{LM}}{m\overline{LN}}$

$\overline{MA} = x$

$\overline{AN} = 8\text{-}x$

$\frac{x}{8-x} = \frac{6}{4}$

$4x = 6(8-x)$

$4x = 48 - 6x$

$4x + 6x = 48$

$10x = 48$

$x = \frac{48}{10}$

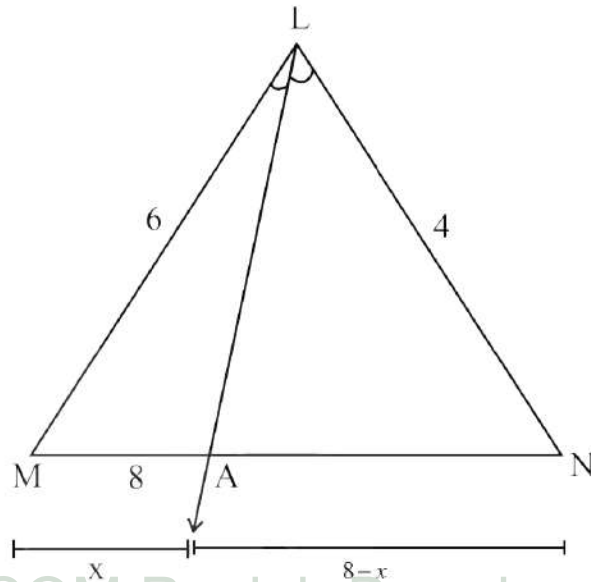
$x = 4.8cm$

$m\overline{MA} = 4.8cm$

$\overline{MN} = \overline{MA} + \overline{AN}$

$8 = 4.8 + \overline{AN}$

$8 - 4.8 = \overline{AN}$

$\overline{AN} = 3.2cm$

Q.6 In Isosceles ΔPQR Shown in the figure, find the value of x and y

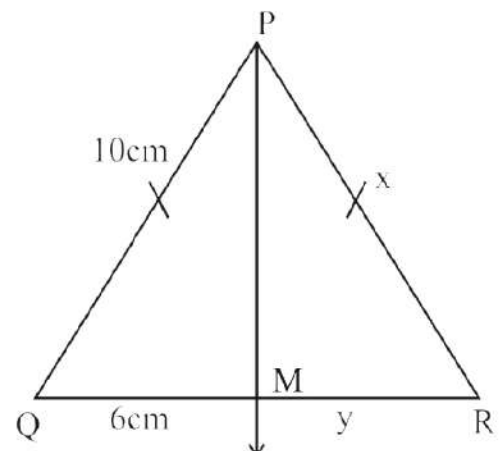
As we know that it is isosceles triangle

So

$\overline{PQ} = \overline{RP}$

$10 = x$

Or

$x = 10cm$

$\overline{PM} \perp \overline{QR}$

So it bisects the side and bisects the angle also

SO $\overline{QM} = \overline{MR}$

$6 = y$

Or

$y = 6\text{cm}$

Unit 14: Ratio and Proportion

Overview

Theorem 14.1.1

A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.

Given:

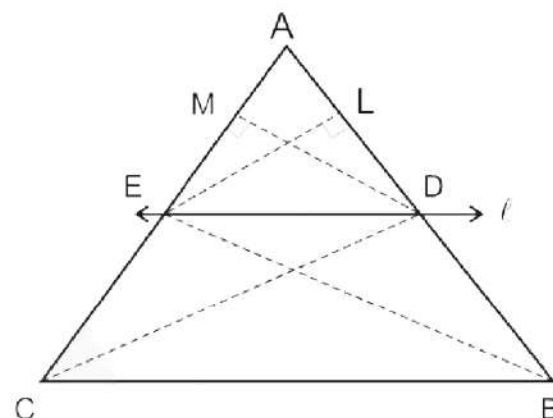
In ΔABC, the line ℓ is intersecting the sides $\overline{AC}$ and $\overline{AB}$ at points E and D respectively such that $\overline{ED} \parallel \overline{CB}$

To Prove

$m\overline{AD}:\overline{DB} = m\overline{AE}:m\overline{EC}$

Construction:

Join B to E and C to D .From D draw $\overline{DM} \perp \overline{AC}$ and from E draw $\overline{EL} \perp \overline{AB}$

Proof

Statements	Reasons
In triangles BED and AED, EL is the common perpendicular	
$\therefore$ Area of $\Delta BED = \frac{1}{2} \times m\overline{BD} \times m\overline{EL}$.......(i)	Area of a $\Delta = \frac{1}{2}$ (base)(height)
and Area of $\Delta AED = \frac{1}{2} \times m\overline{AD} \times m\overline{EL}$.......(ii)	
Thus Area of $\frac{\Delta BED}{\Delta AED} = \frac{m\overline{DB}}{m\overline{AD}}$........(iii)	Dividing (i) by (ii)
Similarly $\frac{\text{Area of } \Delta CDE}{\text{Area of } \Delta ADE} = \frac{m\overline{EC}}{m\overline{AE}}$........(iv)	
But $\Delta BED \cong \Delta CDE$	(Areas of triangles with common base and same altitudes are equal. Given that $\overline{ED} \parallel \overline{CB}$, so altitudes are equal).
$\therefore$ From (iii) and (iv) We have $\frac{m\overline{DB}}{m\overline{AD}} = \frac{m\overline{EC}}{m\overline{AE}}$ or $\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$	Taking reciprocal of both sides.
Hence $m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$	

Theorem: 14.1.2 Converse of Theorem 14.1.1

If a line segment intersects the two sides of a triangle in the same ratio, then it is parallel to the third side.

Given

In ΔABC, $\overline{ED}$ intersect $\overline{AB}$ and $\overline{AC}$ such that $m\overline{AD}:\overline{DB} = m\overline{AE}:m\overline{EC}$

To Prove

$\overline{ED} \parallel \overline{CB}$

Construction

If $\overline{ED} \nparallel \overline{CB}$ then draw $\overline{BF} \parallel \overline{DE}$ to meet $\overline{AC}$ Produced at F

Proof

Statements	Reasons
In ΔABF	
$\overline{DE} \parallel \overline{BF}$	Construction
$\therefore \dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AE}}{m\overline{EF}}$ ....(i)	(A line parallel to one side of a triangle divides the other two sides proportionally Theorem 14.1.1)
But $\dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AE}}{m\overline{EC}}$ ....(ii)	Given
$\therefore \dfrac{m\overline{AE}}{m\overline{EF}} = \dfrac{m\overline{AE}}{m\overline{EC}}$	From (i) and (ii)
or $m\overline{EF} = m\overline{EC}$,	
This is possible only if point F is coincident with C.	(Property of real numbers)
$\therefore$ Our supposition is wrong	
Hence $\overline{ED} \parallel \overline{CB}$	

Exercise 15

Q.1 Verify that the Δs having the following measures of sides are right-angled to verify weather the Δs are right angled or not we use Pythagoras Theorem

(Hypotenuse)^2 = (base)^2 + (Perpendicular)^2

(i) a = 5cm
b = 12cm
c = 13cm
$a^2 = 25cm^2$
$b^2 = 144cm^2$
$c = 169cm^2$
Larger Size is Hypotenuse So
169 = 25 + 144
169 = 169
L.H.S = R.H.S
So it is right angled triangle

(ii) a = 1.5cm
b = 2cm
c = 2.5cm
$a^2 = 2.25cm^2$
$b^2 = 4cm^2$
$c^2 = 6.25$
6.25 = 2.25 + 4
6.25 = 6.25
L.H.S = R.H.S
So it is right-angled triangle

(iii) a = 9cm
b = 12cm
c = 15cm
$a^2 = 81cm^2$
$b^2 = 144cm^2$
$c = 225cm^2$
$225cm^2 = 8.1cm + 144cm$
$225cm^2 = 225cm^2$
L.H.S = R.H.S
So it is right angled triangle

(iv) a = 16cm
b = 30cm
c = 34cm
$a^2 = 256cm^2$
$b^2 = 900cm$
$c^2 = 1156cm^2$
1156 = 256 + 900
1156 = 1156
L.H.S = R.H.S
It is right angled triangle

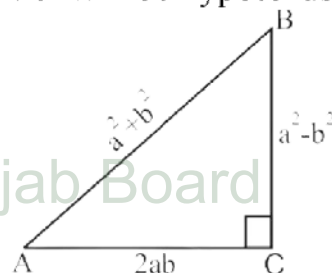
Q.2 Verify that a^2+b^2, a^2-b^2 and 2ab are the measures of the sides of a right angled Triangle where a and b are any two real numbers (a >b)

Let $a = z$ and $b = 1$

$a^2+b^2 = (2)^2+(1)^2 = 4+1 = 5$

$a^2-b^2 = (2)^2-(1)^2 = 4-1 = 3$

$2ab = 2(2)(1) = 4$

Since a^2+b^2 is the largest side so a^2+b^2 will be hypotenuse

So

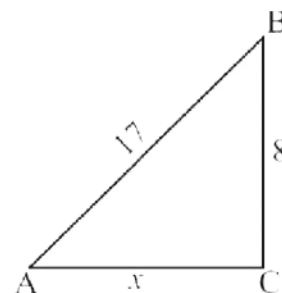
$(\overline{AB})^2 = (\overline{AC})^2 + (\overline{BC})^2$

$(a^2+b^2)^2 = (2ab)^2 + (a^2-b^2)^2$

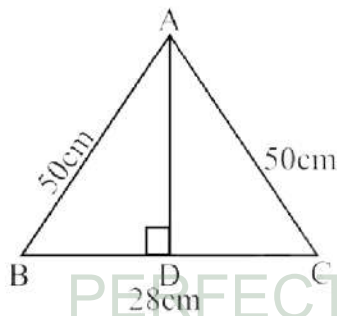
$a^4+b^4+2a^2b^2 = 4a^2b^2+a^4+b^4-2a^2b^2$

$a^4+b^4+2a^2b^2 = a^4+b^4+2a^2b^2$

L.H.S = R.H.S
It is proved that it is a right angled triangle

Q.3 The three sides of a triangle are of measure 8, x and 17 respectively. For what value of x will it become base of right angled triangle by Pythagoras theorem

$$\left(\overline{AB}\right)^2=\left(\overline{AC}\right)^2+\left(\overline{BC}\right)^2$$
$$(17)^2=(x)^2+(8)^2$$
$$289=x^2+64$$
$$289-64=x^2$$
$$x^2=225$$
Taking square root both side
$$\sqrt{x^2}=\sqrt{225}$$
$$x=15$$

Q.4 In an isosceles Δ the base $\overline{BC}=28\,cm$ and $\overline{AB}=\overline{AC}=50\,cm$

If $\overline{AD}\perp\overline{BC}$ then find

(i) Length of $\overline{AD}$

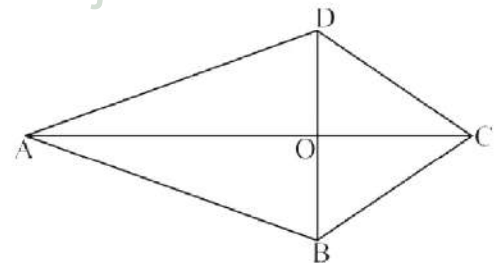
Solution:
$$\overline{AD}\perp\overline{BC}$$
So $\overline{BD}=\overline{CD}$
$$\frac{1}{2}\overline{BC}=\frac{1}{2}(28)$$
$$\frac{1}{2}\overline{BC}=14$$
So
$$\overline{BD}=\overline{CD}=14$$
$$\left(\overline{AB}\right)^2=\left(\overline{BD}\right)^2+\left(\overline{AD}\right)^2$$
$$2500=(14)^2+\left(\overline{AD}\right)^2$$
$$2500=196+\left(\overline{AD}\right)^2$$
\ $$2500-196=\left(\overline{AD}\right)^2$$
$$\left(\overline{AD}\right)^2=2304$$
Taking square root on both side
$$\sqrt{\left(\overline{AD}\right)^2}=\sqrt{2304}$$
$$\overline{AD}=48\,cm$$

(ii) Area of Δ ABC

Area of Δ ABC $=\frac{1}{2}$ (base) (height)
$$=\frac{1}{2}(28)(48)$$
$$=(14)(48)$$
$$=672\text{ cm}^2$$

Q.5 In a quadrilateral ABCD the diagonals $\overline{AC}$ and $\overline{BD}$ are perpendicular to each other.

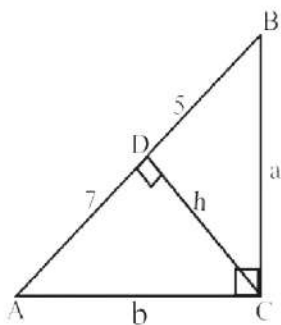
Prove that
$$\left(\overline{AB}\right)^2+\left(\overline{CD}\right)^2=\left(\overline{AD}\right)^2+\left(\overline{BC}\right)^2$$

ΔAOB

$$\left(\overline{AB}\right)^2=\left(\overline{OB}\right)^2+\left(\overline{OA}\right)^2 \longrightarrow \text{(i)}$$
ΔBOC
$$\left(\overline{BC}\right)^2=\left(\overline{OB}\right)^2+\left(\overline{OC}\right)^2 \longrightarrow \text{(ii)}$$
ΔCOD
$$\left(\overline{CD}\right)^2=\left(\overline{OD}\right)^2+\left(\overline{OC}\right)^2 \longrightarrow \text{(iii)}$$
ΔDOA
$$\left(\overline{AD}\right)^2=\left(\overline{OA}\right)^2+\left(\overline{OD}\right)^2 \longrightarrow \text{(iv)}$$
By adding (i) and (iii)
$$\left(\overline{AB}\right)^2+\left(\overline{CD}\right)^2=\left(\overline{OB}\right)^2+\left(\overline{OA}\right)^2+\left(\overline{OD}\right)^2+\left(\overline{OC}\right)^2\rightarrow\text{(v)}$$
By adding (ii) and (iv)
$$\left(\overline{AD}\right)^2+\left(\overline{BC}\right)^2=\left(\overline{OB}\right)^2+\left(\overline{OC}\right)^2+\left(\overline{OA}\right)^2+\left(\overline{OD}\right)^2\rightarrow\text{(vi)}$$
By comparing v and vi

$$\left(\overline{AB}\right)^2+\left(\overline{CD}\right)^2=\left(\overline{AD}\right)^2+\left(\overline{BC}\right)^2$$

Hence proved

Q.6 the ΔABC as shown in the figure $m\angle ACB = 90°$ and $\overline{CD}\perp\overline{AB}$ find the length a, h and b if $m\overline{BD}=5$ units and $m\overline{AD}=7$ units

(i)

ΔACB

$(7+5)^2=(b)^2+(a)^2$

$a^2+b^2=(12)^2$

$a^2+b^2=144$ ______ (i)

ΔADC

$(b)^2=(7)^2+(h)^2$

$b^2-h^2=49$ ________ (ii)

ΔCDB

$a^2=(5)^2+(h)^2$

$a^2-h^2=25$ ________ (iii)

Subtracting ii from iii

$$\begin{array}{r} a^2-\cancel{h^2}=25 \\ \underline{\pm b^2\mp\cancel{h^2}=\pm 49} \\ a^2-b^2=-24 \end{array}$$

$a^2-b^2=-24$ __________ (iv)

Adding equation I and IV

$$\begin{array}{r} a^2+\cancel{b^2}=144 \\ \underline{a^2-\cancel{b^2}=-24} \\ 2a^2=120 \end{array}$$

$2a^2 = 120$

$a^2=\dfrac{\cancel{120}^{60}}{\cancel{2}}$

$a^2 = 60$

Prime factor

$2\times2\times15$

4×15

2	60
2	30
	15

$a^2 = 4 \times 15$

Taking square root both side

$\sqrt{a^2}=\sqrt{4\times15}$

$a=2\sqrt{15}$

Putting the value of a in equation (i)

$\left(2\sqrt{15}\right)^2+b^2=144$

Prime factor

$4\times15+b^2=144$

$60 + b^2 = 144$

$b^2=144-60$

$b^2=84$

$b^2=4\times21$

$2\times2\times21$

4×21

2	84
2	42
21	21
	1

Taking square root both side

$b^2=\sqrt{4\times21}$

$b=2\sqrt{21}$

Putting the value of b in equation (ii)

$\left(2\sqrt{21}\right)^2-h^2=49$

$4\times21-49=h^2$

$h^2 = 84 - 49$

$h^2 = 35$

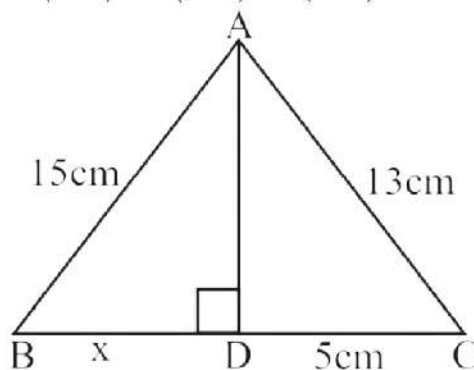
Taking square root both side

$\sqrt{h^2}=\sqrt{35}$

$h=\sqrt{35}$

(ii) Find the value of x in the shown figure

From ΔADC

$\left(\overline{AC}\right)^2=\left(\overline{DC}\right)^2+\left(\overline{AD}\right)^2$

$(13)^2=(5)^2+\left(\overline{AD}\right)^2$

$169=25+\left(\overline{AD}\right)^2$

$169-25=\left(\overline{AD}\right)^2$

$\left(\overline{AD}\right)^2=144$

Taking square root both side

$\sqrt{(\overline{AD})^2} = \sqrt{(144)}$

$\overline{AD} = 12$

From Δ ADB

$(\overline{AB})^2 = (BD)^2 + (\overline{AD})^2$

$(15)^2 = x^2 + (12)^2$

$225 = x^2 + 144$

$225 - 144 = x^2$

$x^2 = 81$

Taking square on both side

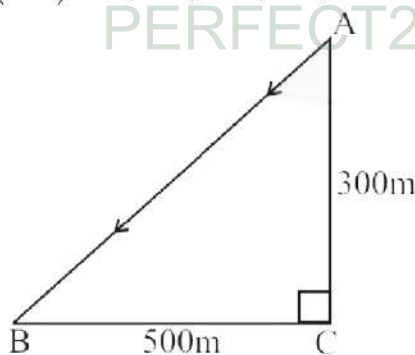
$\sqrt{x^2} = \sqrt{81}$

$x = 9$

Q.7 A plane is at a height of 300m and is 500m away from the airport as shown in the figure How much distance will it travel to land at the airport?

ΔABC is right angle triangle

$(\overline{AB})^2 = (\overline{BC})^2 + (\overline{AC})^2$

$(\overline{AB})^2 = (500)^2 + (300)^2$

$(\overline{AB})^2 = 250000 + 90000$

$(\overline{AB})^2 = 340000$

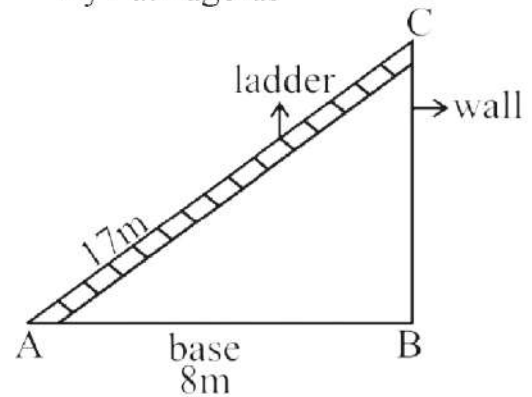
$(\overline{AB})^2 = 10000 \times 34$

Taking square root on both side

$\sqrt{(\overline{AB})^2} = \sqrt{10000 \times 34}$

$AB = 100\sqrt{34}m$

Q.8 A ladder 17m long rests against a vertical wall. The foot of the ladder is 8m away from the base of thewall. How high up the wall will the ladder reach?

By Path agoras

$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2$

$(17)^2 = (8)^2 + (\overline{BC})^2$

$289 = 64 + (\overline{BC})^2$

$284 - 64 = (\overline{BC})^2$

$(\overline{BC})^2 = 225$

Taking square root on both side

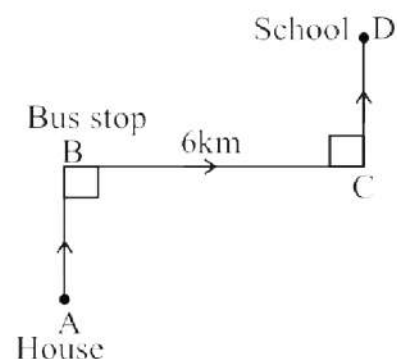
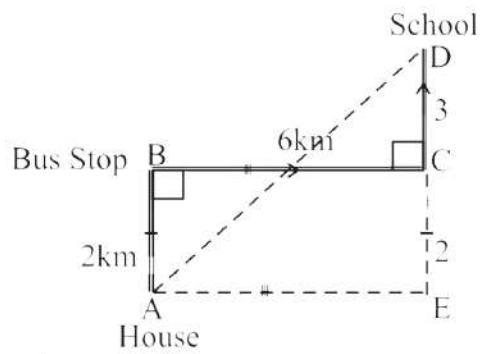
$\sqrt{(\overline{BC})^2} = \sqrt{225}$

$\overline{BC} = 15m$

The height of wall $= \overline{BC} = 15m$

Q.9 A student travels to his school by the route as shown in the figure. Find $m\overline{AD}$, the direct distance from his house to school.

Solution:

As we know that in rectangular opposite sides are equal so

$\overline{AB} = \overline{CE} = 2km$

$\overline{BC} = \overline{AE} = 6km$

$\overline{DE} = \overline{DC} + \overline{CE}$

$\therefore$ We get triangle

Δ ADF which is right angled triangle

$\left(\overline{AD}\right)^2 = \left(\overline{AE}\right)^2 + \left(\overline{ED}\right)^2$

$\left(\overline{AD}\right)^2 = (6)^2 + (3+2)^2$

$\left(\overline{AD}\right)^2 = 36 + (5)^2$

$\left(\overline{AD}\right)^2 = 36 + 25$

$\left(\overline{AD}\right)^2 = 61$

Taking square root on both side

$\sqrt{\left(\overline{AD}\right)^2} = \sqrt{61}$

$\overline{AD} = \sqrt{61}km$

Review Exercise 15

Q.1 Which of the following are true and which are false

(i) In a right angled triangle greater angle is of 90° (True)

(ii) In a right angled triangle right angle is of 60° (False)

(iii) In a right triangle hypotenuse is a side opposite to right angle (True)

(iv) If a,b,c are sides of right angled triangle with c as longer side then c
$c^2 = a^2 + b^2$ (True)

(v) If 3cm and 4cm are two sides of a right angled triangle, the hypotenuse is 5cm (True)

(vi) If hypotenuse of an isosceles right triangle is $\sqrt{2}$ cm then each of other side is of length 2cı
(False)

Q.2 Find the unknown value in each of the following figures.

(i) By Path agoras theorem

(Hypotenuse)^2 = (Base)^2 + (Perpendicular)^2

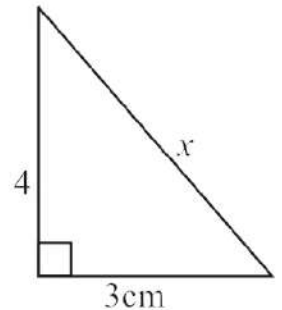
$(x)^2 = (3)^2 + (4)^2$

$x^2 = 9 + 16$

$x^2 = 25$

Taking square root on both side

$\sqrt{x^2} = \sqrt{25}$

$x = 5\,cm$

(ii) By Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

$(10)^2 = (x)^2 + (6)^2$

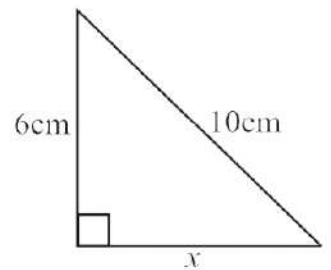
$100 = x^2 + 36$

$100 - 36 = x^2$

$x^2 = 64$

Taking square root on both side

$\sqrt{x^2} = \sqrt{64}$

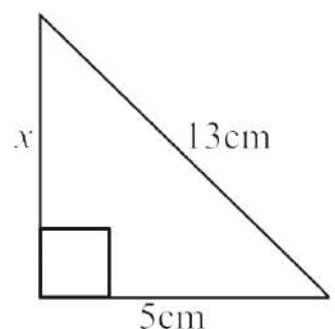
$x = 8\,cm$

(iii) By Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

$(13)^2 = (5)^2 + (x)^2$

$169 = 25 + x^2$

$169 - 25 = x^2$

$x^2 = 144$

Taking square root on both side

$\sqrt{x^2} = \sqrt{144}$

$x = 12\, cm$

(iv) By Path agoras theorem

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$

$(\sqrt{2})^2 = (1)^2 + (x)^2$

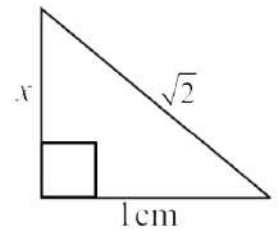
$2 = 1 + x^2$

$2 - 1 = x^2$

$x^2 = 1$

Taking square root on both side

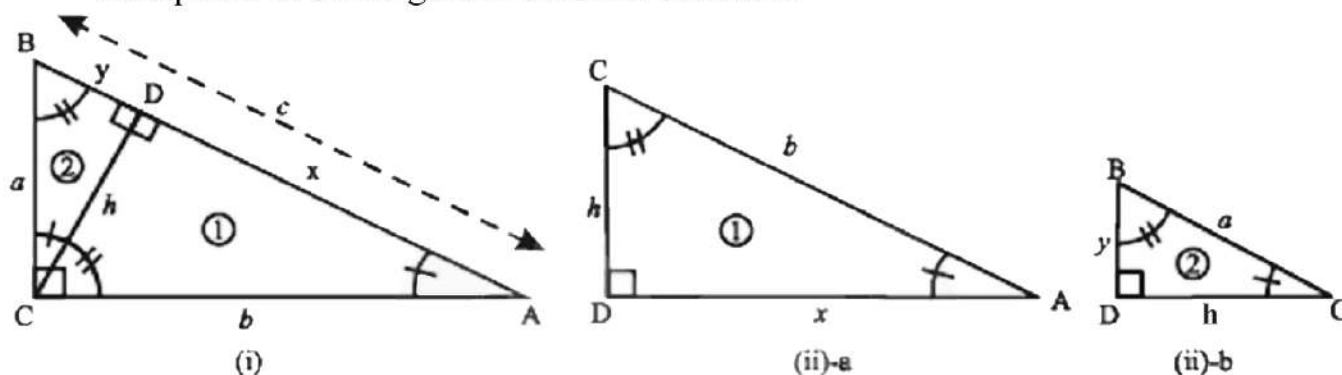
$\sqrt{x^2} = \sqrt{1}$

$x = 1cm$

Unit 15: Pythagoras Theorem

Overview

Theorem 15.1.1

In a right angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides

(i) (ii)-a (ii)-b

Given

Δ ACB is a right angled triangle in which $m\angle C = 90°$ and $m\overline{BC} = a$, $m\overline{AC} = b$ and $m\overline{AB} = c$

To prove

$c^2 = a^2 + b^2$

Construction

Draw $\overline{CD}$ perpendicular from C on $\overline{AB}$

Let $m\overline{CD} = h$, $m\overline{AD} = x$ and $m\overline{BD} = y$. Line segment CD splits ΔABC into two Δs ADC and BDC which are separately shown in the figures (ii) –a and (ii) –b respectively

Proof (using similar Δs)

Statements	Reasons
In Δ ADC $\leftrightarrow$ Δ ACB	Refer to figure (ii)-a and (i)
$\angle A \cong \angle A$	Common – Self Congruent
$\angle ADC \cong \angle ACB$	Construction- given each angle = 90°
$\angle C \cong \angle B$	$\angle C$ and $\angle B$ complements of $\angle A$

$\therefore \Delta ADC \sim \Delta ACB$	Congruency of three angles
$\therefore \frac{x}{b} = \frac{b}{c}$	(Measures of corresponding sides of similar triangles are proportional)
or $x = \frac{b^2}{c}$ ________ (i)	
Again in $\Delta BDC \leftrightarrow \Delta BCA$	Refer to figure (ii)-b and (i)
$\angle B \cong \angle B$	Common – self Congruent
$\angle BDC \cong \angle BCA$	Construction – given each angle = 90°
$\angle C \cong \angle A$	$\angle C$ and $\angle A$ complements of $\angle B$
$\therefore \Delta BDC \sim \Delta BCA$	Congruency of three angles
$\therefore \frac{y}{a} = \frac{a}{c}$	(Corresponding sides of similar triangles are proportional)
or $y = \frac{a^2}{c}$ ________ (ii)	
But $y + x = c$	Supposition
$\therefore \frac{a^2}{c} + \frac{b^2}{c} = c$	By (i) and (ii)
or $a^2 + b^2 = c^2$	Multiplying both side by c
i-e $c^2 = a^2 + b^2$	

Theorem 15.1.2 Converse of Pythagoras Theorem 15.1.1

If the Square of one side of a triangle is equal to the sum of the square of the other two sides then the triangle is a right angled triangle

Given

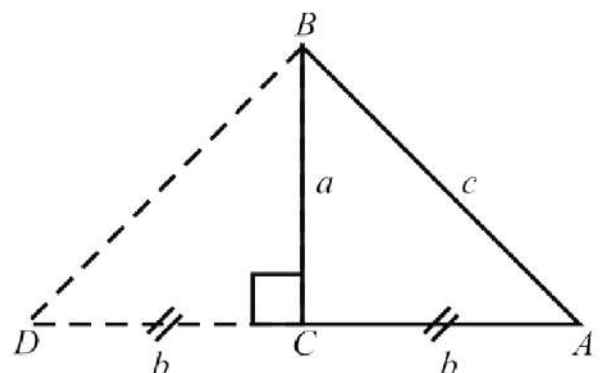
In a ΔABC , $m\overline{AB} = c, m\overline{BC} = a, m\overline{AC} = b$

Such that $a^2 + b^2 = c^2$

To prove

ΔACB is a right angled triangle

Construction

Draw $\overline{CD}$ perpendicular to $\overline{BC}$ Such that

$\overline{CD} \cong \overline{CA}$. Join the points B and D

Proof

Statements	Reasons
ΔDCB is a right angled triangle	Construction
$\therefore \left(m\overline{BD}\right)^2 = a^2 + b^2$	Pythagoras theorem
But $a^2 + b^2 = c^2$	Given
$\therefore \left(m\overline{BD}\right)^2 = c^2$	
or $m\overline{BD} = c$	Taking Square root on both sides
Now in $\Delta DCB \leftrightarrow \Delta ACB$	
$\overline{CD} \cong \overline{CA}$	Construction
$\overline{BC} \cong \overline{BC}$	Common
$\overline{DB} \cong \overline{AB}$	Each side = c
$\therefore \Delta\ DCB \cong \Delta\ ACB$	S.S.S ≅ S.S.S
$\therefore \Delta\ DCB \cong \angle ACB$	(Corresponding angles of congruent triangle)
But $m\angle\ DCB = 90°$	Construction
$\therefore m\angle\ ACB = 90°$	
Hence the Δ ACB is a Right angled triangle	

Exercise 16.1

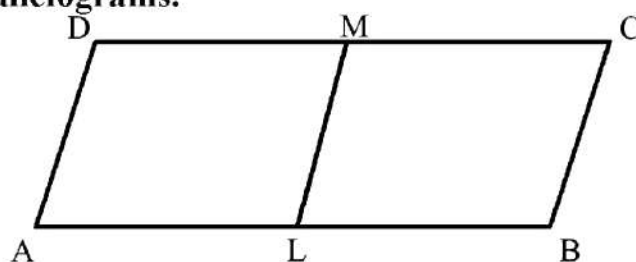
Q.1 Show that the line segment joining the midpoint of opposite sides of a parallelogram divides it into two equal parallelograms.

Given

ABCD is a parallelogram. L is the midpoint of $\overline{AB}$ and M is the midpoint of $\overline{DC}$

To prove

Area of parallelogram ALMD = area of parallelogram LBCM.

Proof

Statements	Reasons
$\overline{AB} \parallel \overline{DC}$	Opposite sides of parallelogram ABCD.
$\overline{AL} \cong \overline{LB}$ ...(i)	L is midpoint of $\overline{AB}$
The parallelograms ALMD and LBCM are on equal bases and between the same parallel lines $\overline{AB}$ and $\overline{DC}$	From equation (i)
Hence area of parallelogram ALMD= area of parallelogram LBCM.	They have equal areas

Q.2 In a parallelogram ABCD, m $\overline{AB}$ =10cm the altitudes Corresponding to Sides AB and AD are respectively 7cm and 8cm Find $\overline{AD}$

$\overline{AB} = 10$ cm

$\overline{DH} = 7$cm

$\overline{MB} = 8$cm

$\overline{AD} = ?$

Formula

Area of parallelogram = base x altitude

$\overline{AB} \times \overline{DH} = \overline{AD} \times \overline{IB}$

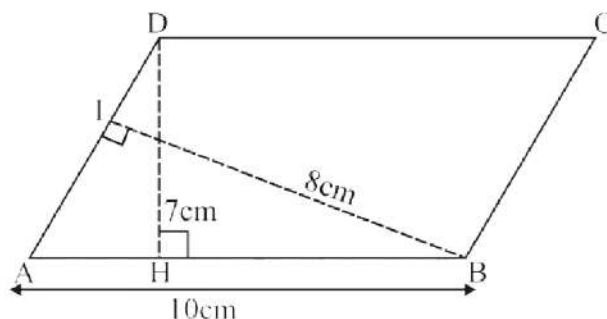
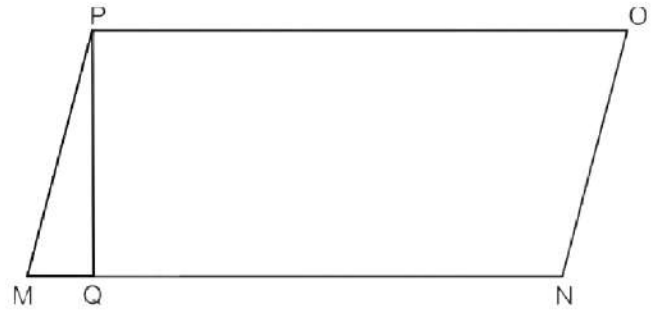
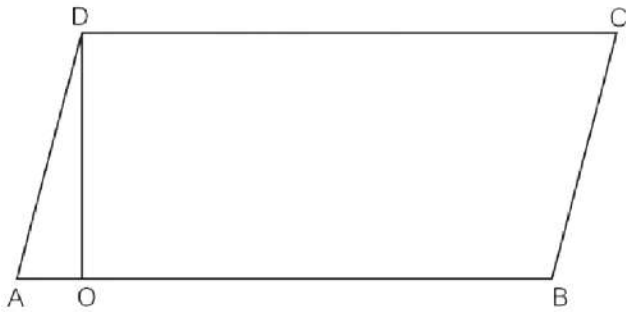
$10 \times 7 = \overline{AD} \times 8$

$\frac{\cancel{70}^{35}}{\cancel{8}^{4}} = \overline{AD}$

$\frac{35}{4} = \overline{AD}$

$\overline{AD} = \frac{35}{4}$

Or

$\overline{AD} = 8.75$cm

Q.3 If two parallelograms of equal areas have the same or equal bases, their altitude are equal

In parallelogram opposite side and opponents angles are Congruent.

Given

Parallelogram ABCD and parallelogram MNOP

OD is altitude of parallelogram ABCD

PQ is altitude of parallelogram MNOP

Area of ABCD $\|^{gm}$ $\cong$ Area of MNOP $\|^{gm}$

To prove

$m\overline{OD} \cong m\overline{PQ}$

Proof

Statements	Reasons
Area of parallelogram ABCD= Area of parallelogram MNOP	Given
Area of parallelogram= base × height	Given
$\overline{AB} \times \overline{OD} = \overline{MN} \times \overline{PQ}$	
We know that	
$\overline{AB} = \overline{MN}$	
So	
$\frac{\cancel{AB}}{\cancel{AB}} \times \overline{OD} = \overline{PQ}$	Proved
$\overline{OD} = \overline{PQ}$	

Theorem 16.1.3

Triangle on the same base and of the same (i.e...equal) altitudes are equal in area

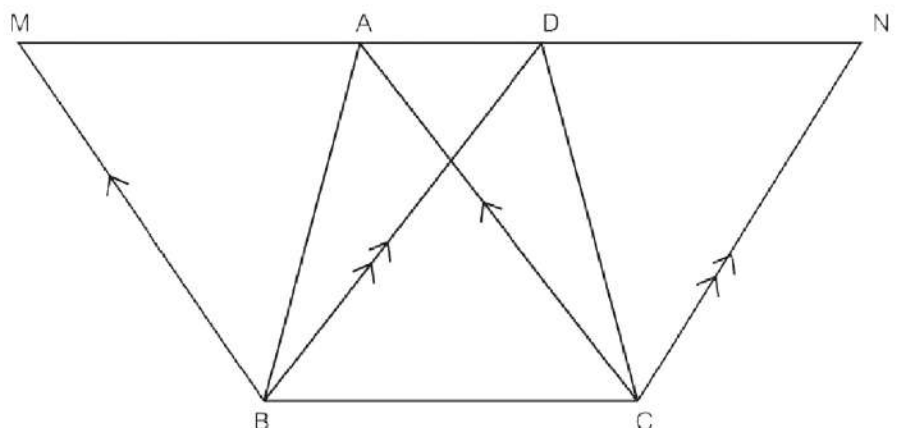
Given

Δ's ABC , DBC on the

Same base $\overline{BC}$ and

having equal altitudes

To prove

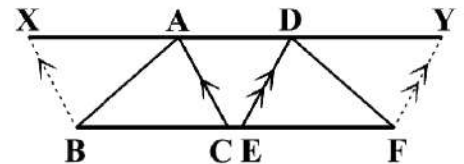
Area of (ΔABC) = area of (ΔDBC)

Construction:

Draw $\overline{BM} \parallel$ to $\overline{CA}$, $\overline{CN} \parallel$ to $\overline{BD}$ meeting $\overline{AD}$ produced in M.N.

Proof

Statements	Reasons
ΔABC and ΔDBC are between the same $\parallel^s$	Their altitudes are equal
Hence MADN is parallel to $\overline{BC}$	
$\therefore$ Area $\parallel^{gm}$ (BCAM)= Area $\parallel^{gm}$ (BCND)	These $\parallel^{gm}$ are on the same base
But $\Delta ABC = \frac{1}{2} \parallel^{gm}$ (BCAM)------(ii)	$\overline{BC}$ and between the same $\parallel^s$
And $\Delta DBC = \frac{1}{2} \parallel^{gm}$ (BCND)-----(iii)	Each diagonal of a $\parallel^{gm}$
Hence area (ΔABC) = Area(ΔDBC)	Bisects it into two congruent triangles From (i) (ii) and (iii)

Theorem 16.1.4

Triangles on equal bases and of equal altitudes are equal in area.

Given

Δs ABC, DEF on equal bases $\overline{BC}$, $\overline{EF}$ and having altitudes equal

To prove

Area (ΔABC) = Area (ΔDEF)

Construction:

Place the Δs ABC and DEF so that their equal bases $\overline{BC}$ and $\overline{EF}$ are in the same straight line BCEF and their vertices on the same side of it .Draw $\overline{BX} \parallel \overline{CA}$ and $\overline{FY} \parallel \overline{ED}$ meeting $\overline{AD}$ produced in X, Y respectively

Proof

Statements	Reasons
ΔABC, ΔDEF are between the same parallels	Their altitudes are equal (given)

$\therefore$ XADY is $\parallel^{gm}$ to BCEF	
$\therefore$ area $\parallel^{gm}$ (BCAX) = A area $\parallel^{gm}$ (EFYD)----(i)	These $\parallel^{gm}$ are on equal bases and between the same parallels
But $\Delta ABC = \frac{1}{2} \parallel^{gm}$ (BCAX)----(ii)	Diagonal of a $\parallel^{gm}$ bisect it
And area of $\Delta DEF = \frac{1}{2}$ area of $\parallel^{gm}$ (EFYD)__ (iii)	
$\therefore$ area (ΔABC) = area (ΔDEF)	From (i),(ii)and(iii)

Exercise 16.2

Q.1

Show that

Given

ΔABC, O is the mid point of $\overline{BC}$

$\overline{OB} \cong \overline{OC}$

To prove

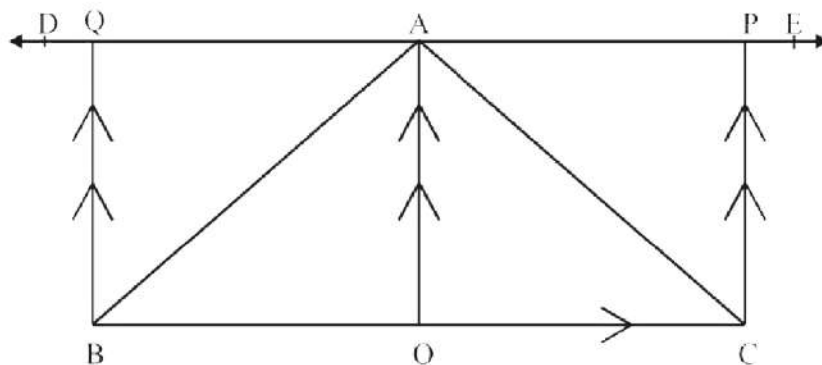
Area ΔABO = area ΔACO

Construction

Draw $\overline{DE} \parallel \overline{BC}$

$\overline{CP} \parallel \overline{OA}$

$\overline{BQ} \parallel \overline{OA}$

Proof

Statements	Reasons
$\overline{BQ} \parallel \overline{OA}$	Construction
$\overline{OB} \parallel \overline{AQ}$	Construction
$\parallel^{gm}$ BOAQ	Base of same
$\parallel^{gm}$ COAP	Parallel line of $\overleftrightarrow{DE}$
$\overline{OB} \cong \overline{OC}$	O is the mid point of $\overline{BC}$
Area of $\parallel^{gm}$ BOAQ= Area of $\parallel^{gm}$ COAP ... (i)	
Area of $\Delta ABO = \frac{1}{2}$ Area of $\parallel^{gm}$ BOAQ	
Area of $\Delta ACO = \frac{1}{2}$ Area of $\parallel^{gm}$ COAP	
Area of ΔABO = Area of ΔACO	Dividing equation (i) both side by (ii)

So median of a triangle divides it into two triangles of equal area.

Q.2 **Prove that a parallelogram is divided by its diagonals into four triangles of equal area.**

Given:

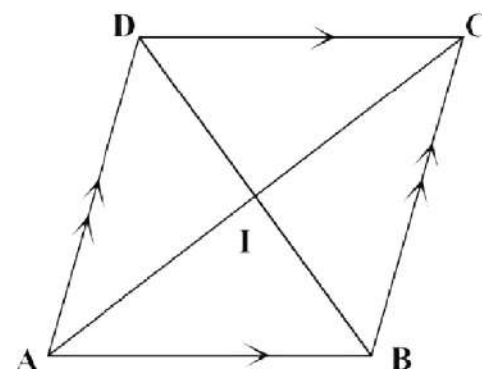
In parallelogram ABCD, $\overline{AC}$ and $\overline{BD}$ are its diagonals, which meet at I

To prove:

Triangles ABI, BCI CDI and ADI have equal areas.

Proof:

Triangles ABC and ABD have the same base $\overline{AB}$ and are between the same parallel lines $\overline{AB}$ and $\overline{DC}$ $\therefore$ they have equal areas.

Or area of ΔABC =area of ΔABD

Or area of Δ ABI + area of Δ BCI= area of Δ ABI+ area of Δ ADI
⇒ Area of Δ BCI = area of Δ ADI ... (i)
Similarly area of Δ ABC = area of Δ BCD
⇒ Area of Δ ABI +area of Δ BCI = area of Δ BCI + area of Δ CDI
⇒ Area of Δ ABI = area of Δ CDI... (ii)

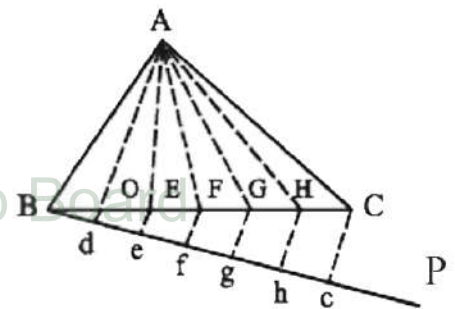
As diagonals of a parallelogram bisect each other I is the midpoint of $\overline{AC}$ so $\overline{BI}$ is a median of Δ ABC
∴ Area of Δ ABI = area of Δ BCI... (iii)
$\Delta CDI \cong \Delta AOI$
$\overline{BI} \cong \overline{DI}$
Area of Δ ABI = area of Δ BCI = area of Δ CDI= area of Δ ADI

Q.3 Divide a triangle into six equal triangular parts

Given
ΔABC
To prove
To divide ΔABC into six equal part triangular parts
Construction
Take $\overrightarrow{BP}$ any ray making an acute angle with $\overline{BC}$ draw six arcs of the same radius on $\overrightarrow{BP}$ i.e $m\overline{Bd} = mde = mef = mfg = mgh = mhc$

Join c to C and parallel line segments as

$\overline{cC} \| \overline{hH} \| \overline{gG} \| \overline{fF} \| \overline{eE} \| \overline{do}$

Join A to O,E,F,G,H
Proof
Base $\overline{BC}$ of ΔABC has been divided to six equal parts.
We get six triangles having equal base and same altitude
Their area is equal
Hence ΔBOA=ΔOEA=ΔEFA=ΔFGA=ΔGHA=ΔHCA

Review Exercise 16

Q.1 Which of the following are true and which are false?

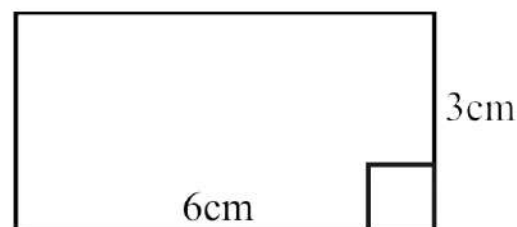
(i) Area of a figure means region enclosed by bounding lines of closed figures. (True)
(ii) Similar figures have same area. (False)
(iii) Congruent figures have same area. (True)
(iv) A diagonal of a parallelogram divides it into two non-congruent triangles. (False)
(v) Altitude of a triangle means perpendicular from vertex to the opposite side (base). (True)
(vi) Area of a parallelogram is equal to the product of base and height. (True)

Q.2 Find the area of the following.

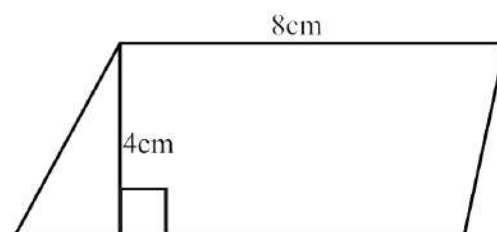
(i)

Given
Length of rectangle = $\ell = 3cm$
Width of rectangle = w = 6cm
Required:
Area of rectangle =?
Solution:
Area of rectangle = length $\times$ width
$= 3cm \times 6cm$
$\Rightarrow$ Area of rectangle = 18 cm^2

(ii)

4cm

Given
Length of square = $\ell = 4cm$
Required:
Area of square =?
Solution:
Area of square = $\ell \times \ell$
$= \ell^2$
$= (4cm)^2$
$\Rightarrow$ Area of square = $16cm^2$

(iii)

Given
Height of parallelogram = 4cm
Base of parallelogram = 8cm
Required:
Area of parallelogram = ?
Solution:
Area of parallelogram = b $\times$ h
$= 8cm \times 4cm$

$\Rightarrow$ area of parallelogram = 32 cm^2

(iv)

Given:

Height of triangle = h = 10 m

Base of triangle = b = 16cm

Required:

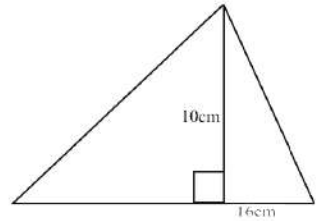
Area of triangle =?

Solution:

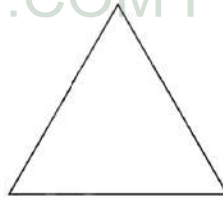
Area of triangle = $\frac{1}{2} \times b \times h$

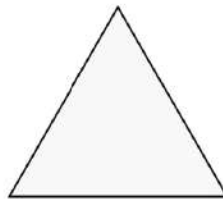
$= \frac{1}{\not{2}} \times {}^{8}\not{16}\ cm \times 10cm$

$= 8cm \times 10\ cm$

$= 80cm^2$

Q.3 Define the following

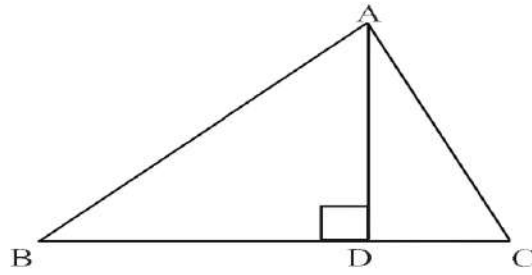
(i) Area of a figure

The region enclosed by the bounding lines of a closed figure is known as area of the figure.

(ii) Triangular Region

A triangular region is the union of a triangle and its interior i-e three line segments forming the triangle and its interior

(iii) Rectangular Region

A rectangular region is the union of a rectangle and its interior. A rectangular region can be divided into two or more than two triangular regions in many ways.

(iv) Altitude or Height

If one side of a triangle is taken as its base, the perpendicular distance form one vertex opposite side is called altitude of triangle. $\overline{AD}$ is its altitude.

Unit 16: Theorems Related With Area

Overview

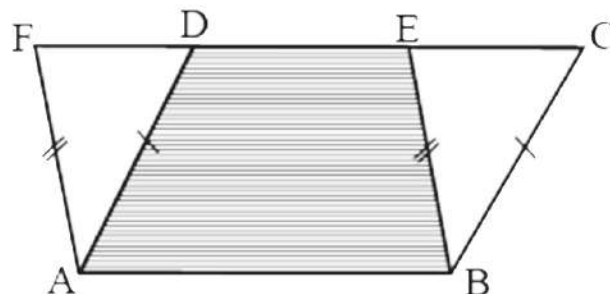
Theorem 16.1.1

Parallelograms on the same base and between the same parallel lines (or of the same altitude) are equal in area

Given

Two parallelograms ABCD and ABEF having the same base $\overline{AB}$ between the same parallel lines $\overline{AB}$ and $\overline{DE}$

To prove

Area of parallelogram ABCD=area of parallelogram ABEF

Proof

Statements	Reasons
Area of (parallelogram ABCD) = Area of (Quad. ABED) + Area of (Δ CBE) … (1)	[Area addition axiom]
Area of (parallelogram ABEF) = Area of (Quad. ABED) + Area of (Δ DAF) … (2)	[Area addition axiom]
In Δ s CBE and DAF	
m $\overline{CB}$ = m $\overline{DA}$	[opposite sides of a Parallelogram]
m $\overline{BE}$ = m $\overline{AF}$	[opposite sides of a Parallelogram]
m$\angle$CBE = m$\angle$DAF	$\left[\because \overline{BC} \parallel \overline{AD}, \overline{BE} \parallel \overline{AF}\right]$
Δ CBE $\cong$ Δ DAF	[S.A.S Cong.axiom]
Area of (ΔCBE)= area of (ΔDAF)… (3)	[Cong. Area axiom]
Hence area of (Parallelogram ABCD) = area of (parallelogram ABEF)	From (1),(2) and (3)

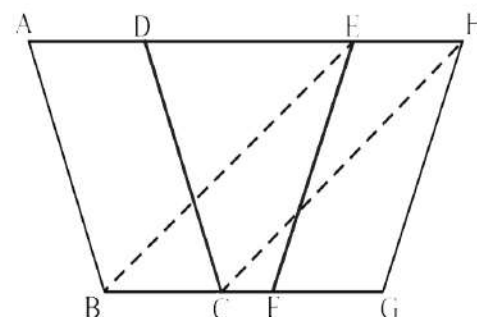
Theorem 16.1.2

Parallelograms on equal bases and having the same (or equal) altitude area equal in area.

Given :

Parallelogram ABCD, EFGH are on equal base $\overline{BC}$, $\overline{FG}$ having equal altitudes

To prove

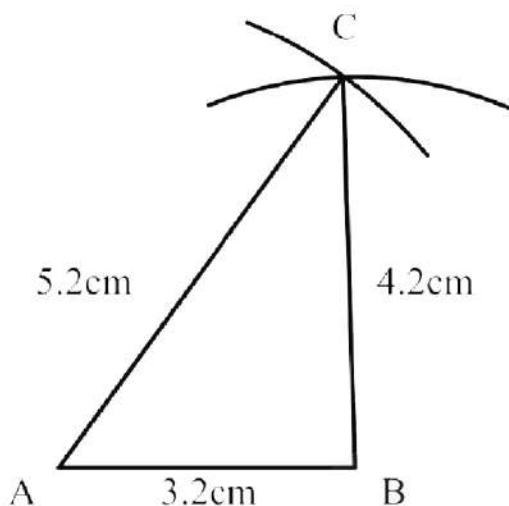
Area of (Parallelogram ABCD)= area of (parallelogram EFGH)

Construction

Place the parallelogram ABCD and EFGH So that their equal bases $\overline{BC}, \overline{FG}$ are in the straight line BCFG. Join $\overline{BE}$ and $\overline{CH}$

Proof

Statements	Reasons
The give 11^{mg} ABCD and EFGH are between the same parallels	
Hence ADEH is a straight line $\parallel$ to $\overline{BC}$	Their altitudes are equal (given)
$\therefore$ m$\overline{BC}$ = m$\overline{FG}$ =m$\overline{EH}$	
Now m$\overline{BC}$ =m$\overline{EH}$ and they are $\parallel$	Given
$\therefore \overline{BE}$ and $\overline{CH}$ are both equal and $\parallel$	EFGH is a parallelogram
Hence EBCH is a Parallelogram	A quadrilateral with two opposite side congruent and parallel is a parallelogram
Now $\parallel^{gm}$ ABCD = $\parallel^{gm}$ EBCH –(i)	Being on the same base $\overline{BC}$ and between the same parallels
But $\parallel^{gm}$ EBCH = $\parallel^{gm}$ EFGH – (ii)	Being on the same base $\overline{EH}$ and between the same parallels
Hence area $\parallel^{gm}$ (ABCD)= Area $\parallel^{gm}$ (EFGH)	From (i) and (ii)

Exercise 17.1

Q.1 Construct a ΔABC in which

(i) $m\overline{AB} = 3.2cm$ $m\overline{BC} = 4.2cm$ $m\overline{CA} = 5.2cm$

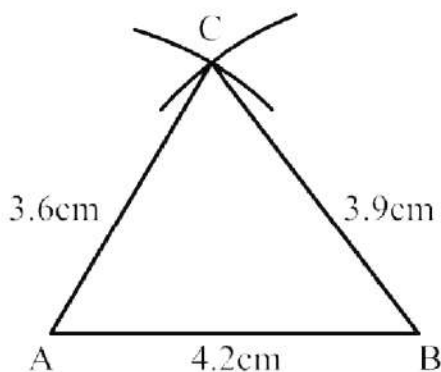
i. Draw a line segment $m\overline{AB} = 3.2cm$

ii. Taking A as centre draw an arc of radius 5.2cm.

iii. Taking B as centre draw an arc of radius 4.2cm to cut at point C.

iv. Join C to A and C to B.

Thus ΔABC is the required triangle.

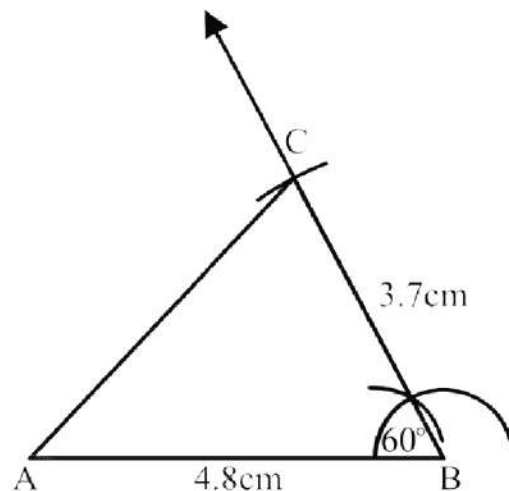
(ii) $m\overline{AB} = 4.2cm$ $m\overline{BC} = 3.9cm$ $m\overline{CA} = 3.6cm$

i. Draw a line segment $m\overline{AB} = 4.2cm$

ii. Taking A as centre draw an arc of radius 3.6cm.

iii. Taking B as centre draw an arc of radius 3.9cm to cut at point C.

iv. Join C to A and C to B.

Thus ΔABC is the required triangle.

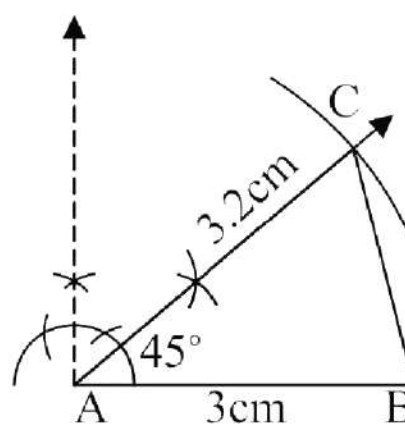
(iii) $m\overline{AB} = 4.8cm$ $m\overline{BC} = 3.7cm$ $m\angle B = 60^\circ$

i. Draw a line segment $m\overline{AB} = 4.8cm$.

ii. Taking B as centre draw an angle of 60°.

iii. Taking B as centre draw an arc of radius 3.7cm cutting terminal side of 60° at C.

iv. Join C to A.

Thus ΔABC is the required triangle.

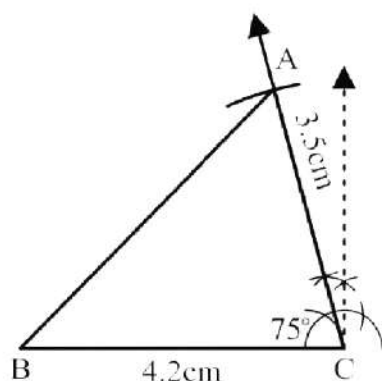
(iv) $m\overline{AB} = 3cm$ $m\overline{AC} = 3.2cm$ $m\angle A = 45^\circ$

i. Draw a line segment $m\overline{AB} = 3cm$.

ii. Taking A as centre draw an angle of 45°.

iii. Taking A as centre draw an arc of radius 3.2cm to cut the terminal side of angle at C.

iv. Join C to B.

Thus ΔABC is the required triangle.

(v) $m\overline{BC}=4.2cm$ $m\overline{CA}=3.5cm$ $m\angle C=75^\circ$

i. Draw a line segment $m\overline{BC}=4.2cm$.

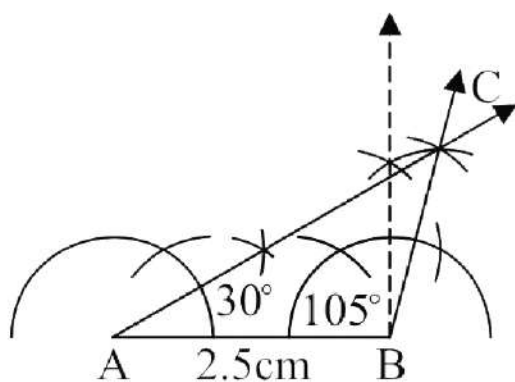
ii. Taking C as centre draw an angle of 75°.

iii. Taking C as centre draw an arc of radius 3.5cm.

iv. Cutting the terminal side of angle at A.

v. Join A to B.

Thus ΔABC is the required triangle.

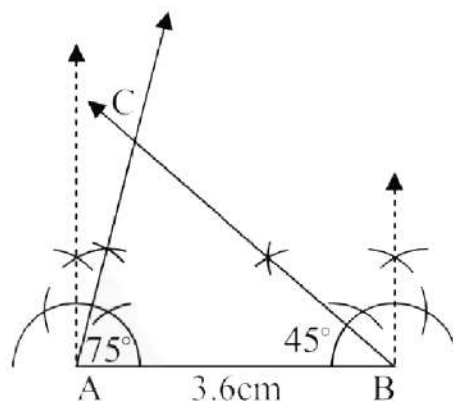
(vi) $m\overline{AB}=2.5cm$ $m\angle A=30^\circ$ $m\angle B=105^\circ$

i. Draw a line segment $m\overline{AB}=2.5cm$.

ii. Taking A as centre draw an angle of 30°.

iii. Taking B as centre draw an angle of 105°.

iv. Terminal sides of these two angles meet at C.

Thus ΔABC is the required triangle.

(vii) $m\overline{AB}=3.6cm$ $m\angle A=75^\circ$ $m\angle B=45^\circ$

i. Draw a line segment $m\overline{AB}=3.6cm$.

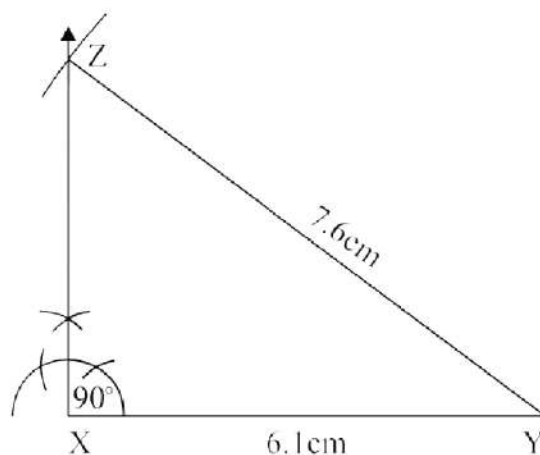
ii. Taking A as centre draw an angle of 75°.

iii. Taking B as centre draw an angle of 45°.

iv. Terminal sides of these two angles meet at point C.

Thus ΔABC is the required triangle.

Q.2 Construct a ΔXYZ in which

(i) $m\overline{YZ}=7.6cm$ $m\overline{XY}=6.1cm$ $m\angle X=90^\circ$

i. Draw a line segment $m\overline{XY} = 6.1cm$.

ii. Taking X as Centre draw an angle of 90^o.

iii. Taking Y as Centre draw an arc of radius 7.6cm to cut terminal sides of angle at Z.

iv. Join Y to Z.

Thus ΔXYZ is the required triangle.

(ii) $m\overline{ZX} = 6.4cm$ $m\overline{YZ} = 2.4cm$ $m\angle Y = 90^o$

i. Draw a line segment $m\overline{YZ} = 2.4cm$.

ii. Taking Y as centre draw an angle of 90°.

iii. Taking Z as centre draw an arc of radius 6.4cm. Which cuts the terminal side of angle at X.

iv. Join X and Z.

Thus ΔXYZ is the required triangle.

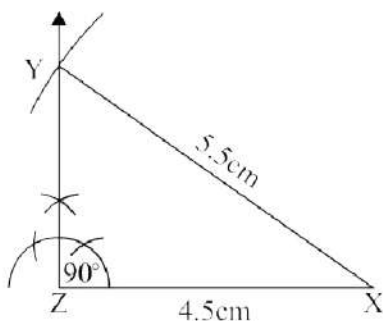
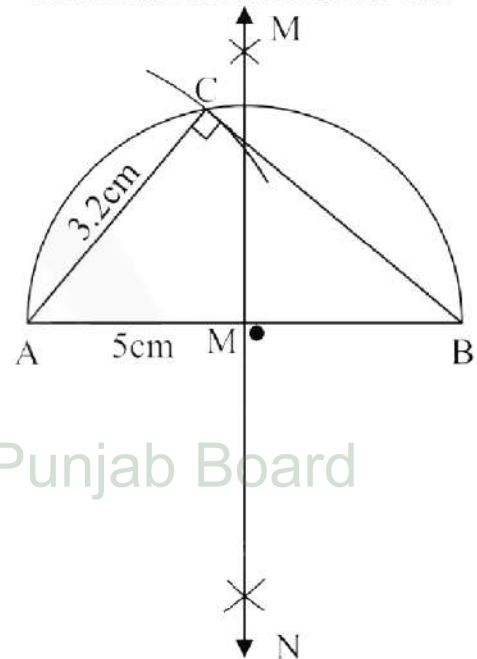
(iii) $m\overline{XY} = 5.5cm$ $m\overline{ZX} = 4.5cm$ $m\angle Z = 90^o$

i. Draw a line segment 4.5cm.

ii. Taking Z as centre draw an angle of 90^o.

iii. Taking X as centre draw an arc of radius 5.5cm. Which cut the terminal side angle at Y.

iv. Join Y to X.

Thus ΔXYZ is the required triangle.

Q.3 Construct a right angled Δ measure of whose hypotenuse is 5cm and one side is 3.2 cm

Construction:

i. Draw a line segment $m\overline{AB}$=5cm.

ii. Bisect $\overline{AB}$ at M.

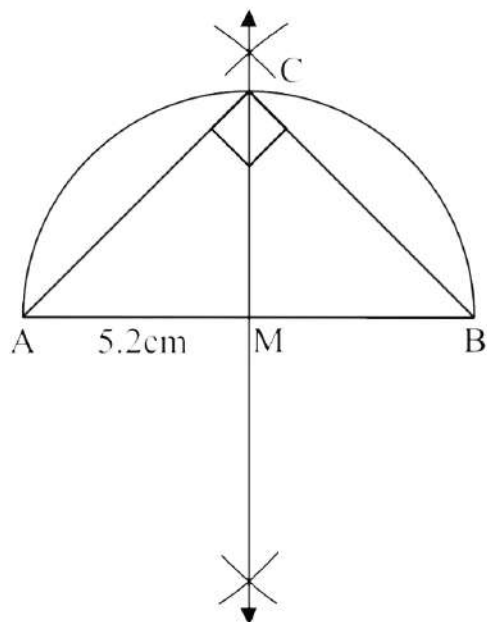
iii. Taking M as centre take a radius $\overline{AM}$ or $\overline{BM}$ and draw a semicircle.

iv. Taking A as centre draw an arc of radius 3.2cm cutting semicircle at C.

v. Join C to A and C to B.

Thus ΔABC is the required right angled triangle.

Q.4 Construct right angled isosceles triangle whose hypotenuse is

(i) 5.2cm long

Construction:

i. Draw a line segment $m\overline{AB} = 5.2cm$.

ii. Bisect $\overline{AB}$ at point M.

iii. With M as centre draw a semi circle of radius $\overline{AM}$ or $\overline{BM}$ which intersects the right bisector at C.

iv. Join A to C and B to C.

ΔABC is the required right angled isosceles triangle with $m\angle C = 90^\circ$.

(ii) 4.8cm long

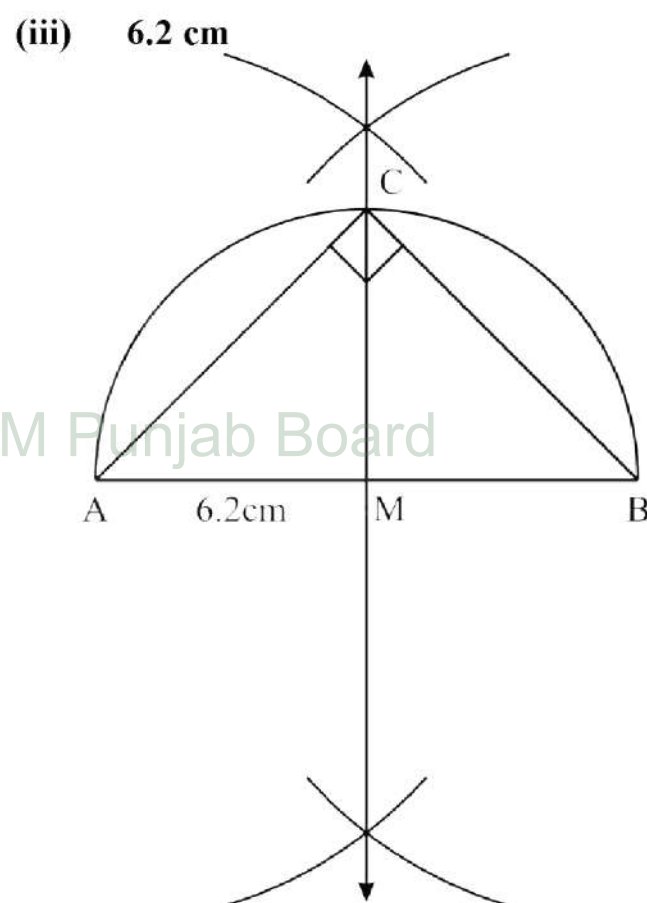
i. Take a line segment $m\overline{AB} = 4.8cm$.

ii. Bisect $\overline{AB}$ at point M.

iii. Taking M as centre draw a semi circle of radius $\overline{AM}$ or $\overline{MB}$ which intersects the right bisector at C.

iv. Join A to C and B to C.

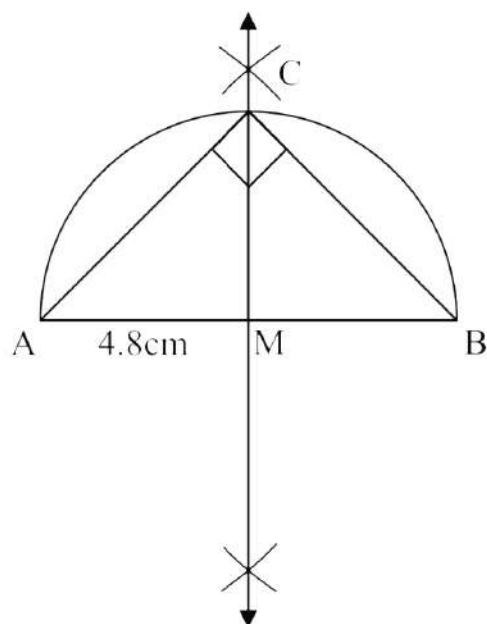
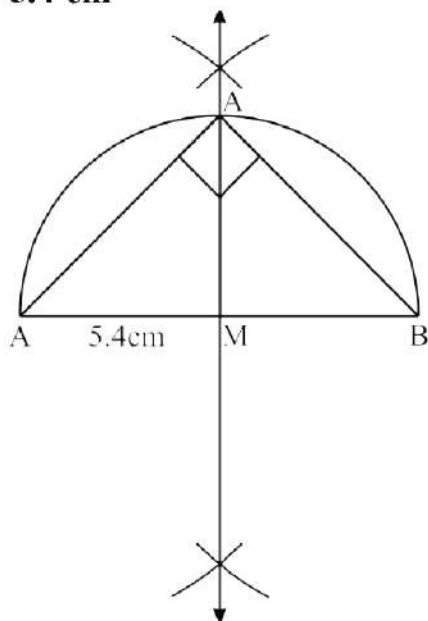
Thus ABC is the right angled isosceles triangle with $\angle C = 90^\circ$.

(iii) 6.2 cm

i. Take a line segment $m\overline{AB} = 6.2cm$.

ii. Bisect $\overline{AB}$ at point M.

iii. Taking M as a centre draw a semi circle of radius $\overline{AM}$ or $\overline{BM}$ which intersects the right bisector at C.

iv. Join A to C and B to C.

Thus ΔABC is the right angled isosceles triangle with $\angle C = 90^\circ$.

(iv) **5.4 cm**

Construction:

i. Take a line segment $m\overline{AB} = 5.4cm$.

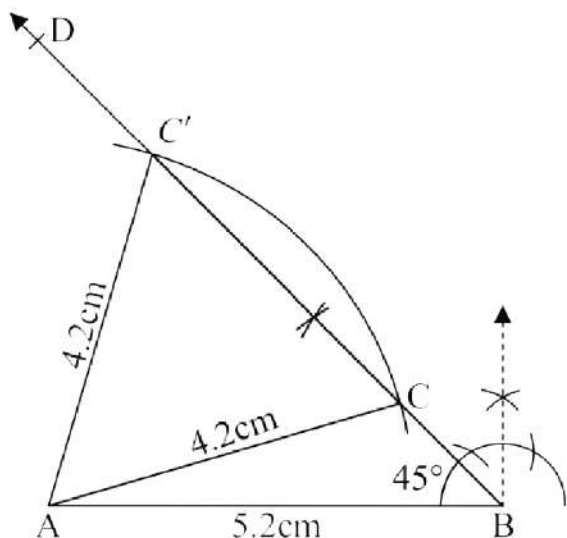
ii. Bisect $\overline{AB}$ at point M.

iii. Taking M as a centre draw a semi circle of radius $\overline{AM}$ or $\overline{BM}$ which intersects the right bisector at C.

iv. Join A to C and B to C.

Thus ΔABC is the right angled isosceles triangle with $\angle C = 90^\circ$.

Q.5 **(Ambiguous case) Construct a Δ ABC in which**

(i) $m\overline{AC} = 4.2cm$ $m\overline{AB} = 5.2cm$ $m\angle B = 45^\circ$

Construction:

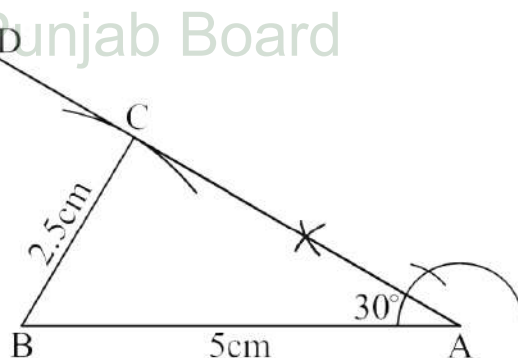
i. Draw a line segment $m\overline{AB} = 5.2cm$.

ii. At the end point B of $\overline{BA}$ make $\angle B = 45^\circ$.

iii. With centre at A and radius 4.2cm draw an arc which cuts $\overrightarrow{BD}$ in two distinct points C and C`.

iv. Draw $\overline{AC}$ and $\overline{AC'}$.

∴ ΔABC and ΔABC` are required triangles.

(ii) $m\overline{BC} = 2.5cm$ $m\overline{AB} = 5cm$ $m\angle A = 30^\circ$

Construction:

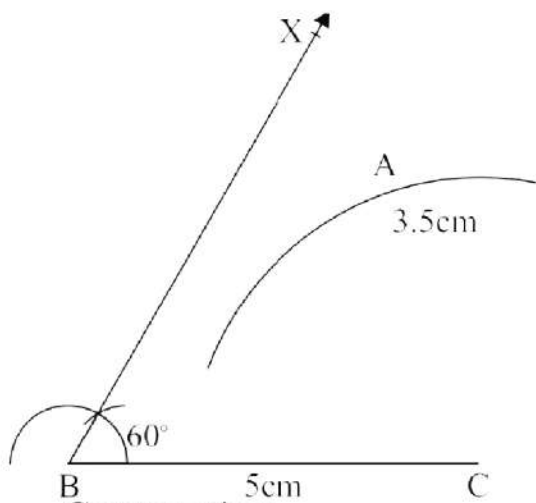
i. Take a line segment $m\overline{AB} = 5cm$.

ii. At the end point A of $\overline{AB}$ make $m\angle A = 30^\circ$.

iii. Taking B as centre draw an arc of radius 2.5cm which touch as $\overrightarrow{AD}$ at point C.

iv. Join B to C.

∴ ΔABC is required triangle.

(iii) $m\overline{BC} = 5cm$ $m\overline{AC} = 3.5cm$ $m\angle B = 60^\circ$

Construction:

i. Take a line segment $m\overline{BC} = 5cm$.

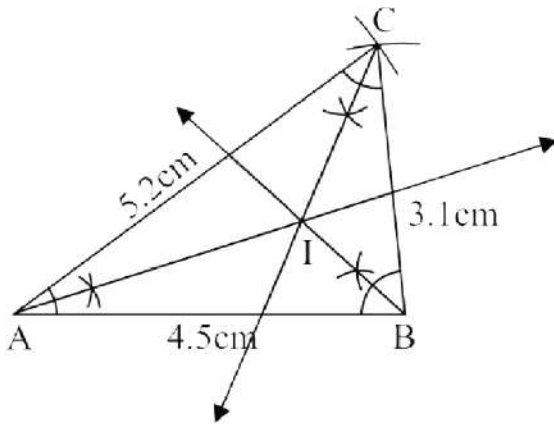
ii. At the end point B of $\overline{BC}$ make an angle of $\angle B = 60^\circ$.

iii. Taking C as centre draw an arc of radius 3.5cm which does not touches or intersects $\overrightarrow{BX}$ at any point.

$\therefore$ ΔABC is not possible.

Exercise 17.2

Q.1 Construct the following Δ's ABC. Draw the Bisector of their angle and verify their Concurrency.

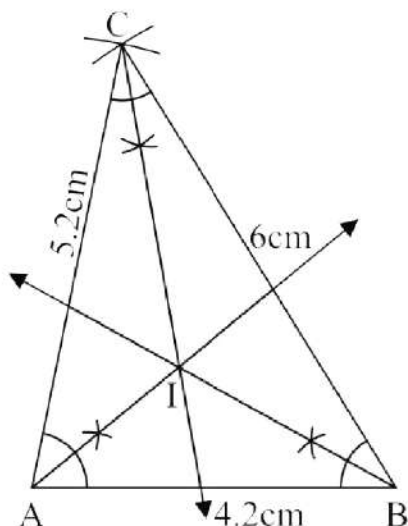
(i) $mAB = 4.5cm \quad m\overline{BC} = 3.1cm \quad m\overline{CA} = 5.2cm$

i. Draw a line segment $m\overline{AB} = 4.5cm$.

ii. Taking B as centre draw an arc of $m\overline{BC} = 3.1cm$.

iii. Taking A as centre draw a arc $m\overline{AC} = 5.2cm$ to cut C.

iv. Join C to B and C to A.

v. Draw the angle bisectors of $\angle A, \angle B$ and $\angle C$ meeting each other at the point I.
All the angle bisectors pass through point I. hence angle bisectors of ΔABC are concurrent.

(ii) $mAB = 4.2cm \quad m\overline{BC} = 6cm \quad m\overline{CA} = 5.2cm$

i. Draw a line segment $\overline{AB} = 4.2cm$.

ii. Taking A as centre draw an arc of radius $5.2cm$.

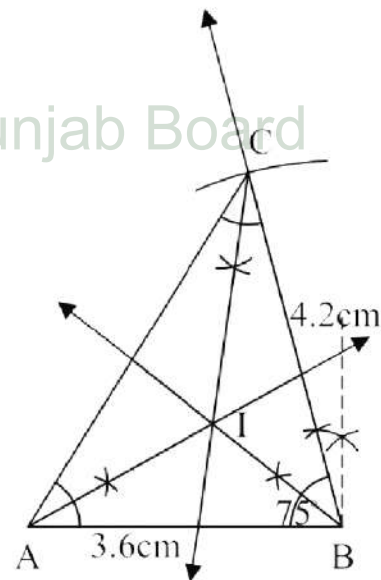
iii. Taking B as centre draw another arc of radius $6cm$ to intersect the first arc at C.

iv. Draw $\overline{AC}$ and $\overline{BC}$. Thus ΔABC is the required triangle.

v. Draw the bisectors of $\angle A$ and $\angle B$ meeting each other at point I.

vi. Now draw the bisector of third $\angle C$.

vii. We observe that the third angle bisector also passes through the point I.
Hence the angle bisectors of the ΔABC are concurrent at I.

(iii) $mAB = 3.6cm \quad m\overline{BC} = 4.2cm \quad m\angle B = 75^\circ$

i. Draw a line segment $m\overline{AB} = 3.6$cm .

ii. Taking B as center draw an angle of 75° .

iii. Taking B as centre draw an arc of radius $4.2cm$ to intersect the terminal sides of angle at C.

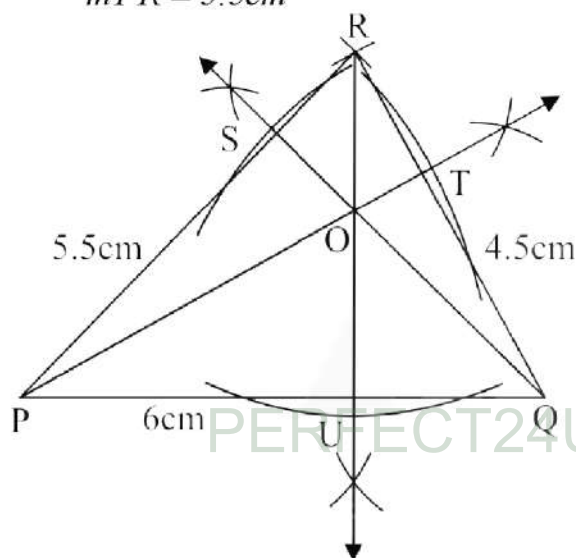
iv. Draw $\overline{AC}$ to complete ΔABC.

v. Draw the bisector of $\angle A$ and $\angle B$ meeting each other at point I.

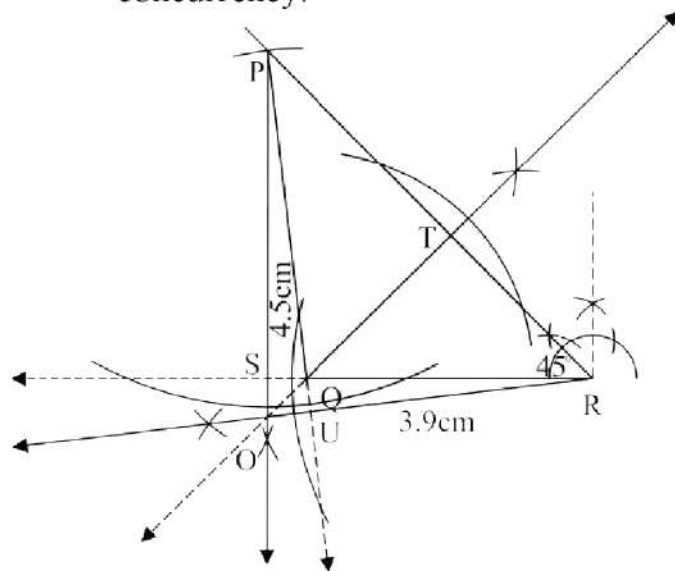
vi. Now draw the bisector of the third angle $\angle C$.

vii. We observe that third angle bisector also passes through the point I.
Hence the angle bisectors of the ΔABC are concurrent at I which lies within the triangle.

Q.2 Construct the following triangles PQR. Draw their altitudes and show that they are concurrent.

(i) $m\overline{PQ} = 6cm, m\overline{QR} = 4.5cm$ and $m\overline{PR} = 5.5cm$

i. Draw a line segment $m\overline{PQ} = 6cm$.
ii. Taking P as centre draw an arc of radius $5.5cm$.
iii. Taking Q as centre draw another arc of radius $4.5cm$ to intersect the first arc at R.
iv. Join P to R and Q to R to complete ΔPQR.
v. From vertex P drop $\overline{PT} \perp \overline{QR}$.
vi. Form vertex Q drop $\overline{QS} \perp \overline{PR}$.
vii. Now from third vertex R drop $\overline{RU} \perp \overline{PQ}$.
viii. We observe that third altitude also passes through the point of intersection O of the first two.
Hence three altitudes of ΔPQR are concurrent at O.

(ii) $m\overline{PQ} = 4.5cm$ $m\overline{QR} = 3.9cm$ $m\angle R = 45°$

Required:
i. To construct ΔPQR.
ii. To draw altitudes and verify their concurrency.

Construction:
i. Draw a line segment $m\overline{QR} = 3.9cm$.
ii. Taking R as centre draw an angle of 45°.

iii. Taking Q as centre draw an arc of radius $4.5cm$ which intersects the terminal side of angle at P.
iv. Join P to Q to complete the ΔPQR.
v. From vertex P drop $\overline{PS} \perp \overline{RQ}$ produced.
vi. From vertex Q drop $\overline{QT} \perp \overline{PR}$.
vii. Form vertex R drop $\overline{RU} \perp \overline{PQ}$ produced.
Hence the three altitudes of ΔPQR are concurrent at point O.

(iii) $m\overline{RP} = 3.6cm \quad m\angle Q = 30° \quad m\angle P = 105°$

Sum of three angles in a triangle is 180° so,

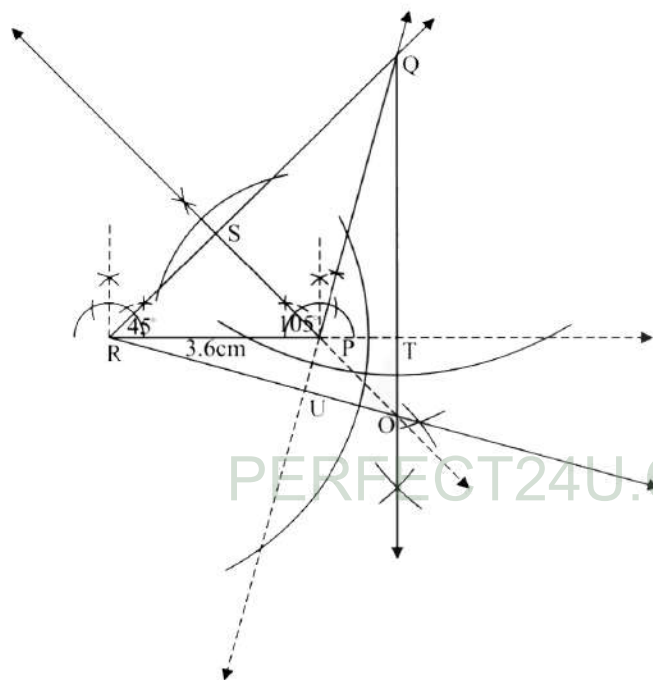
$\angle P + \angle Q + \angle R = 180°$

$105 + 30 + \angle R = 180°$

$135 + \angle R = 180°$

$\angle R = 180° - 135°$

$\angle R = 45°$

So

Construction:

i. Draw a line segment $m\overline{RP} = 3.6cm$.

ii. Taking R as centre, construct an angle of $45°$.

iii. Taking P as centre draw an angle of 105°.

iv. Terminal arms of both angles meet in point Q forming ΔPQR.

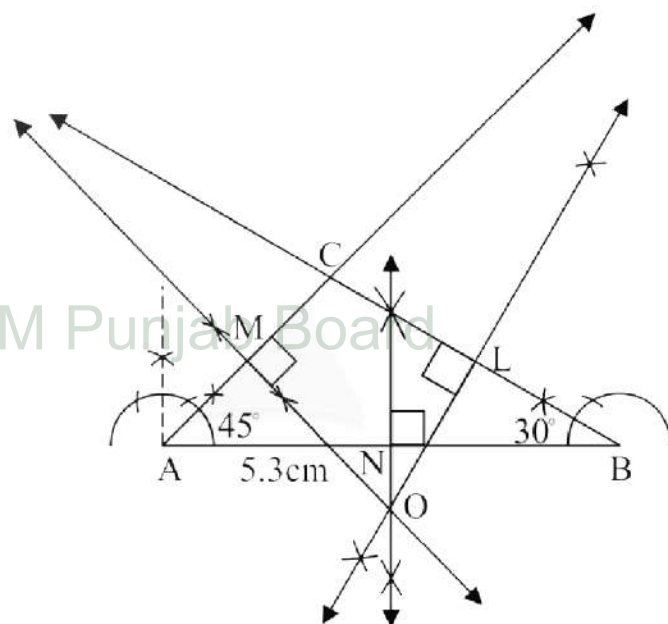
v. From vertex P drop $\overline{PS} \perp \overline{RQ}$.

vi. From vertex Q drop $\overline{QT} \perp \overline{RP}$ produced.

vii. Form vertex R drop $\overline{RU} \perp \overline{QP}$ produced.

Hence the three altitudes of ΔPQR are concurrent at point O.

Q.3 Contract the following triangles ABC draw the perpendicular bisector of three sides and verify their concurrency. Do they meet inside the triangle?

(i) $\overline{AB} = 5.3cm \quad m\angle A = 45° \quad m\angle B = 30°$

Construction:

i. Draw a line segment $m\overline{AB} = 5.3cm$.

ii. At the end point A of $\overline{AB}$ make $m\angle A = 45°$.

iii. At the end point B of $\overline{AB}$ make $m\angle B = 30°$.

iv. Terminal sides of two angles meet at C. The ABC is requiredΔ.

v. Draw perpendicular bisectors of $\overline{AB}, \overline{BC}$ and $\overline{CA}$ meeting each other in the point O.

Hence the three perpendicular bisectors of sides of ΔABC are concurrent at O outside the triangle.

(ii) $m\overline{BC} = 2.9cm \quad m\angle A = 30^\circ \quad m\angle B = 60^\circ$

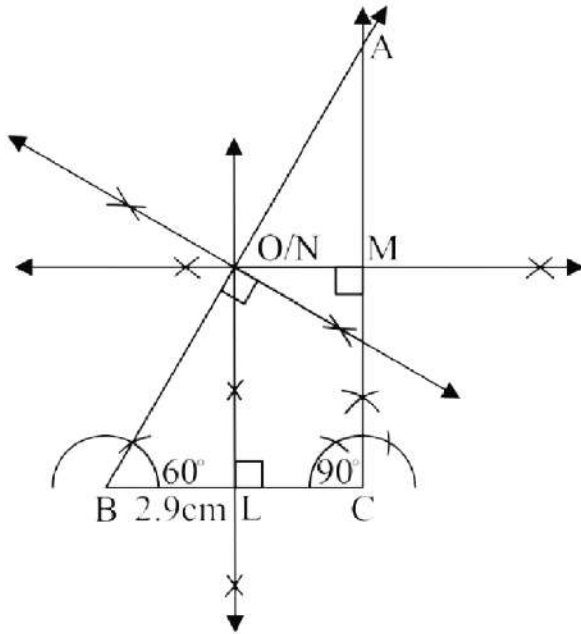
The sum of three angles in a triangle is 180° then

$\angle A + \angle B + \angle C = 180^\circ$

$30 + 60 + \angle C = 180^\circ$

$90 + \angle C = 180^\circ$

$\angle C = 180^\circ - 90^\circ$

$\angle C = 90^\circ$

Construction:

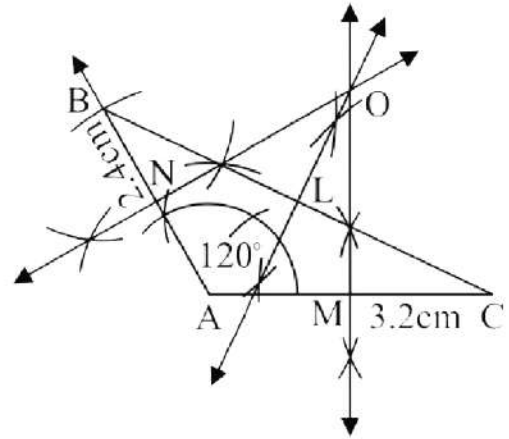
i. Draw a line segment $m\overline{BC} = 2.9cm$.

ii. At the end point B of $\overline{BC}$ make $m\angle B = 60^\circ$.

iii. At the end point C of $\overline{BC}$ make $m\angle C = 90^\circ$.

iv. Terminal sides of two angles meet at A. The ABC is requiredΔ.

v. Draw perpendicular bisectors of $\overline{AB}, \overline{BC}$ and $\overline{CA}$ meeting each other at the point O.
Hence the three perpendicular bisectors of sides of ΔABC are concurrent at O, at the mid point of hypotenuse.

(iii) $m\overline{AB} = 2.4cm \quad m\overline{AC} = 3.2cm \quad m\angle A = 120^\circ$

Construction:

i. Take $\overline{AC} = 3.2cm$.

ii. At A draw an angle of 120°.

iii. Taking centre A draw an arc of radius 2.4cm which cuts the terminal side of angle A at point B.

iv. Join C to B, ΔABC is the triangle.

v. Draw perpendicular bisectors of $\overline{AB}, \overline{BC}$ and $\overline{CA}$ meeting each other at the point O outside the triangle.
Hence all the three perpendicular bisectors are concurrent.

Q.4 Construct the following $\Delta s\ XYZ$. Draw their three medians and show that they are concurrent.

(i) $m\overline{YZ} = 4.1cm \quad m\angle Y = 60^\circ \quad m\angle X = 75^\circ$

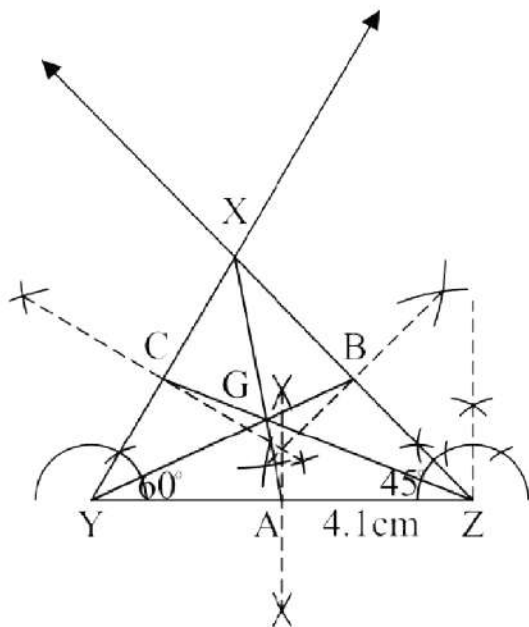
Sum of three angles in a triangle is 180° then

$m\angle X + m\angle Y + m\angle Z = 180^\circ$

$75 + 60 + m\angle Z = 180^\circ$

$135 + m\angle Z = 180^\circ$

$m\angle Z = 180^\circ - 135^\circ$

$m\angle Z = 45^\circ$

Construction:

i. Take $m\overline{YZ} = 4.1cm$.

ii. Taking Z as centre draw an angle of 45°.

iii. Taking Y as centre draw an angle of 60°.

iv. The terminal sides of these angles meet at X.
Then XYZ is required∆.

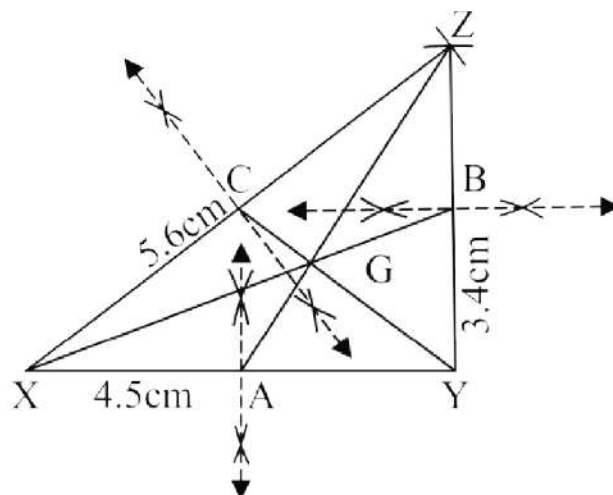
v. Draw perpendicular bisectors of the sides
$\overline{XZ}, \overline{XY}$ and $\overline{YZ}$ of ∆XYZ and make their midpoints B,C and A respectively.

vi. Join Y to B, midpoint of XZ to get $\overline{YB}$ as median.

vii. Join Z to C midpoint of XY to get $\overline{ZC}$ as median.

viii. Join X to A midpoint of YZ to get $\overline{XA}$ as median.

All median intersect at point G. Hence the median are concurrent at G.

(ii) $m\overline{XY} = 4.5cm$ $m\overline{YZ} = 3.4cm$ $m\overline{ZX} = 5.6cm$

Construction:

i. Take $m\overline{XY} = 4.5cm$.

ii. Taking Y as centre draw an arc of radius 3.4cm.

iii. Taking X as center draw another arc of radius 6.5cm to cut at point Z.

iv. Join X to Z and Y to Z.

v. Draw perpendicular bisectors of the sides $\overline{XY}, \overline{YZ}$ and $\overline{XZ}$ of ∆XYZ and make their mid point A, B and C.

vi. Join Y to mid point C to get median $\overline{YC}$.

vii. Join Y to mid point B to get median $\overline{XB}$.

viii. Join Z to mid point A to get median $\overline{ZA}$.

All medians intersect at point G. Hence medians are concurrent at G.

(iii) $m\overline{ZX} = 4.3cm$ $m\angle X = 75°$ and $m\angle Y = 45°$

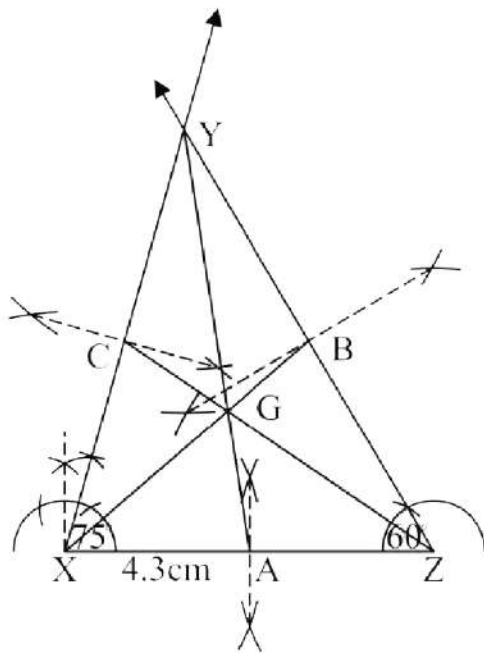
Sum of three angles in a triangle is 180° then

$m\angle X + m\angle Y + m\angle Z = 180°$

$75 + 45 + m\angle Z = 180°$

$120° + m\angle Z = 180°$

$m\angle Z = 180° - 120°$

$m\angle Z = 60°$

Construction:

i. Take $m\overline{ZX} = 4.3cm$.

ii. Taking Z as centre draw an angle of 60°.

iii. Taking X as centre draw an angle of 75°.

iv. The terminal sides of these angles meet at Y.
Then XYZ is required∆.

v. Draw perpendicular bisectors of the sides $\overline{XZ}, \overline{YZ}$ and $\overline{XY}$ of ∆XYZ and make their midpoints A,B and C respectively.

vi. Join X to midpoint B to get $\overline{XB}$ as median.

vii. Join Z to midpoint C to get $\overline{ZC}$ as median.

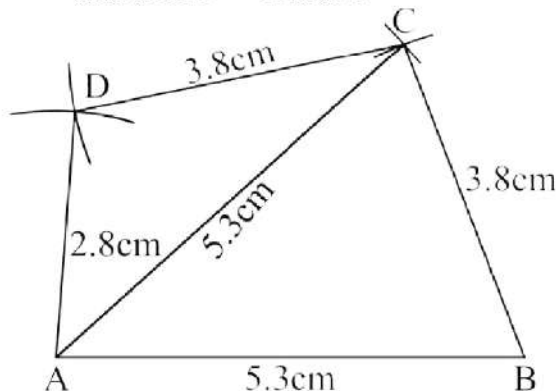
viii. Join Y to midpoint A to get $\overline{YA}$ as median.
All median intersect at point G.
Hence the median are concurrent at G.

Exercise 17.3

Q.1

(i) Construction a quadrilateral ABCD, having

$m\overline{AB} = \overline{AC} = 5.3cm \quad m\overline{BC} = m\overline{CD} = 3.8cm$

and $m\overline{AD} = 2.8cm$.

Construction:

i. Draw a line segment $\overline{AB} = 5.3cm$.

ii. Taking B as centre draw an arc of radius $\overline{BC} = 3.8cm$.

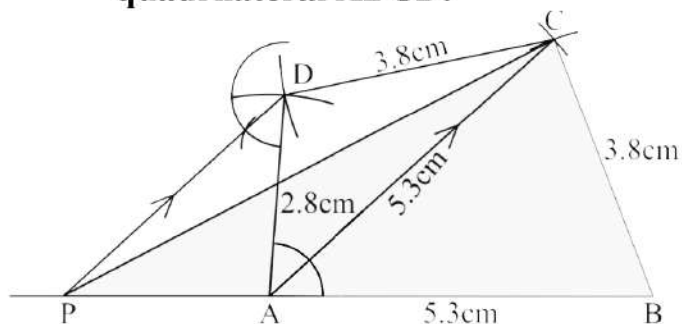
iii. Taking A as centre draw an arc of radius $\overline{AC} = 5.3cm$ to cut at C.

iv. Taking C as centre draw an arc of radius $\overline{CD} = 3.8cm$.

v. Taking A as centre draw an arc of radius $\overline{AB} = 2.8cm$ to cut at D.

vi. Join B to C, C to D, A to C and A to D.

ABCD is the required quadrilateral.

(ii) On the side $\overline{BC}$ construct a Δequal in area to the quadrilateral ABCD.

Construction:

i. Join A to C.

ii. Through D draw $\overline{DP} \| \overline{CA}$ meeting $\overline{BA}$ produced at P.

iii. Join $\overline{PC}$.

iv. Then PBC is required triangle.

$\Delta s\ APC, ADC$ stand on the same base AC and same parallels AC and PD.

Hence

$\Delta APC = \Delta ADC$

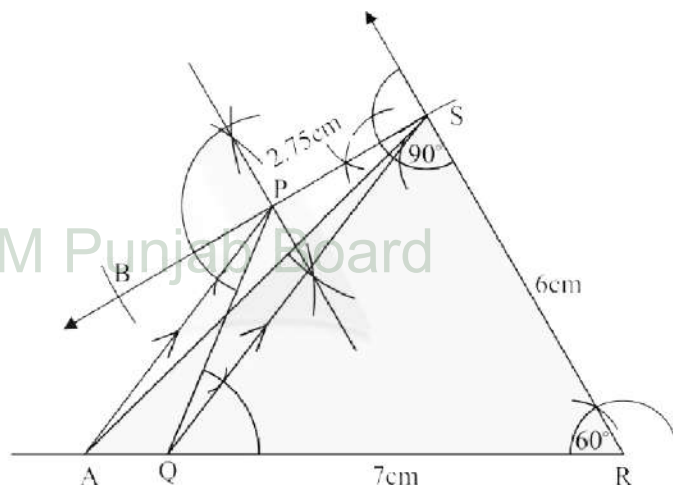
$\Delta APC + \Delta ABC = \Delta ADC + \Delta ABC$

or ΔPBC =quadrilateral ABCD

Q.2 Construct a Δ equal to the quadrilateral PQRS, having

$m\overline{QR} = 7cm \quad m\overline{RS} = 6cm$

$m\overline{SP} = 2.75cm \quad m\angle QRS = 60^\circ$

and $m\angle RSP = 90^\circ$.

Construction:

i. Draw a line segment $\overline{QR} = 7cm$.

ii. At point R draw an angle of 60°.

iii. Taking R as center draw an arc of radius of 6cm to cut at S.

iv. At point S draw an angle 90°.

v. Taking S as centre draw an arc of radius of 5.5cm, cutting the terminal side of 90° at point B.

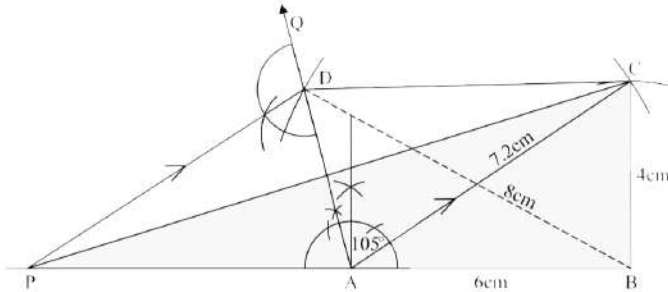
vi. Find the mid point of $m\overline{SB}$ at point P.

vii. Join P to Q.

viii. Draw $\overline{PA}$ parallel to $\overline{SQ}$

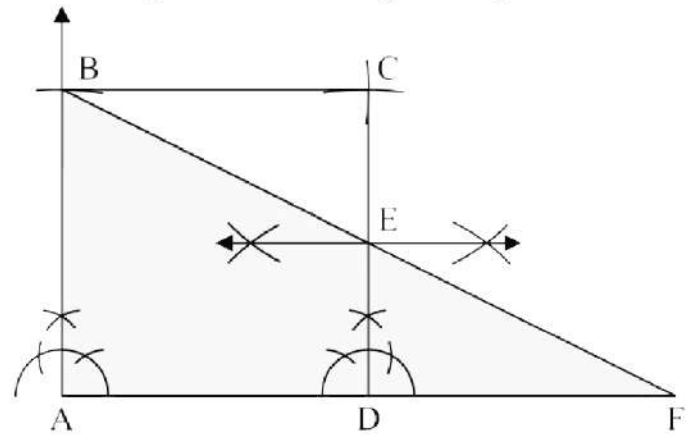
ix. Join A to S.

x. ΔARS is required triangle equal in area to quadrilateral PQRS.

Q.3 Construct a Δ equal in area to quadrilateral ABCD having
$m\overline{AB} = 6cm$ $m\overline{BC} = 4cm,$
$\overline{AC} = 7.2cm$ $m\angle BAD = 105^\circ$
and $m\overline{BD} = 8cm$ **.**

Construction:

i. Draw a line segment $\overline{AB} = 6cm$.
ii. Taking A as centre draw an arc of radius 7.2cm.
iii. Taking B as centre draw an arc of radius $4cm$ to cut at C. Join C to A and C to B.
iv. Taking A as centre make an angle $\angle QAB = 105^\circ$.
v. Taking B as centre make an arc of radius 8cm to cut at D point.
vi. Join D to C to complete the ABCD quadrilateral.
vii. Draw $\overline{DP} \parallel \overline{CA}$ o meet $\overrightarrow{BA}$ produced at P.
viii. Join C to P.

Thus ΔPBC is the required triangle.

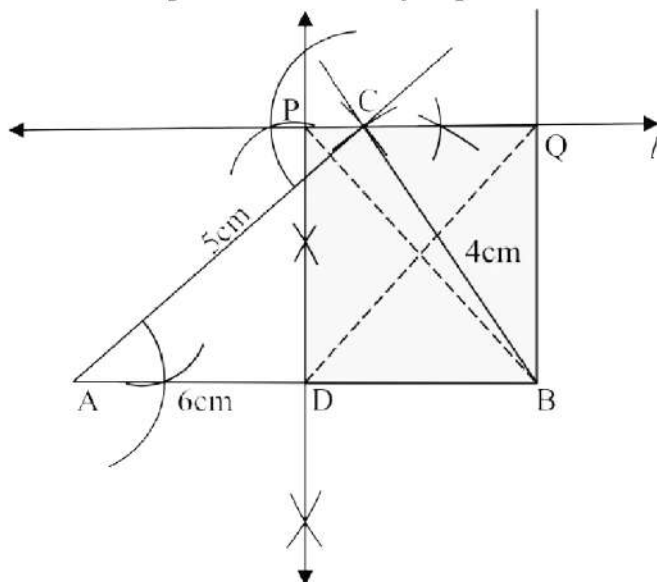
Q.4 Construct a right angled triangle equal in area to given square.

Construction:

Let measurement of each side of square is 3.8cm.

i. Construct a square ABCD with each side 3.8cm long.
ii. Bisect $\overline{CD}$ at E .
iii. Join B to E and produced it to meet $\overline{AD}$ produced in F.

ΔABF is required triangle equal in area to square ABCD.

Exercise 17.4

Q.1 Construct a Δ with sides 4cm, 5cm and 6cm and construct a rectangle having its area equal to that of the Δ measure its diagonals. Are they equal

Construction:

i. Draw a line segment $\overline{AB} = 6cm$.

ii. Taking A as centre draw an arc of radius 5cm.

iii. Taking B as centre draw an arc of radius 4cm to cut at C. Join A to C and B to C.

iv. ABC is the requiredΔ.

v. Draw a line *l* through C parallel to $\overline{AB}$.

vi. Draw the ⊥ bisector of $\overline{AB}$ in D and cutting the line at P.

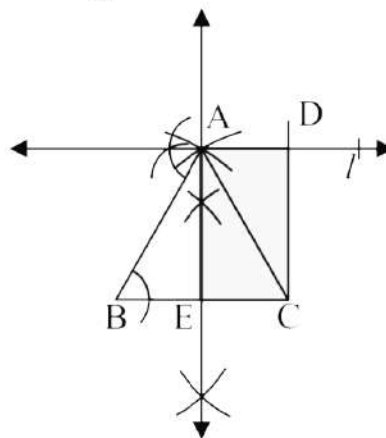
vii. On the line *l*, cut $\overline{PQ}$ equal to $\overline{DB}$.

viii. Join B to Q.

ix. PQBD is the required rectangle.

x. The length of each diagonal measured to be 4.5cm.

xi. The length of each diagonal is same.

Q.2 Transform an isosceles Δ into a rectangle.

Construction:

i. Draw a line segment $\overline{BC}$.

ii. With B as centre draw in arc of suitable radius.

iii. With C as centre draw another are of same radius which cuts the first arc at point A.

iv. Join A to B and A to C.

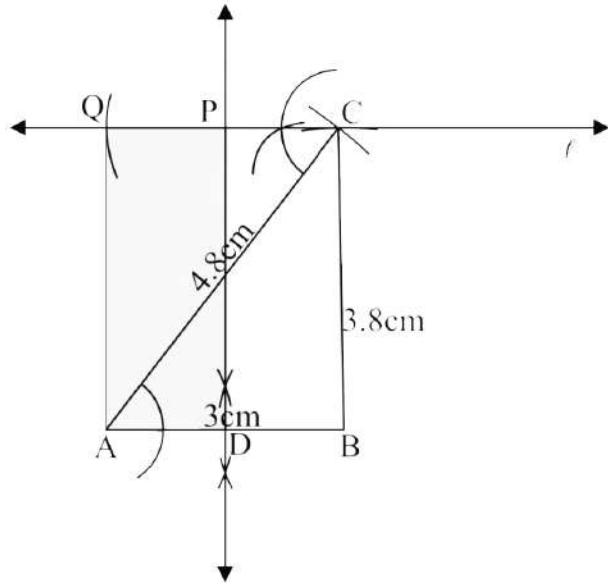
v. ΔABC is the isosceles Δ with $m\overline{AB} = m\overline{AC}$.

vi. Draw the perpendicular bisector of $\overline{BC}$ passing through point A.

vii. Through A draw a line $l \parallel \overrightarrow{BC}$.

viii. On *l* cut $\overline{AD}$ equal to $\overline{EC}$ and the Join C with D.

ix. CDAE is the required rectangle equal in area to ΔABC.

Q.3 Construct a ABC such that $m\overline{AB} = 3cm$, $m\overline{BC} = 3.8cm$ and $m\overline{AC} = 4.8cm$ **. Construct a rectangle equal in area to the ΔABC, and measure its sides.**

Construction:

i. Draw a line segment $\overline{AB} = 3cm$.

ii. Taking B as centre draw an arc of radius $\overline{BC} = 3.8cm$.

iii. Taking A as centre draw an arc of radius $\overline{AC} = 4.8cm$ to cut at C.

iv. Join C to A and C to B.

v. ABC is the requiredΔ.

vi. Through C draw a line l parallel $\overline{AB}$.

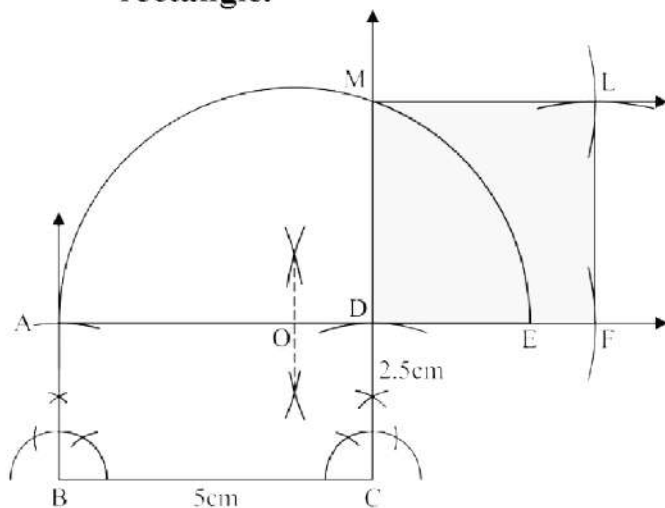
vii. Draw the $\perp$ bisector of $\overline{AB}$ cutting the line l in P.

viii. On ℓ cut $\overline{PQ} \cong \overline{DA}$.

ix. PQAD is the required rectangle measure of sides of rectangle PQAD

$m\overline{PD} = 3.8\text{cm}$ $m\overline{AD} = 1.5\text{cm}$

Exercise 17.5

Q.1 Construct a rectangle whose adjacent sides are 2.5cm and 5cm respectively. Construct a square having area equal to the given rectangle.

Construction:

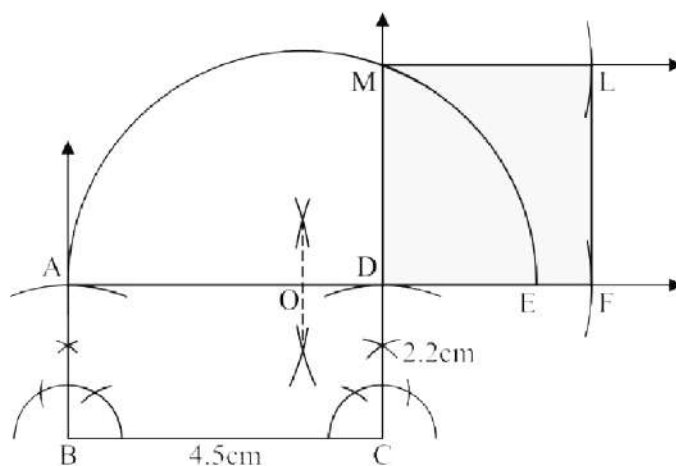
i. Make the rectangle ABCD with given lengths of sides.

ii. Produce AD to point E such that $m\overline{DE} = m\overline{DC}$.

iii. Bisect $\overline{AE}$ at O.

iv. With O as centre and $\overline{OA}$ radius draw a semicircle cutting $\overline{CD}$ produced in M.

v. With $\overline{DM}$ as side complete the square $DFLM$.

Q.2 Construct a square equal in area to a rectangle whose adjacent sides are 4.5cm and 2.2cm respectively. Measure the sides of the square and find its area and compare with the area of the rectangle.

Construction:

i. Make the rectangle ABCD with given sides.

ii. Produce AD and cut $m\overline{DE} = m\overline{DC}$.

iii. Bisect $\overline{AE}$ at O.

iv. With O as centre and $\overline{OA}$ radius draw a semicircle cutting $\overline{CD}$ produced in M.

v. With $\overline{DM}$ as side complete the square $DF\angle M$.

vi. Side of the square (average) = 3.15cm

Area $= 3.15 \times 3.15 = 9.9cm^2$

Area of rectangle $= 2.2 \times 4.5 = 9.9cm^2$

Area of rectangle = Area of square

Q.3 In Q2 above verify by measurement that the perimeter of the square is less then that of the rectangle.

Perimeter of rectangle = 2 [length + briclth]

= 2 [4.5 + 2.2]

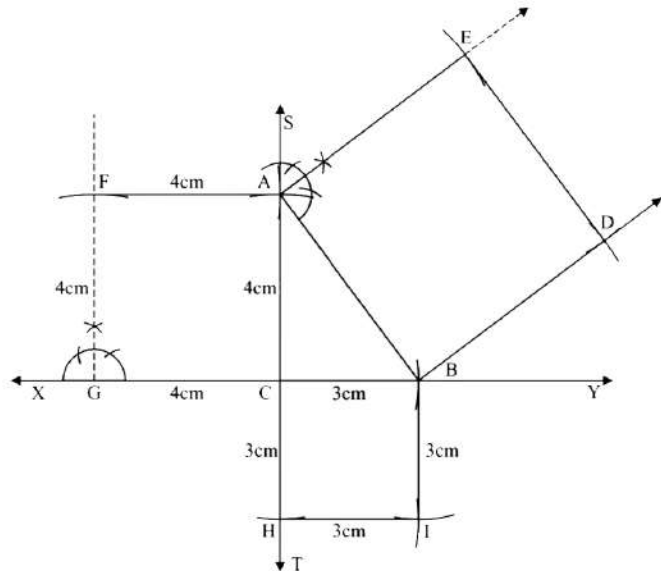
= 2 [6.7]

= 13.4 cm

Perimeter of square = 4 × l

= 4 × 3.2

= 12.8 cm

Q.4 Construct a square equal in area to the sum of two squares having sides 3cm and 4cm respectively.

Construction:

i. Draw a line segment $\overleftrightarrow{XY}$.

ii. Draw a line perpendicular $\overrightarrow{ST}$ at point C.

iii. Cut of $\overline{CB} = 3cm$ and $\overline{CG} = 4cm$.

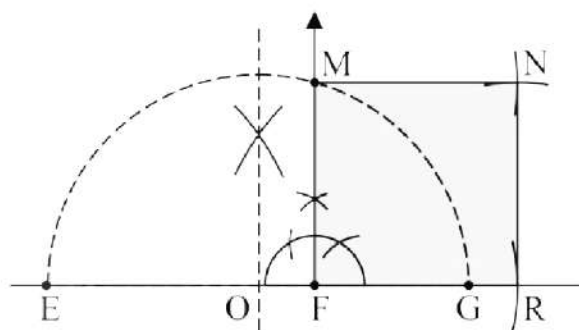
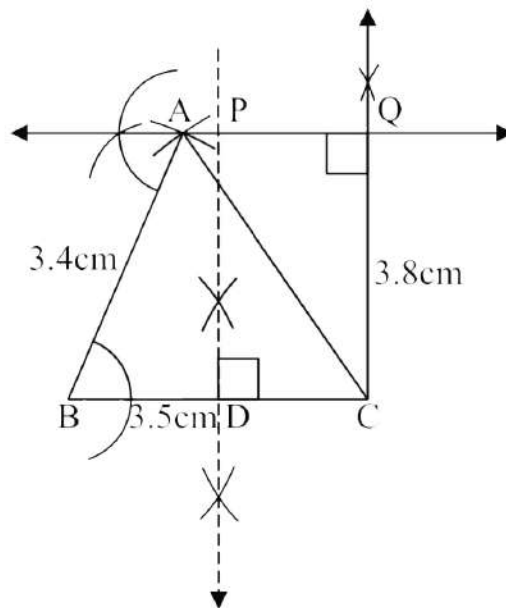
iv. $\overline{CG}$ is the side of square complete the square ACGF.

v. $\overline{CB}$ is the side of square complete the square CBIH.

vi. Join B to A.

vii. $\overline{AB}$ is the side of square so, complete the square ABDE.

viii. ABDE is the required square. Using Pythagoras theorem to prove.

Q.5 Construct a Δ having base 3.5cm and other two sides equal to 3.4cm and 3.8cm respectively. Transform it into a square of equal area

Construction:

i. Draw $\overleftrightarrow{PAQ} \parallel \overline{BC}$

ii. Draw perpendicular bisector of $\overline{BC}$, bisector it at D and meeting $\overleftrightarrow{PAQ}$ at P.

iii. Draw $\overline{CQ} \perp \overline{PQ}$ meeting it in Q.

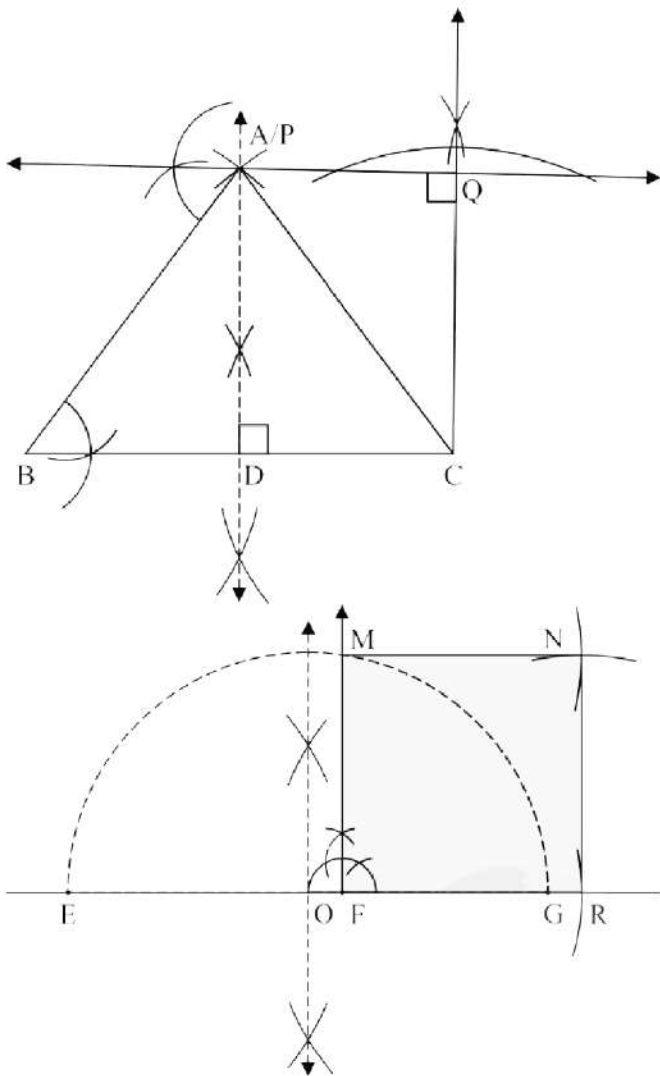
iv. Take a line EFG and cut radius $\overline{EF} = \overline{DP}$ and $\overline{FG} = \overline{DC}$.

v. Bisect $\overline{EG}$ at O.

vi. With O as centre and radius $= \overline{OE}$ draw a semi-circle.

vii. At F draw $\overline{FM} \perp \overline{EG}$ meeting the semi-circle at M.

viii. With $\overline{MF}$ as a side, complete the required square FMNR.

Q.6 Construct a Δ having base 5 and other sides equal to 5cm and 6cm construct a square equal in area to given Δ.

Construction:

i. Draw $\overleftrightarrow{PAQ} \parallel \overline{BC}$

ii. Draw perpendicular bisector of $\overline{BC}$, bisector it at D and meeting $\overleftrightarrow{PAQ}$ at P.

iii. Draw $\overline{CQ} \perp \overline{PQ}$ meeting it in Q.

iv. Take a line EFG and cut radius $\overline{EF} = \overline{DP}$ and $\overline{FG} = \overline{DC}$.

v. Bisect $\overline{EG}$ at O.

vi. With O as centre and radius $= \overline{OE}$ draw a semi-circle.

vii. At F draw $\overline{FM} \perp \overline{EG}$ meeting the semi-circle at M.

viii. With $\overline{MF}$ as a side, complete the required square FMNR.

Revised Exercise 17

Q.1 Fill in the blanks to make the statements true:

(i) The side of right angled triangle opposite to 90° is called __________.

(ii) The line segment joining a vertex of a triangle which is to the mid point of its opposite side is called a __________.

(iii) A line drawn from a vertex of a triangle which is _________ to its opposite side is called an attitude of the triangle.

(iv) The bisectors of the three angles of a triangle are__________.

(v) The points of concurrency of right bisector of the three sides of the triangle are _________ from its vertices.

(vi) Two or more triangle are said to be similar if they are equiangular and measures of their corresponding sides are__________.

(vii) The altitudes of a rights triangle are concurrent at the ______of the right angle.

Answer Key

(Fill in the Blank)

i	Hypotenuse	v	Equidistant
ii	Median	vi	Proportional
iii	Perpendicular	vii	Vertex
iv	Concurrent		

Q.2 Multiple Choice Questions. (Choose the correct answer).

(i) The triangle having two sides congruent is called
(a) Scalene (b) Right angled
(c) Equilateral (d) Isosceles

(ii) A quadrilateral having each angle equal to 90° is called
(a) Parallelogram (b) Rectangle
(c) Trapezium (d) Rhombus

(iii) The right bisector of the three sides of a triangle are
(a) Congruent (b) Collinear
(c) Concurrent (d) Parallel

(iv) The __________ altitudes of an isosceles triangle are congruent.
(a) Two (b) Three
(c) Four (d) None of these

(v) A point equidistant from the end points of a line – segments is on its__________.
(a) Bisector (b) Right - bisector
(c) Perpendicular (d) Median

(vi) __________ congruent triangles can be made by joining the mid-point of the sides of a triangle.
(a) Three (b) Four
(c) Five (d) Two

(vii) The diagonals of parallelogram __________ each other.
(a) Bisect (b) Trisect
(c) Bisect at right angle (d) None of these

(viii) **The medians of a triangle cut each other in the ration__________.**

(a) 4:1 **(b)** 3:1

(c) 2:1 **(d)** 1:1

(ix) **One angle on the base of an isosceles triangle is 30°. What is the measure o its vertical angle__________.**

(a) 30° **(b)** 60°

(c) 90° **(d)** 120°

(x) **If the three attitudes of a triangle are congruent then, the triangle will be__________.**

(a) Isosceles **(b)** Equilateral

(c) Right angled **(d)** Acute angled

(xi) **If two medians of a triangle are congruent then the triangle will be__________.**

(a) Isosceles **(b)** Equilateral

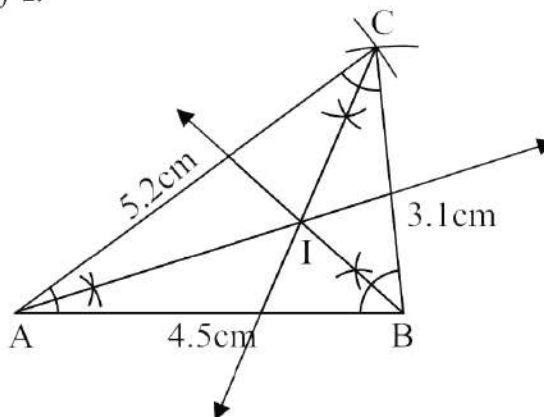
(c) Right angled **(d)** Acute angled

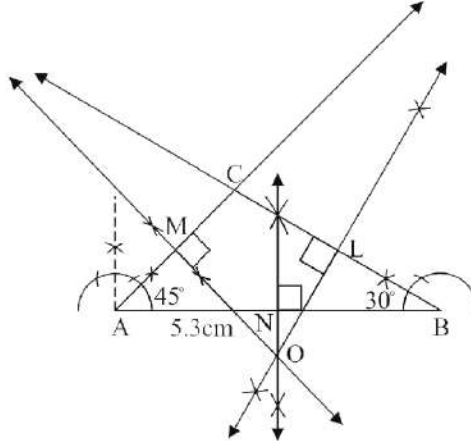
Answer Key
(MCQ'S)

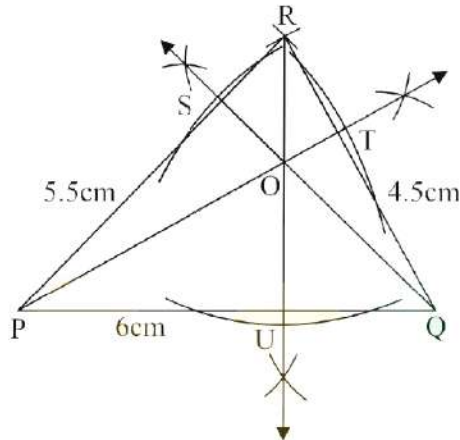
i	d	vii	a
ii	b	viii	c
iii	c	ix	d
iv	a	x	
v		xi	a
vi	b		

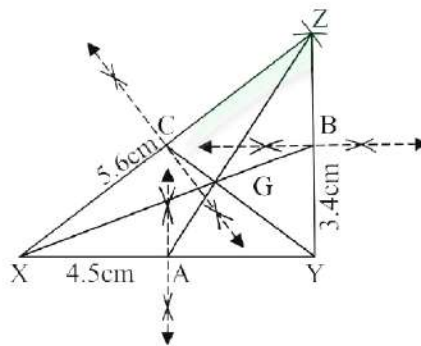
Q.3 **Define the following.**

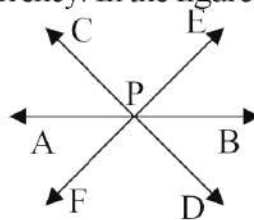
(i) **Incentre**

The point where the internal bisectors of the angles of a triangle meet is called incentre of a triangle. It is denoted by I.

(ii) **Circumcentre**

The point of concurrency of the three perpendicular bisectors of the sides of a triangle is called circumcentre of a triangle. It is denoted by O.

(iii) Orthocenter

The point of concurrency of three altitudes of a triangle is called orthocenter of a triangle. It is denoted by O.

(iv) Centroid

The point of concurrency of three medians of a triangle is called centroid of a triangle. It is denoted by G.

(v) Point of concurrency

Three are more than three lines are said to be concurrent if these lines pass through the same point and that point is called the point of concurrency. In the figure, *P* is the point of concurrency.

Unit 17: Practical Geometry – Triangles

Overview

Right bisector of a line segment

A line l is called a right bisector of a line segment if l is perpendicular to the line segment and passes through its mid-point.

Angle bisector

Median of a triangle

A line segment joining a vertex of a triangle to the mid-point of the opposite side is called a median of the triangle.

Altitude of a triangle

A line segment from a vertex of a triangle, perpendicular to the line containing the opposite side, is called an altitude of the triangle.